Power Engineering Letters

Participation of Every Generator to Loads, Currents, and Power Losses

Dragoslav Rajičić, *Life Senior Member, IEEE*, and Mirko Todorovski, *Senior Member, IEEE*

***Abstract*—Among other things, in a detailed analysis of circumstances in power systems of great importance is the knowledge on contribution of individual generators to loads, and to currents and power losses in network elements. This work proposes an efficient method for obtaining these details. The method uses results of a power flow calculation, utilizes specific power system model, and applies the superposition theorem. The method does not contain problematic assumptions or simplifications and it is easy to apply. It allows getting important facts enabling us to establish transparent and non-discriminatory relationships between network operators, electricity producers and consumers.**

***Index Terms*—Power flow, power losses, power system modeling, superposition theorem.**

I. Introduction

In deregulated environment it is important to create confidence between network operators, producers and consumers. However, this cannot be achieved without detailed knowledge of circumstances in the system. This was recognized in the early stage of deregulation [1]. Primarily, it is important to know contribution of every generator to every load, and to currents and power losses in every line and transformer in the network.

The topic is in the focus of the researchers for more than two decades. A review of various loss allocation methods along with their advantages and weaknesses is given in [2]. The main difficulty existing in all methods, which seems unresolved satisfactorily to present time, is the presence of a nonlinear relationship between losses and delivered power.

This work proposes a method that starts from a solved power flow problem for a given system using one of the well-known power flow solvers (e.g., [3], [4]). However, the power flow problem is not the subject of this work. After this initial step we propose to represent each load by an equivalent admittance and each generator by an equivalent current generator or as combination of an equivalent current generator and additional admittance. The parameters of these equivalent represent are calculated using bus voltages obtained with the power flow solver.

This enables the application of superposition principle, accounting for generator current injections one by one, and use of appropriate solution methods from general circuit theory. In this way we can determine participation of each generator to current in every network element including loads as well.

Finally, power flows in network elements are expressed as products of conjugate relating currents and corresponding voltages at both ends known from the initial step, similarly to what was done in [5]. Therefore, power flows in network elements are expressed as a linear combination of generators' currents and known voltages. Consequently, there are no nonlinear expressions and problems with non-separability of losses.

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Dragoslav Rajičić retired, was with the Electrical Engineering Faculty, Ss. Cyril and Methodius University, Skopje 1000, Republic of North Macedonia (e-mail: dragoslav@ieee.org).

Mirko Todorovski is with the Faculty of Electrical Engineering and Information Technologies, Ss Cyril and Methodius University, Skopje 1000, Republic of North Macedonia (e-mail: mirko@feit.ukim.edu.mk).

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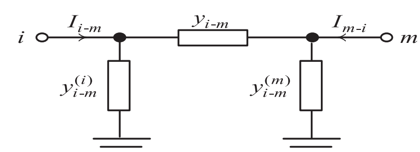

Fig. 1. General π-equivalent circuit of line or transformer.

II. Preparation Stage

The proposed method was developed supposing that all three phase system elements are symmetrical and balanced. So, only the positive sequence currents and voltages are non-zero meaning that we may ignore negative and zero sequence.

For the system under study, with *n* buses and *ng* generators (one at each of first *ng* buses), the following data are given:

- Parameters of π-equivalent circuit of every line and transformer in the network (see Fig. 1). We use these parameters to form bus admittance matrix **YBUS**.
- Active power PG_i and reactive power QG_i for generator at PQ bus *i* or active power PG_i and bus voltage magnitude $|V_i|$ for generator at PV bus *i*.
- Voltage magnitude and voltage angle at the slack bus.
- Active power PL_l and reactive power QL_l for load at bus *l*.

Using appropriate power flow method, we calculate vector of total bus voltages **V**, as well as active and reactive power of the slack bus generator.

III. System Model and Superposition Application

In the following procedure we represent the system under study by $n \times n$ admittance matrix $\mathbf{Y}$ and by $ng \times ng$ diagonal matrix $\mathbf{J}$ which diagonal elements are currents of current generators. At the beginning we take $\mathbf{Y} = \mathbf{YBUS}$ and set all elements of $\mathbf{J}$ equal to zero.

We represent load at bus *l* by equivalent admittance

$$y_l = -(PL_l - jQL_l)/|V_l|^2, \tag{1}$$

and add it to the diagonal element of matrix $\mathbf{Y}$ at position (*l,l*) setting $Y_{l,l(\text{new})} = Y_{l,l(\text{old})} + y_l$.

When representing generator at bus *k* we discern two cases:

a) If the generator produces active power and do not absorb reactive power, i.e., if $PG_k > 0$ and $QG_k \geq 0$ (no matter whether bus *k* is PQ, PV or the slack), then element of $\mathbf{J}$ at position (*k*, *k*) is equal to the total generator current

$$J_{k,k} = (PG_k - jQG_k)/V_k^*. \tag{2}$$

b) If the generator produces active power and absorbs reactive power, i.e., if $PG_k > 0$ and $QG_k < 0$, then we represent it as composed of two parts: i) current generator with current equal to $J_{k,k} = PG_k/V_k^*$, and ii) load represented by admittance $y_k = -jQG_k/|V_k|^2$, which is added to element at position (*k,k*) of matrix $\mathbf{Y}$ by setting $Y_{k,k(\text{new})} = Y_{k,k(\text{old})} + y_k$.

For check-up purposes we can form $n \times 1$ vector $\mathbf{I}$ where the first *ng* elements are equal to the diagonal elements of matrix $\mathbf{J}$, while the rest are zeros. Then by solving $\mathbf{Y} \cdot \mathbf{V} = \mathbf{I}$ we get the same elements of vector $\mathbf{V}$ as obtained by the power flow method.

According to the superposition theorem (known in circuit theory) we calculate bus voltages in *ng* cases where in each case only one of the current generators is switched on. Saving these voltages in $n \times ng$ matrix $\mathbf{v}$, they can be obtained from

$$\begin{bmatrix} v_{1,1} & v_{1,2} & \cdots & v_{1,ng} \\ v_{2,1} & v_{2,2} & \cdots & v_{2,ng} \\ \vdots & \vdots & \vdots & \vdots \\ v_{n,1} & v_{n,2} & \cdots & v_{n,ng} \end{bmatrix} = \mathbf{Z} \cdot \begin{bmatrix} J_{1,1} & 0 & \cdots & 0 \\ 0 & J_{2,2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & J_{ng,ng} \end{bmatrix}, \tag{3}$$

where $n \times ng$ matrix $\mathbf{Z}$ holds first *ng* columns of matrix $\mathbf{Y}^{-1}$.

Note that column *k* of matrix $\mathbf{v}$ contains bus voltages in case only generator at bus *k* is switched on. Also, note that the sum of elements in row *i* of matrix $\mathbf{v}$ is equal to the voltage at bus *i* (i.e., V_i) in case when all generators are switched on.

With known voltage $v_{l,k}$ and admittance y_l contribution of generator at bus *k* to load current at bus *l* is

$$IL_{l,k} = y_l \cdot v_{l,k}, \tag{4}$$

and corresponding power contribution is

$$SL_{l,k} = V_l \cdot (IL_{l,k})^* = V_l \cdot y_l^* \cdot v_{l,k}^*. \tag{5}$$

Of course, total load current at bus *l* can be expressed as

$$IL_l = \sum_{k=1}^{ng} IL_{l,k} = y_l \cdot \sum_{k=1}^{ng} v_{l,k} = y_l \cdot V_l. \tag{6}$$

In addition, for a network element directly connecting buses *i* and *m* (marked as element *i-m* with π-equivalent circuit as in Fig. 1), using voltages from column *k* of matrix $\mathbf{v}$, we calculate contribution of generator *k* to current at bus *i* with direction from bus *i* to bus *m* as

$$I_{i-m,k} = (v_{i,k} - v_{m,k}) \cdot y_{i-m} + v_{i,k} \cdot y_{i-m}^{(i)}, \tag{7}$$

and current at bus *m* with direction from bus *m* to bus *i* as

$$I_{m-i,k} = (v_{m,k} - v_{i,k}) \cdot y_{i-m} + v_{m,k} \cdot y_{i-m}^{(m)}. \tag{8}$$

With currents from (7) and (8), and total voltages at buses *i* and *m*, we can express contribution of generator at bus *k* to power losses in element *i-m* as

$$\Delta S_{i-m,k} = V_i \cdot I_{i-m,k}^* + V_m \cdot I_{m-i,k}^*. \tag{9}$$

It is essential to stress that in order to obtain currents $I_{i-m,k}$ and $I_{m-i,k}$, by (7) and (8), we use corresponding bus voltages from matrix $\mathbf{v}$, but in order to obtain power losses $\Delta S_{i-m,k}$ by (9) we also use corresponding bus voltages from vector $\mathbf{V}$.

When real part of $\Delta S_{i-m,k}$ is negative it means that generator *k* adds to a decrease of active power losses compare to the case in which the other generators are switched on.

Total power losses in element *i-m* can be express as

$$\begin{aligned} \Delta S_{i-m} &= \sum_{k=1}^{ng} \Delta S_{i-m,k} = \sum_{k=1}^{ng} \left(V_i \cdot I_{i-m,k}^* + V_m \cdot I_{m-i,k}^*\right) \\ &= V_i \cdot \sum_{k=1}^{ng} I_{i-m,k}^* + V_m \cdot \sum_{k=1}^{ng} I_{m-i,k}^* \\ &= V_i \cdot I_{i-m,\text{total}}^* + V_m \cdot I_{m-i,\text{total}}^*, \end{aligned} \tag{10}$$

Where in subscript "total" means that current relates to the case with all generators switched on.

From (4) and (5) we calculate contribution to currents/powers of any generator to any load. Furthermore, using (9) it is possible to calculate contribution of any generator to power losses in any network element. By summing contributions of individual generator to losses in all network elements we get the generator participation in total network losses.

IV. Example and Results

Fig. 2 shows single line diagram of a simple six-bus test system taken from [6]. All branches in the figure are labeled with a triplet of numbers specifying branch *R*, *X* and *B*, expressed in pu, calculated with base power of 100 MVA and base voltage of 230 kV. There are 3 identical loads $SL_4 = SL_5 = SL_6 = (100 + j15)$ MVA and 3 generators $SG_1 = (228.58 - j32.18)$ MVA, $SG_2 = (50 + j75.71)$ MVA and $SG_3 = (50 + j24.20)$ MVA. The voltage set points of the generators at buses 1, 2, and 3 are 1.07 pu, 1.05 pu and 1.05 pu, respectively.

After solving this network, we obtain bus voltages, total generator currents, total load currents and equivalent load admittances as in Table I, while the total network losses are $(28.581+j22.737)$ MVA.

It is important to note that generator 1 has negative reactive power. According to the explanation given in Section III we

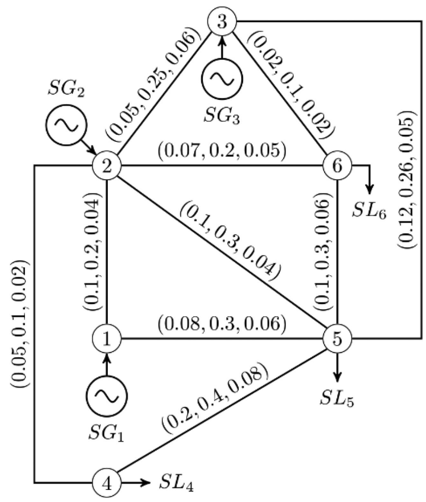

Fig. 2. Six-bus test network.

TABLE I
BUS VOLTAGES, GENERATOR/LOAD CURRENTS AND LOAD ADMITTANCES

Bus	*V* (pu)	*IG* (pu)	*IL* (pu)	*y* (pu)
1	1.070	2.136	*j*0.301	–*j*0.281
2	1.017–*j*0.262	0.281–*j*0.817		
3	1.009–*j*0.289	0.394–*j*0.353		
4	0.937–*j*0.324		–0.904+*j*0.473	1.017–*j*0.153
5	0.972–*j*0.289		–0.903+*j*0.423	0.972–*j*0.146
6	0.970–*j*0.326		–0.880+*j*0.450	0.955–*j*0.143

TABLE II
PARTICIPATION OF GENERATORS TO BRANCH LOSSES (MVA)

	Generator		
Branch	1	2	3
1-2	9.524+*j*25.844	3.649–*j*1.110	1.134–*j*0.614
1-5	6.825+*j*21.849	0.079+*j*1.186	0.833–*j*0.542
2-3	–1.582–*j*3.725	1.102–*j*1.117	0.540–*j*1.470
2-4	4.312+*j*2.289	–0.510+*j*2.619	0.280+*j*1.171
2-5	–0.746–*j*2.693	0.747–*j*0.137	0.271–*j*0.619
2-6	0.268–*j*1.615	0.308+*j*0.251	0.403–*j*1.213
3-5	–1.653–*j*3.430	1.187–*j*0.830	0.668–*j*0.632
3-6	0.227–*j*0.354	0.168–*j*0.005	0.166+*j*1.012
4-5	–2.010–*j*4.722	1.745–*j*1.649	0.511–*j*1.186
5-6	–1.723–*j*3.517	1.388–*j*1.143	0.471–*j*1.161
Total	13.442+*j*29.925	9.863–*j*1.935	5.277–*j*5.254
Total	28.581+*j*22.737		

represent this generator by current generator with current $J_{1,1} = PG_1/V_1{}^* = 2.136$ pu and by equivalent admittance $y_1 = -jQG_1/|V_1|^2 = -j0.281$ pu, which is added to the element at position (1,1) of matrix **Y**. The other two generators are represented only by current generators with currents calculated by (2). Note, in the system model, each load is represented by the equivalent admittance *y*, as given with (1).

In Table II we present participation of generators to power losses for all network elements (branches). It is interesting to note that generator 1 contributes negative active power losses to several of the network elements, meaning that its presence and mode of operation reduces their losses. For example, the total active power losses of element 2-3 are 0.06 MW, but we now know that they are such a low because of the presence of generator 1. Low losses in branches 3-5, 4-5 and 5-6 can also be attributed to generator 1. Of course, the sum of all losses in all network elements is equal to the total system losses. Their breakdown in Table II give us an insight for the causes.

V. CONCLUSION

The proposed method is developed starting from the results of power flow calculations. Knowledge of bus voltages (i.e., vector **V**) facilitate creation of a specific power system model characterized by $n \times n$ admittance matrix **Y** and $ng \times ng$ diagonal matrix of generator currents **J**, as explained in Section III.

The use of this model and application of superposition theorem enables us to calculate bus voltages (stored in matrix **v**) in cases when the generators are switched on one by one. The knowledge of matrix **v** and vector **V** enables the calculation of contributions of individual generators to load currents, as well as currents and power losses in each network element.

When calculating power losses in network elements the proposed method does not use quadratic expressions. Therefore, there are no problems with non-separability of losses. In addition, there are no assumptions whatsoever, nor simplifications nor need for normalization of allocated losses. The sum of allocated power losses (active and reactive) exactly matches with the total losses in the network obtained by power flow method. Also, without any modifications the method handles cases where reactive power direction at one end of a network element is opposite to the reactive power direction at the other end.

The method is applicable to radial and meshed networks. It is non-iterative and exact, giving no privilege to any network user. It enables to obtain information that can be of great assistance in creation of transparent and nondiscriminatory relations between the network operator and network users.

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