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Decomposing total power flows into contributions originating from power sources in loads, generators, and network elements

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A R T I C L E I N F O

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A B S T R A C T

To thoroughly analyze the conditions in power lines, transformers or loads, it is essential to have knowledge of their active and reactive power flow contributions from each power source. Understanding these contributions allows for a detailed analysis of the impact of active and reactive power flow contribution from each power source on active and reactive power losses in each network element, as well as the determination of power participation of each source to every load. This paper provides the necessary equations and an adequate procedure for calculating power flow contributions. The method is suitable for meshed network and considers shunt elements, avoiding problematic simplifications. Furthermore, it ensures impartiality towards any network user, making it well-suited for application in deregulated environments. By offering valuable information useful, this method supports the creation of transparent and non-discriminatory agreements between network operator and users.

1. Introduction

Building trust and establishing proper relationships between the system operator, power providers, and electricity users is crucial in a deregulated market. However, this cannot be done without thorough knowledge of the system's conditions, which was acknowledged at the beginning of the deregulation process (e.g., [1,2]). Since then, numerous articles have suggested ways to get crucial information (e.g., [3-19]). All providers and consumers should have equal access to transmission and distribution networks, which is essential for fair competition in generation and supply. It is crucial to value each generator's contribution to each load, as well as the currents and power losses in each line and transformer in the network.

Due to the continual integration of Distributed Energy Resources and the use of improved metering and communication infrastructure, power distribution networks are changing from passive to active systems. This in turn changes power flows in the network and has an impact on network losses. Many distribution networks today have many sources that are located at various distances from the loads. Therefore, loss allocation (LA) methodologies should be able to determine how each member contributed to overall network losses. Researchers have been concentrating on this subject in the past decades.

In [4-6], a review of several loss allocation methods is provided, along with a discussion of their advantages and drawbacks.

In [7], a two-stage technique is used. Firstly, losses are allocated to loads only and no distributed power generation (DGs) are considered. Afterwards, the network is solved again taking DGs into account, and the loss change is allocated to DGs solely. Any discrepancy between total and allocated losses is broken down to DGs according to their apparent power.

There is a series of techniques that deal with branch flows expressed as current [8], power [9], and energy [10] by decomposing branch quantities into nodal injections. As a result of their inherent use of squares of sums of branch contributions, these methods are somewhat arbitrary because it is impossible to separate losses into sums of terms solely attributable to generation or load due to crossed terms in quadratic expressions.

Again, in [11] losses are allocated in two independent steps and assigned to loads and DGs. Since allocated losses and total losses do not match, a third step involving the normalization of allocated losses is necessary.

The game theory-based techniques are also utilized to achieve justness in loss allocation. Shapley values are used to model the problem as a cooperative game in which each player is accountable for the system's losses [12,13]. Because the game dimension grows with the factorial of the number of participants, these solutions can only be recommended to systems with a small number of players (loads and DGs). To get around

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Nomenclature

i, j, k, l, m bus indices
k-l network element directly connecting bus k to bus l with positive direction from bus k to bus l
g power source at bus g or a branch as a reactive power source
G_k, L_k generator and load at bus k, respectively
TAPF, TRPF total active and reactive power flow, respectively
APFC, RPFC active and reactive power flow contribution, respectively
A_k, B_k, C_k, D_k sets defined in subSection 2.3
Γ_k set of sources delivering active power to bus k
Ω_k set of sources delivering reactive power to bus k of branch k-l, excluding the branch as a source
$\Omega_k^{'}$ set of sources delivering reactive power to bus k of branch k-l, including $CQS_{k-l,k-l}^{(k)} = QS_{k-l}^{(k)}$
Ω_l set of sources contributing in $Q_{k-l}^{''}$
Q_{k-l} total reactive power flow reaching bus k (Fig. 4-6)
$Q_{k-l}^{''}$ total reactive power flow reaching bus l, not including influence of $QS_{k-l}^{(l)}$ (Fig. 4-6)
$P_{k-l}^{'}$, $Q_{k-l}^{'}$ total active and reactive power flows in k-l outflowing bus k, respectively (Fig. 3-6)
$P_{k-l}^{''}$, $Q_{k-l}^{''}$ total active and reactive power flows reaching bus l through k-l, respectively (Fig. 3-6)
$PS_{k-l}^{(k)}$,$QS_{k-l}^{(k)}$ total active and reactive power flows in shunt element of k-l at bus k, respectively (Fig. 3-6)
P_{G_k}, Q_{G_k} total generator active and reactive power at bus k, respectively
P_{L_k}, Q_{L_k} total load active and reactive power at bus k, respectively
$CQ_{k-l,g}$ reactive power contribution flowing to bus k, originated from source g (Fig. 4-6)
$CP_{k-l,g}^{'}$, $CQ_{k-l,g}^{'}$ active and reactive power flow contributions from bus k towards bus l, originated from source g, respectively (Fig. 3-6)
$CP_{k-l,g}^{''}$, $CQ_{k-l,g}^{''}$ active and reactive power flow contributions reaching bus l, originated from source g, respectively (Fig. 3-6)
$CQ_{k-l,g}^{'''}$ reactive power flow contribution reaching bus l, originated from source g, not including influence of $QS_{k-l}^{(l)}$ (Fig. 4-6)
$CQS_{k-l,g}^{(k)}$ reactive power flow contributions in shunt element at bus k, originated from source g, (Fig. 4-6)
$CQ_{k-l,i-m}^{'}$, $CQ_{k-l,i-m}^{''}$ reactive power flow contributions in k-l originated from network element i-m, at bus k and bus l, respectively

the dimensionality constraint an analytical equation is developed using the Shapley value. However, DGs are modeled as negative loads and loss normalization is used.

The "exact method" given in [14,15], which is exclusively applicable to systems without DGs, avoids the issue with crossed terms. In [16] it is expanded to incorporate DGs. In [14–16] branch losses are calculated as a product of the branch voltage drop and branch conjugate current contributions. In such a way, there is no quadratic expansion at all, the losses are written as linear functions of the branch current contributions and there are no issues with crossed terms. This overcomes difficulties encountered elsewhere.

Finally, there are papers that discuss methods based on circuit theory. In [17–18], the concept of loss allocation using the bus impedance matrix and bus current injections is explored. In [17], the participation of generators and loads in the total losses is calculated, while in [18], authors calculate the branch loss components attributed to generators and loads based on currents in branch series impedances. However, in the latter case, losses caused by branch shunt admittances are distributed equally among all users. A circuit theory-based loss allocation method specifically for active distribution systems is developed in [19]. This method adopts a branch-oriented approach to decompose power loss cross-terms among contributing load points. The loss components are computed at the bus level, but no specific information is provided regarding branch losses.

In [20], a novel methodology for allocating losses in active distribution networks is introduced. This approach combines virtual contribution theory with power flow tracing to devise a bidirectional loss allocation technique. By creating a virtual contribution matrix, it quantifies how generators and loads utilize the distribution network. However, each load and generator is only assigned a value representing its contribution to overall network losses.

Introducing a probabilistic loss sensitivity framework, the paper [21] aims to assess the impact of random power changes on power distribution systems, particularly pertinent with the integration of renewable energy and increased adoption of electric vehicles. It derives an analytical expression to approximate line loss variations resulting from uncertainties in power fluctuations.

The methodology proposed in this paper has the potential to build upon the concepts introduced in recent works, such as [20–21], by providing comprehensive insights into the contributions originating from power sources in loads, generators, and all network components including lines and transformers. This could pave the way for future research endeavors, including dynamic loss allocation and the investigation of methods to leverage various distributed energy resources for enhancing the operational efficiency and economic performance of power networks. Such a framework could enable real-time loss monitoring and facilitate optimal asset management.

The method proposed in this paper starts from a solved power flow problem for a given system using one of the well-known power flow solvers (e.g., [22]). Therefore, how to solve the power flow problem is not the subject of this work.

In fact, we count as known the direction and magnitude of total active and reactive power flows in generators, loads, and at both ends of every line and transformer, as well as in all shunt elements. Having this in mind, the paper proposes equations and procedures to calculate active and reactive power flow contributions from each generator to each network element and load, and related active and reactive power losses in network elements.

It is essential to put the accent on that this approach deals with meshed networks and can manage shunt elements. Also, we respect that power flow at one end of a line or transformer is not equal to the power flow at another end of the same line or transformer.

Also, the paper shows that local load (no matter of its quantity) may not only have power flow contributions from the nearest power source, even in case local generator can completely supply the load (as it was earlier mentioned in [2]).

2. Suppositions and definitions

2.1. Suppositions

- All three phase power system elements are symmetrical and balanced.
- We represent lines and transformers by conventional π-equivalent circuits, with resistless shunt elements.

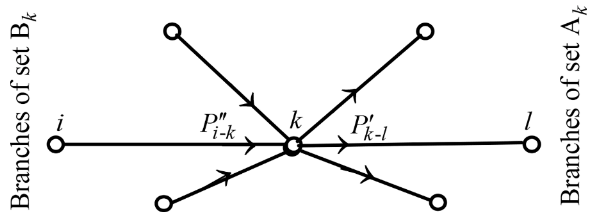

Fig. 1. General case of bus k with sets A_k and B_k.

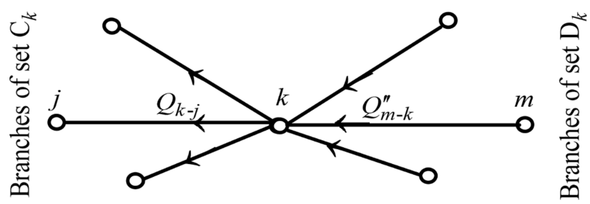

Fig. 2. General case of bus k with sets C_k and D_k.

- Power flow computation has been conducted previously. Therefore, we know directions and total active and reactive power flows in each generator and load, in addition to the values of P'_{k-l}, Q'_{k-l}, P''_{k-l}, Q''_{k-l}, $QS^{(k)}_{k-l}$, $QS^{(l)}_{k-l}$ in π-equivalent circuits of every line or transformer.

2.2. Term definitions

- *Network element* is a line, transformer, capacitor, or reactor.
- *Branch* is a network element, as well as a generator or load, where generator G_k or load L_k is considered as a branch between bus k and the ground.
- *Total branch power flow* is entire power flow through the branch.
- *Source* is a branch generating any power.
- *Power flow contribution* (active or reactive) is a part of branch total power flow originated from an individual power source.

2.3. Sets definitions

By knowing the direction of TAPFs and TRPFs in every branch, it becomes possible to categorize branches into in sets A_k, B_k, C_k, and D_k for each bus k. The sets definitions are (Figs. 1–6):

A_k is set of branches incident to bus k, where the total active power flow direction of each branch is away from bus k.
B_k is set of branches incident to bus k, where the total active power flow direction of each branch is towards bus k.
C_k is set of branches incident to bus k, where the total reactive power flow direction of each branch is away from bus k.
D_k is set of branches incident to bus k, where the total reactive power flow direction of each branch is towards bus k.

Fig. 1 depicts a general case of bus k, with the incident branches and the sets A_k and B_k. In Fig. 2, an analogous scenario of bus k is depicted, emphasizing the incident branches and the sets C_k and D_k.

Prior to initiating the proposed procedure, for every bus k we gather the incident branches in sets A_k, B_k, C_k, and D_k. Subsequently, we calculate APFCs and RPFCs separately.

3. Equations for active power flow contributions

By employing the knowledge of the APFCs in all branches of set B_k, we can calculate the APFCs in all branches of set A_k. This procedure is referred to as processing bus k for APFCs.

In formulating the necessary equations, we take into account the existing series impedances, assuming that shunt elements in π-equivalent circuits of lines and transformers do not generate or consume active power. Fig. 3 shows conventional π-equivalent circuit with the related TAPFs and accompanying APFCs.

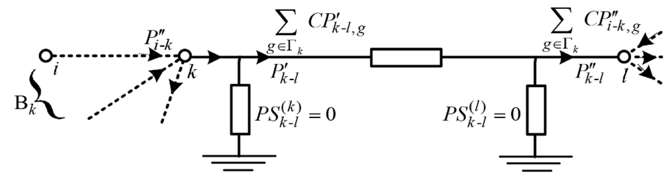

Fig. 3. Line or transformer π-equivalent circuit, including corresponding TAPFs and APFCs.

In order to obtain equations for APFCs associated with bus k, we use the following two facts (see Fig. 3):

1. The sum of all APFCs in branch $k-l \in A_k$ at bus k is equal to the TAPF in this branch at bus k (Fig. 3), i.e.,

$$\sum_{g\in\Gamma_k} CP'_{k-l,g} = P'_{k-l}, \forall k-l \in A_k. \tag{1}$$

2. The sum of all APFCs in all branches of set B_k at bus k is equal to the sum of TAPFs in these branches at bus k (Fig. 3), i.e.,

$$\sum_{i-k\in B_k}\left(\sum_{g\in\Gamma_k} CP''_{i-k,g}\right) = \sum_{i-k\in B_k} P''_{i-k}. \tag{2}$$

We select branches in sets A_k and B_k based on their TAPF direction. Therefore, all quantities in (1) and (2) are positive. The same principle applies to the quantities in (10) and (11).

When calculating APFCs in a branch, we consider two cases: a) the branch is a line or transformer, and b) the branch is a load.

3.1. Branch $k-l \in A_k$ is a line or transformer

By dividing the left-hand side of (1) by the left-hand side of (2), and dividing the right-hand side of (1) by the right hand side of (2), we obtain an equation that allows us to quantify the active power flow contributions in branch $k-l \in A_k$ at bus k (Fig. 3)

$$\sum_{g\in\Gamma_k} CP'_{k-l,g} = \frac{P'_{k-l}}{\sum\limits_{i-k\in B_k} P''_{i-k}} \cdot \sum_{i-k\in B_k}\left(\sum_{g\in\Gamma_k} CP''_{i-k,g}\right). \tag{3}$$

From (3), by selecting only the branches in set B_k that have APFC of source g, we can determine the APFC in branch $k-l \in A_k$ (quantified at bus k) that originates from source g. The expression is as follows:

$$CP'_{k-l,g} = \frac{P'_{k-l}}{\sum\limits_{i-k\in B_k} P''_{i-k}} \cdot \sum_{i-k\in B_k} CP''_{i-k,g}. \tag{4}$$

Regarding (3) and (4), we may point out the following:

This approach considers that APFC from any source may reach bus k through any number of branches in set B_k. Accordingly, the proposed procedure is applicable to both meshed networks and radial networks.

Since APFCs from source g can reach bus k through any number of branches in set B_k, corresponding parts of these contributions will be present in all branches $k-l \in A_k$. The same is valid for contributions from the source at bus k, irrespective of its size. This is a consequence of the fact that not any power flow contribution flowing into bus k has priority and their distribution between branches of set A_k is proportional to corresponding total power flows.

When developing equations for APFCs referring to bus l, we consider

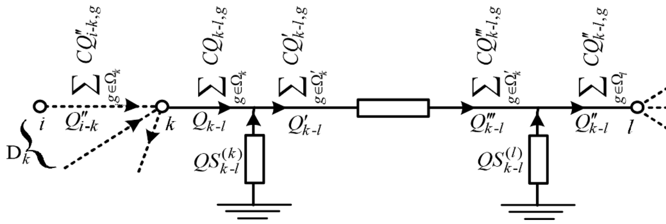

Fig. 4. Line *k-l* π-equivalent circuit, including related TRPFs and RPFCs.

series impedance in corresponding π-equivalent circuit of the line or transformer (see Fig. 3). From identity

$$\frac{1}{P''_{k-l}}\cdot\sum_{g\in\Gamma_k} CP''_{k-l,g} = \frac{1}{P'_{k-l}}\cdot\sum_{g\in\Gamma_k} CP'_{k-l,g} \tag{5}$$

for active power flow contributions reaching bus *l*, we get:

$$\sum_{g\in\Gamma_k} CP''_{k-l,g} = \frac{P''_{k-l}}{P'_{k-l}}\cdot\sum_{g\in\Gamma_k} CP'_{k-l,g}, \tag{6}$$

from which we can isolate the contribution that originated solely from source g as follows:

$$CP''_{k-l,g} = \frac{P''_{k-l}}{P'_{k-l}}\cdot CP'_{k-l,g}. \tag{7}$$

3.2. Branch $L_k \in A_k$ is a load

If branch $L_k \in A_k$ is a load, from (3) and (4) we obtain equations for active power flow components on the load as

$$\sum_{g\in\Gamma_k} CP_{L_k,g} = \frac{P_{L_k}}{\sum\limits_{i-k\in B_k} P''_{i-k}}\cdot\sum_{i-k\in B_k}\left(\sum_{g\in\Gamma_k} CP''_{i-k,g}\right), \tag{8}$$

$$CP_{L_k,g} = \frac{P_{L_k}}{\sum\limits_{i-k\in B_k} P''_{i-k}}\cdot\sum_{i-k\in B_k} CP''_{i-k,g}. \tag{9}$$

4. Equations for reactive power flow contributions

By having knowledge of the reactive power flow components in all branches of set D_k, we can calculate RPFCs in all branches of set C_k. This procedure is denoted as processing bus k for RPFCs.

Similarly to Section 3, in this context, we rely on the following two unquestionable equalities (Fig. 4):

$$\sum_{g\in\Omega_k} CQ_{k-l,g} = Q_{k-l}, \forall k-l\in C_k, \tag{10}$$

$$\sum_{i-k\in D_k}\left(\sum_{g\in\Omega_k} CQ''_{i-k,g}\right) = \sum_{i-k\in D_k} Q''_{i-k}. \tag{11}$$

Then, as in SubSection 3.1, from (10) and (11) we can derive the following

$$\sum_{g\in\Omega_k} CQ_{k-l,g} = \frac{Q_{k-l}}{\sum\limits_{i-k\in D_k} Q''_{i-k}}\cdot\sum_{i-k\in D_k}\left(\sum_{g\in\Omega_k} CQ''_{i-k,g}\right). \tag{12}$$

Note that (12) is valid irrespective of the branch type.

For the branch $k-l\in C_k$, we develop equations for RPFCs at bus k and RPFCs at bus l, taking into account that *k-l* can be a line, transformer, load, or generator that consumes reactive power.

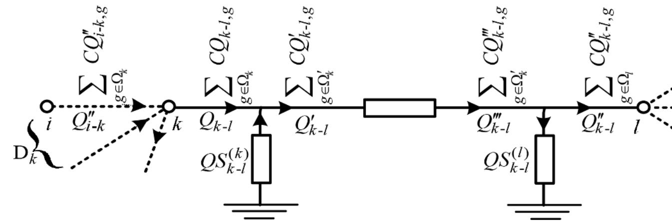

Fig. 5. π-equivalent circuit scheme of transformer $k-l\in C_k$ with turns ratio greater than unity (pu).

4.1. Line k-l which is in set C_k

In this case $QS^{(k)}_{k-l}$ is towards bus k and $QS^{(l)}_{k-l}$ is towards bus l (Fig. 4) and we consider both shunt elements as reactive power sources. The following equations are valid in this scenario:

$$Q_{k-l} = Q'_{k-l} - QS^{(k)}_{k-l}, \tag{13}$$

$$Q''_{k-l} = Q'_{k-l} - QS^{(l)}_{k-l}, \tag{14}$$

$$\sum_{g\in\Omega'_k} CQ'_{k-l,g} = \sum_{g\in\Omega_k} CQ_{k-l,g} + CQS^{(k)}_{k-l,k-l}, \tag{15}$$

$$\sum_{g\in\Omega_l} CQ''_{k-l,g} = \sum_{g\in\Omega'_k} CQ''_{k-l,g} + CQS^{(l)}_{k-l,k-l}, \tag{16}$$

$$CQS^{(k)}_{k-l,k-l} = QS^{(k)}_{k-l}, \tag{17}$$

$$CQS^{(l)}_{k-l,k-l} = QS^{(l)}_{k-l}. \tag{18}$$

Additionally, following the analogy with (6) we may write:

$$\sum_{g\in\Omega_k} CQ''_{k-l,g} = \frac{Q''_{k-l}}{Q'_{k-l}}\cdot\sum_{g\in\Omega'_k} CQ'_{k-l,g}. \tag{19}$$

Then, by considering (14) and (19) we can express (16) as follows:

$$\sum_{g\in\Omega_l} CQ''_{k-l,g} = \frac{Q'_{k-l} - QS^{(l)}_{k-l}}{Q'_{k-l}}\cdot\sum_{g\in\Omega'_k} CQ'_{k-l,g} + CQS^{(l)}_{k-l,k-l}, \tag{20}$$

with $\sum\limits_{g\in\Omega_k} CQ_{k-l,g}$ from (12) and $\sum\limits_{g\in\Omega'_k} CQ'_{k-l,g}$ from (15).

4.2. Transformer k-l with a turns ratio greater than unity (pu)

In this case $QS^{(k)}_{k-l}$ is towards bus k and $QS^{(l)}_{k-l}$ is in the direction away from bus l (Fig. 5).

The following equations are valid (12), (13), (15) and (19), as well as

$$Q''_{k-l} = Q'''_{k-l} + QS^{(l)}_{k-l}. \tag{21}$$

Because the shunt element at bus l is not a source, for RPFCs on the end segment of *k-l*, by analogy to (19), we have

$$\sum_{g\in\Omega'_k} CQ''_{k-l,g} = \frac{Q''_{k-l}}{Q'''_{k-l}}\cdot\sum_{g\in\Omega'_k} CQ'''_{k-l,g}, \tag{22}$$

and

$$\sum_{g\in\Omega'_k} CQS^{(l)}_{k-l,g} = \frac{QS^{(l)}_{k-l}}{Q'''_{k-l}}\cdot\sum_{g\in\Omega'_k} CQ''_{k-l,g}. \tag{23}$$

Respecting (19) and (21) we can express (22) and (23) as follows:

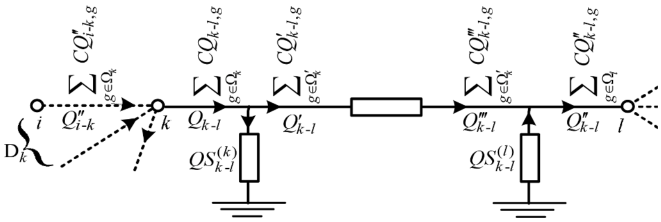

Fig. 6. π-equivalent circuit scheme of transformer $k-l \in C_k$ with turns ratio less than unity (pu).

$$\sum_{g\in\Omega_k} CQ^{''}_{k-l,g} = \frac{Q^{''}_{k-l}}{Q^{'}_{k-l}} \cdot \sum_{g\in\Omega^{'}_k} CQ^{'}_{k-l,g}, \tag{24}$$

$$\sum_{g\in\Omega^{'}_k} CQS^{(l)}_{k-l,g} = \frac{QS^{(l)}_{k-l}}{Q^{'}_{k-l}} \cdot \sum_{g\in\Omega^{'}_k} CQ^{'}_{k-l,g}, \tag{25}$$

with $\sum_{g\in\Omega_k} CQ_{k-l,g}$ from (12) and $\sum_{g\in\Omega^{'}_k} CQ^{'}_{k-l,g}$ from (15).

4.3. Transformer k-l with a turns ratio less than unity (pu)

In this case $QS^{(k)}_{k-l}$ is in the direction away from bus k and $QS^{(l)}_{k-l}$ is in the direction towards bus l, as shown in Fig. 6.

Here, the following equations are valid: (12), (14), (16), (19), and

$$Q_{k-l} = Q^{'}_{k-l} + QS^{(k)}_{k-l}. \tag{26}$$

In this case, by adapting (19) we may write:

$$\sum_{g\in\Omega_k} CQ^{'}_{k-l,g} = \frac{Q^{'}_{k-l}}{Q_{k-l}} \cdot \sum_{g\in\Omega_k} CQ_{k-l,g}. \tag{27}$$

Since the shunt element at bus k consumes reactive power, we have:

$$\sum_{g\in\Omega_k} CQS^{(k)}_{k-l,g} = \frac{QS^{(k)}_{k-l}}{Q_{k-l}} \cdot \sum_{g\in\Omega_k} CQ_{k-l,g}. \tag{28}$$

Finally, for RPFCs in the last segment of *k-l* we use (20).

4.4. Branch of set C_k is a load (L_k) or generator (G_k)

When $L_k \in C_k$, we can utilize (12) to obtain RPFCs on load L_k as follows:

$$\sum_{g\in\Omega_k} CQ_{L_k,g} = \frac{Q_{L_k}}{\sum_{i-k\in D_k} Q^{''}_{i-k}} \cdot \sum_{i-k\in D_k} \left(\sum_{g\in\Omega_k} CQ^{''}_{i-k,g} \right), \tag{29}$$

and for the contribution from generator g, we have:

$$CQ_{L_k,g} = \frac{Q_{L_k}}{\sum_{i-k\in D_k} Q^{''}_{i-k}} \cdot \sum_{i-k\in D_k} CQ^{''}_{i-k,g}. \tag{30}$$

In case where $G_k \in C_k$ consumes reactive power, we also use (29) and (30), replacing $CQ_{L_k,g}$ with $CQ_{G_k,g}$ and Q_{L_k} with Q_{G_k}.

5. Special cases

5.1. Direction of $Q^{''}_{l-k}$ is towards bus k and direction of $Q^{''}_{k-l}$ is towards bus l

In this case, the network element *k-l* does not carry reactive power from any other source. Instead, it acts a source of reactive power, exporting it via both end buses. Therefore, because the TRPFs at bus k and at bus l are the only RPFCs in this network element, then *k-l* should be treated as an element of set D_k with $\sum_{g\in\Omega^{'}_k} CQ^{''}_{l-k,g} = CQ^{''}_{l-k,l-k} = Q^{''}_{l-k}$ and as an element of set D_l with

$$\sum_{g\in\Omega_l} CQ^{''}_{k-l,g} = CQ^{''}_{k-l,k-l} = Q^{''}_{k-l}.$$

5.2. Direction of $Q^{'}_{k-l}$ is away from bus k and direction of $Q^{''}_{k-l}$ is away from bus l

In this case, the network element *k-l* consumes reactive power from other sources through both end buses. Therefore, at bus k network element *k-l* should be treated as an element of set C_k and at bus l should be treated as an element of set C_l.

6. Power losses in network element caused by power flow contributions

Having knowledge of APFCs and RPFCs in all network elements originating from various sources, allows us to calculate the contribution of any specific source to power losses in any line or transformer. Specifically, for active and reactive power losses in line or transformer *k-l* caused by APFC and RPFC from source *g*, we can write:

$$LossP_{k-l,g} = CP^{'}_{k-l,g} - CP^{''}_{k-l,g}, \tag{33}$$

$$LossQ_{k-l,g} = CQ_{k-l,g} - CQ^{''}_{k-l,g}. \tag{34}$$

In case (34) gives a negative result, it means that source *g* contributes to a reduction in power losses in *k-l*.

By summing the active power losses $LossP_{k-l,g}$ for all network elements, we determine the contribution of source *g* to the active power losses in the entire network. Similarly, the same principle applies to reactive power losses, where we can calculate the contribution of source *g* to the total reactive power losses in the network.

In addition, with the knowledge of APFCs and RPFCs in all network elements, we can calculate power losses associated with a specific pair consisting of source *g* and load at bus *m*. To accomplish this, we first identify all network elements transferring power from the source to the load. Then, using (33) and (34), we can calculate the power losses caused by active and reactive power flow contributions from source *g* in each of these network elements. By summing the power losses in respective network elements, we obtain the power losses related to the given source-to-load pair.

Additionally, using the results for power losses related to source-to-load pairs (as described in the previous paragraph), we can determine the network power losses associated with the load at bus *m*. This can be achieved by summing power losses associated to pairs involving load at bus *m*.

7. Procedure overview

In this section, we describe two pseudo-code algorithms that offer effective solutions for the calculation of APFC and RPFC. At the beginning we perform power flow computation using Matpower [22], which we also use as a source of input data for several test cases. These algorithms are easily translatable into Python code, as we have demonstrated in our paper. To facilitate further exploration and implementation, we have made the corresponding code available on GitHub [23].

The equations formulated in Sections 3 and 4 are individually employed on each branch within the network. They assume that for a single branch *k-l* both active and reactive power flows are directed from bus k to bus l. However, depending on the way the network input data are entered sometimes these powers may be negative. Therefore, before calculating APFC and RPFC the following preparatory steps are performed:

Algorithm 1
Compute APFC.

1: Read solved power flow results
2: Make set of all buses $\mathcal{B}$ and set of all branches $\mathcal{L}$
3: **for each** $k-l \in \mathcal{L}$ **do**
4:     **if** $P'_{k-l} < 0$ **then** swap branch orientation
5: **end for**
6: **for each** $k \in \mathcal{B}$ **do**
7:     Make sets A_k and B_k (Fig. 1)
8:     **if** $P_{L_k} < 0$ **then** convert load to generator
9:     **if** $P_{G_k} < 0$ **then** convert generator to load
10: **end for**
11: $\mathcal{L}' = \{G_k, k \in \mathcal{B}\}$ ▻ Set of branches with known APFC
12: $\mathcal{B}' = \emptyset$ ▻ Set of processed buses
13: iteration = 0
14: **while** $\mathcal{L}' \neq \mathcal{L}$ **do**
15:     iteration ← *iteration* + 1
16:     **for** $\boldsymbol{k} \notin \mathcal{B}'$ **do** ▻ Loop through buses
17:         **if** $i-k \in \mathcal{L}'$ **for each** $i-k \in B_k$ **then** ▻ All APFC known
18:             **for each** $k-l \in A_k$ **do**
19:                 Calculate APFC for branch *k-l* using (4)-(7)
20:                 $\mathcal{L}' \leftarrow \mathcal{L}' + \{k-l\}$
21:             **end for**
22:             $\mathcal{B}' \leftarrow \mathcal{B}' + \{k\}$
23:         **end if**
24:     **end for**
25: **end while**

Swap branch orientation when active or reactive power flows are negative (lines 3–5 in Algorithm 1, lines 3–6 in Algorithm 2),

Convert loads to generators and vice versa when respective active or reactive powers are negative (lines 6–10 in Algorithm 1 and lines 7–11 in Algorithm 2),

In cases when a branch draws reactive power at its both ends it is replaced with reactive power loads equal to the corresponding branch flows (line 5 in Algorithm 2).

Both algorithms are iterative, and we monitor their progress using two sets: $\mathscr{L}'$ and $\mathscr{B}'$. The set $\mathscr{L}'$ consists of branches whose APFC or RPFC are known, while the set $\mathscr{B}'$ comprises the buses that have already been processed. It is important to note that these sets are subsets of the set of all branches, $\mathscr{L}$, and the set of all buses, $\mathscr{B}$.

At the start of Algorithm 1, the set $\mathscr{L}'$ exclusively contains the generator branches. This is because the generator branches are the only sources of active power in the network, and their APFC values are equal to their active power outputs. Furthermore, the set $\mathscr{B}'$ is empty during this initialization phase (lines 11–12).

The iterative loop (lines 14–25) continues until the set $\mathscr{L}'$ is equal to the set $\mathscr{L}$. This means that the procedure terminates once the APFC values have been calculated for all branches in the network. Within each iteration, we iterate through the unprocessed buses (lines 16–24) and search for a bus k for which the APFC values for all branches from the set B_k are known. When this condition is met, we can then calculate the APFC values for all branches in the set A_k. After calculating the APFC for each branch, they are individually added to the set $\mathscr{L}'$. Finally, at the end of the iteration, bus k is included in the set $\mathscr{B}'$.

Algorithm 2 follows a similar flow to Algorithm 1 with some minor distinctions. In the beginning, not only the RPFC values for generators are known, but also the RPFC values for all branches that inject reactive power at both ends. These branches have RPFC values determined solely from their own flows. Hence, at the start, the set $\mathscr{L}'$ is expanded to include such branches, as described in lines 13–17. The iterative loop (lines 20–31) continues until the RPFC values for all branches have been calculated. During this process, we search for a bus k for which the RPFC values for all branches in the set D_k are known. Subsequently, we calculate the RPFC values for all branches in the set C_k. Similar to the previous algorithm, the processed branches are added to the set $\mathscr{L}'$, while the processed buses are added to the set $\mathscr{B}'$.

8. Case studies

8.1. Case 1: A simple test system comprising three buses

First, let us present the results obtained using the method on a simple power system depicted in Fig. 7. In this system, each line is represented by a π-equivalent circuit with a total series impedance (12+j40.9) Ω and a total shunt admittance of j277 μS. The generator at bus 2 produces 20 MW of active power and consumes 5 Mvar of reactive power. Loads at bus 2 and 3 are equal and their power consumption is $P_{L_2} + jQ_{L_2} = P_{L_3} + jQ_{L_3} = (50 + j20)$MVA. The voltage at bus 1 is V_1=115 kV. The power flow calculations yield the following results: V_2= (101.020–j10.811) kV, V_3=(100.433–j13.540) kV, $QS^{(1)}_{1-2}$=$QS^{(1)}_{1-3}$=1.83166 Mvar, $QS^{(2)}_{1-2}$=$QS^{(2)}_{2-3}$=1.42959 Mvar, $QS^{(3)}_{1-3}$ = $QS^{(3)}_{2-3}$=1.42242 Mvar. Total line losses in the network amount to (4.727+j6.743) MVA and the total generator power at bus 1 is $P_{G_1} + jQ_{G_1}$= (84.727+j51.743) MVA. Table 1 presents the results of total active and reactive power flows at both line buses.

The reason we selected this simple system is to demonstrate, within a reasonably small space, all the results that can be obtained using the proposed method. These results are presented in Tables 2–5.

In this case, the active power sources consist of generators located at bus 1 and bus 2. On the other hand, the reactive power sources

Algorithm 2
Compute RPFC.

Read solved power flow results
Make set of all buses $\mathcal{B}$ and set of all branches $\mathcal{L}$
for each $k-l\ \in \mathcal{L}$ **do**
  if $\boldsymbol{Q_{k-l}} < \mathbf{0}$ and $Q''_{kl} < 0$ **then** swap branch orientation
  if $\boldsymbol{Q_{k-l}} > \mathbf{0}$ and $Q''_{kl} < 0$ **then** replace the branch with loads $\boldsymbol{Q_{k-l}}$ and $-Q''_{k-l}$ at buses k and l, respectively
end for
for each $k\ \in \mathcal{B}$ **do**
  Make sets C_k and D_k (Fig. 2)
  if $Q_{L_k} < 0$ **then** convert load to generator
  if $Q_{G_k} < 0$ **then** convert generator to load
end for
$\mathcal{L}' = \{G_k, k \in \mathcal{B}\}$ ⊳ Set of branches with known RPFC
for each $k-l\ \in \mathcal{L}$ **do**
  if $\boldsymbol{Q_{k-l}} < \mathbf{0}$ and $Q''_{kl} > 0$ **then** ⊳ Inserting Q at k and l
    $\mathcal{L}' \leftarrow \mathcal{L}' + \{k-l\}$ ⊳ RPFC are from k-l only
  end if
end for
$\mathcal{B}' = \emptyset$ ⊳ Set of processed buses
iteration = 0
while $\mathcal{L}' \neq \mathcal{L}$ **do**
  iteration ← *iteration* + 1
  for $\boldsymbol{k} \notin \mathcal{B}'$ **do** ⊳ Loop through buses
    if $i-k \in \mathcal{L}'$ **for each** $i-k \in D_k$ **then** ⊳ All RPFC known
      for each $k-l \in C_k$ **do**
        Calculate RPFC for branch k-l using (13)-(30)
        $\mathcal{L}' \leftarrow \mathcal{L}' + \{k-l\}$
      end for
      $\mathcal{B}' \leftarrow \mathcal{B}' + \{k\}$
    end if
  end for
end while

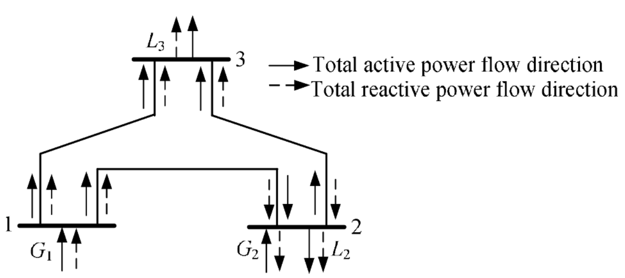

Fig. 7. Single-line diagram of the simple system with directions of total line active and reactive power flows.

Table 1
Total Power Flows in Lines.

Line	Total active and reactive power flows (MVA)	
k-l	$P'_{k-l} + jQ'_{k-l}$	$P''_{k-l} + jQ''_{k-l}$
1–2	38.608 + j26.148	36.545 + j22.378
1–3	46.119 + j25.595	43.506 + j19.945
2–3	6.545 – j2.622	6.494 + j0.055

Table 2
Origin of Active and Reactive Power Flow Contributions in Loads.

Load	Origin of active and reactive power flow contributions (MVA)				
	Generator 1	Generator 2	Line 1–2	Line 1–3	Line 2–3
Load at 2	32.315+j15.662	17.685+j0	0 + j2.241	0 + j0	0 + j2.098
Load at 3	47.703+j17.286	2.297+j0	0 + j0	0 + j2.659	0 + j0.055
Gen. at 2	0 + j3.9154	20+j0	0 + j0.5602	0 + j0	0 + j0.5244

encompass the generator at bus 1, as well as line 1–2, line 1–3, and line 2–3.

The active and reactive power flow contributions acquired in the loads at bus 2 and bus 3, along with the reactive power contributions in the generator at bus 2 (which consumes reactive power), sourced from specific origins, are outlined in Table 2. From the table, it is evident that despite the capacity of the load at bus 2 to absorb all active power generated at bus 2, a portion of the active power generated (specifically 2.297 MW) is transmitted to the load at bus 3. This observation aligns with the second note provided after Eq. (4).

Table 3 displays the results regarding the active and reactive power

Table 3
Origin of active and reactive power flow contributions in lines.

Line and bus	Origin of active and reactive power flow contribution (MVA)				
	Generator 1	Generator 2	Line 1–2	Line 1–3	Line 2–3
1–2 at 1	38.608+j26.148	0 + j0	0 + j0	0 + j0	0 + j0
1–2 at 2	36.545+j19.577	0 + j0	0 + j2.801	0 + j0	0 + j0
1–3 at 1	46.119+25.595	0 + j0	0 + j0	0 + j0	0 + j0.0
1–3 at 3	43.506+j17.286	0 + j0	0 + j0	0 + j2.659	0 + j0.0
2–3 at 2	4.230+j0	2.315+j0	0 + j0	0 + j0	0 + j2.622
2–3 at 3	4.197+j0	2.297+j0	0 + j0	0 + j0	0 + j0.055

flow contributions at each line side originating from specific sources. Observing Table 3, we note that in line 2–3, there are active power flow contributions from both the generator at bus 1 and the generator at bus 2. Additionally, it is evident that line 2–3 injects reactive power into bus 2 and bus 3. It's worth noting that line 2–3 does not transmit reactive power but rather acts as a source of reactive power.

Table 4 presents the line power losses attributed to power flow contributions originating from specific sources. A negative sign in Tables 3 to 5 indicates a reduction in losses.

As shown in Table 2, a load may receive power flow contributions from multiple sources. Table 3 provides a comprehensive display of power flow contributions at both ends of the line. In Table 4, the participation of power flow contributions originating from specific sources in line power losses is shown.

At both ends of a line, we observe power flow contributions originating from a specific source and flowing to a particular load. With these contributions and utilizing Eqs. (33) and (34), we can compute line power losses caused by these contributions. Subsequently, for a given pair of source and load, we can select a set of lines containing these contributions, and by summing the corresponding losses in these lines, we obtain the losses associated with the source-to-load pair. The relevant results are presented in Table 5.

Please note that if we are only focused on active power flow contributions in L_2, we do not need to go through the entire procedure. It is sufficient to calculate as follows:

- from (4) $CP'_{1-2,G_1} = P'_{1-2} = 38.608\text{MW}$,
- from (7) $CP''_{1-2,G_1} = P''_{1-2} \cdot CP'_{1-2,G_1} / P'_{1-2} = 36.545\text{MW}$,
- from (9) $CP_{L_2,G_1} = P_{L_2} \cdot CP''_{1-2,G_1} / (P''_{1-2} + P_{G_2}) = 32.315\text{MW}$ and $CP_{L_2,G_2} = P_{L_2} \cdot P_{G_2} / (P''_{1-2} + P_{G_2}) = 17.685\text{MW}$.

8.2. *Case 2: First modification of the system from case 1*

In this case, we use a modified version of the system introduced in Case 1. The only change is in the active power output of the generator at bus 2, which now generates 50 MW. Our specific interest is in calculating the active power flow contributions to load 2. Utilizing the proposed method, we find that the active power flow contribution from the generator at bus 1 to load 2 is 12.412 MW, while the contribution from the generator at bus 2 to load 2 is 37.588 MW. Consequently, even when the active power output of the generator at bus 2 matches the active power demand of load 2, we still observe a significant contribution from the generator at bus 1 to load 2. Furthermore, the generator at bus 2 not only supplies load 2 but also load 3.

8.3. *Case 3: Second modification of the system from case 1*

In this case, we implement a different modification to the system outlined in Case 1. Here, the modifications involve setting $P_{G_2} = 50\text{MW}$ and $P_{L_2} = 80\text{MW}$, while all other data remain unchanged.

In this case, the total injected power at buses (including bus 2) remains the same as in Case 1. Consequently, all bus voltages and total power flows in lines also remain unchanged from Case 1. However, this consistency does not extend to the power flow contributions. To illustrate this point, we focus solely on the results for active power flow contributions in the load at bus 2. Here, the active power flow contribution from the generator at bus 1 to the load at bus 2 is 33.7813 MW, while the active power flow contribution from the generator at bus 2 to the load at bus 2 is 46.2187 MW. It is evident that although the total power flows remain the same as in Case 1, the corresponding power flow contributions differ significantly.

For instance, let's compare the active power flow contributions in line 2–3 at bus 2. In Case 1, they are 4.230 MW from the generator at bus 1 and 2.315 MW from the generator at bus 2. However, in this case, the corresponding contributions are 2.7637 MW from the generator at bus 1 and 3.7813 MW from the generator at bus 2. Consequently, changes in active power flow contributions lead to alterations in their involvement in total line power losses. This holds true even though there are no changes in total line power flows or total line power losses.

Table 5
Source to load pairs power losses (MVA).

Source to load pair	Active and reactive line power losses		
	Line 1–2	Line 1–3	Line 2–3
Gen. 1 to load 2	1.824+j5.257	0 + j0	0 + j0
Gen. 1 to load 3	0.239+j0	2.613+j8.309	0.033+j0
Gen. 1 to generator 2	0 + j1.314	0 + j0	0 + j0
Gen. 2 to load 3	0 + j0	0 + j0	0.018+j0
Line 1–2 to load 2	0–j2.241	0 + j0	0 + j0
Line 1–2 to generator 2	0–j0.560	0 + j0	0 + j0
Line 1–3 to load 3	0 + j0	0–j2.659	0 + j0
Line 2–3 to load 2	0 + j0	0 + j0	0–j2.0973
Line 2–3 to load 3	0 + j0	0 + j0	0–j0.0550
Line 2–3 to generator 2	0 + j0	0 + j0	0–j0.5243
Total line power losses	2.063+j3.770	2.613+j5.650	0.051–j2.677

Table 6
Active power flow contributions in some loads of IEEE 30 bus test system.

Load at bus	Active power flow contribution in some loads (MW) and their origin			
	G_1	G_2	G_{13}	G_{27}
2	3.28	18.42		
3	2.40			
4	3.77	3.41	0.42	
7	5.08	17.29	0.27	0.16
8	8.10	15.45	0.71	5.75

Table 4
Participation of sources in line power losses (MVA).

Line	Generator 1	Gener. 2	Line 1–2	Line 1–3	Line 2–3	Total
1–2	2.063+j6.571	0 + j0	0–j2.801	0 + j0	0 + j0	2.063+j3.770
1–3	2.613+j8.309	0 + j0	0 + j0	0–j2659	0 + j0	2.613+j5.650
2–3	0.033+j0	0.018+j0	0 + j0	0 + j0	0–j2.677	0.051–j2.677
Total network power losses						4.727+j6.743

Table 7
Reactive power flow contributions in some loads of IEEE 30 bus test system.

Load at bus	Reactive power flow contributions in some loads (Mvar) and their origin														
	G_2	G_{13}	G_{22}	G_{27}	G_{23}	S_5	L_{1-2}	L_{1-3}	L_{2-4}	L_{2-5}	L_{2-6}	L_{5-7}	L_{6-7}	L_{28-8}	L_{28-6}
2	12.70														
3	0.33						0.46	0.40							
4	0.74	0.22			0.03		0.21	0.18	0.23						
7	5.29	0.17	0.59	0.20	0.02	0.18	0.16	0.14	0.18	1.80	0.21	0.93	0.93		0.11
8	10.20	1.33	4.59	5.75	0.15		1.24	1.07	1.39		1.60			1.85	0.82

8.4. Case 4: Ieee 30 bus test system

Partial results of the method applied to a 30-bus system (IEEE 30) are presented in Tables 6 and 7, specifically presenting the active and reactive power contributions to several loads. The column headings employ abbreviations, with G, S, and L representing generators, shunts, and branches respectively. The subscripts assigned to generators and shunts indicate the bus numbers they are connected to, while the branch subscript indicates the orientation of the branch.

Table 6 reveals that only generators contribute to the active power of the loads. In contrast, the reactive power is supplied not only by generators but also by shunts and branch capacitances, as displayed in Table 7. For instance, we observe that the active power of the load at bus 2 is provided by generators 1 and 2, generating 3.28 MW and 18.42 MW respectively. Conversely, the reactive power of the load at bus 4 is supplied by generators 2, 13, and 23, generating 0.74 Mvar, 0.22 Mvar, and 0.03 Mvar respectively. Additionally, shunts of branches 1–2, 1–3, and 2–4 contribute 0.21 Mvar, 0.18 Mvar, and 0.23 Mvar respectively. Notably, Table 7 does not include generator 1, as it exhibits a negative reactive power output, classifying it as a consumer of reactive power.

For comprehensive input data and the complete set of results for the 30-bus system, please refer to [23].

9. Conclusion

The developed equations enable accurate calculation of power flow contributions, both active and reactive, from any power source to any network element and load. Additionally, these equations facilitate the determination of losses in network elements associated with these contributions. Consequently, the method provides the capability to calculate network power losses associated related to every source-to-load pair, as well as power losses in the network attributed to any specific load or any source.

Furthermore, the proposed procedure demonstrates that if power flow contributions, whether active or reactive, from any number of power sources (including local source) reach bus k, regardless of the number of branches involved, each of these contributions will have a corresponding portion present in every branch of set A_k.

Therefore, the proposed method helps in obtaining important and detailed information about the system's status, facilitating a very responsible process of reaching a fair, transparent and non-discriminatory agreement between network operators and network users.

The authors applied the proposed method to numerous power systems featuring diverse network topologies, varying numbers of buses, sources, loads, and network elements. Throughout these applications, no issues were encountered. Further research is necessary to enhance the proposed method, allowing it to handle unbalanced networks, accommodating scenarios where the system's parameters and conditions are not perfectly symmetrical.

CRediT authorship contribution statement

Mirko Todorovski: Writing – review & editing. **Dragoslav Rajičić:** .

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The paper cites a GitHub repository where the data and code used to generate the results can be found. Additionally, the url link has been included in the "Attach Files" section.

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