EXPERIMENTAL AND NUMERICAL MODEL FOR ANALYSIS OF THE WATER HAMMER IN PUMP WATER SUPPLY SYSTEM

GOCE TASESKI [1], NIKOLA KRSTOVSKI [2]

[1] *Faculty of Civil Engineering Skopje, N. Macedonia,* taseski@gf.ukim.edu.mk

[2] *Faculty of Civil Engineering Skopje, N. Macedonia,* nikolatudence@gmail.com

1 Abstract

Water supply systems in such transient regimes should be hydraulically analyzed by means of basic equations for unsteady flow. For the needs of this research, by using the method of characteristics, a custom mathematical model HTM is developed for the purpose of water supply network analysis in conditions of unsteady flow, i.e. with this mathematical model, the characteristics of the water hammer in closed systems for water transport under pressure can be seen. In addition, a physical model in ratio 1:1 has been made for in-situ analyses of pressure change in occurrence of water hammer in the water supply system.

Keywords: Water supply system; water hammer; mathematical model; experimental (in-situ) model

2 Introduction

Water supply systems are complex systems which may consist of reservoirs, water treatment, water supply pipes which can be gravitational or pump-type, pressure regulators, valves, tanks and household water supply installations [1]. This complexity of water supply systems leads to water flow under pressure where flow and pressure change during time – unsteady water flow.

The phenomenon of water hammer in water supply networks is an inevitable occurrence which is most often initiated by pump stations, valves, hydro-mechanical equipment etc., and is distributed in the entire water supply network, especially noticeable in the areas of big height, pipeline sections with small hydrostatic pressure, etc. However, the water hammer parameters which can cause damage to the pipelines, hydro-mechanical equipment and to cause certain water pollution in the system are the most significant to engineers. The water hammer occurrence can have significant influence on water quality through the influence of cavitation, movement of particular microscopic particles may occur which are deposited as biofilm along the pipe volume – occurrence of "red water" in the water supply system. Also, in the occurrence of cavitation, if there are certain irregularities existing in pipe connection and small cracks in them, the possibility of underground waters infiltration in the water supply system cannot be excluded, which can amount from several liters to hundreds of liters depending on the opening size. The air captured in pipelines has also been shown to cause a lot of harm in the ductile pipes from the aspect of corrosion appearance inside the pipes, by which there is a direct influence on their quality [1].

Unsteady flow, i.e. water hammer occurrence in water supply systems represents challenge for scientific research from the aspect of making mathematical models for simulation of water hammer, which afterwards would be applied in projecting and managing water supply systems. Therefore, for the purposes of this research, by applying the method of characteristics, the HTM (Hydraulic Transient Model) mathematical model has been made for analysis of the water hammer in pumping water supply system, and the results obtained from the analytical model are calibrated and verified on a physical model – in-situ on real water supply system.

3 Numerical Model

There are many factors [1] which have influence on the flow under pressure, i.e. on the phenomenon of water hammer, such as: geometrical characteristics of the pipeline, material from which the pipeline is made, physical and pressure-deformable characteristics of water, distribution of flow speed in the cross section of the pipeline, dissipation of energy due to friction.

Starting points in the mathematical description of the water hammer are the basic equations in fluid mechanics:

The equation for maintenance of movement quantity, which, for unsteady flow in closed systems under pressure such convective acceleration is ignored, has the following form [1]:

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+\frac{\lambda}{2D}V|V|=0 \tag{1}$$

Continuity equation which, for unsteady flow of elastic fluid in elastic environment with assumed that pipe disposition is very small regarding the change in piezometric head, and instead of derivation the inclination of the pipe is introduced. Also, it is assumed that fluid thickness changes very little regarding piezometric head, has the following form:

$$\frac{\partial P}{\partial t}+V\frac{\partial P}{\partial x}-Vsin\alpha+V\frac{a^2}{g}\frac{\partial V}{\partial x}=0 \tag{2}$$

In Eq. (1) and (2), P denotes the static pressure at the centerline of the pipeline at location x and time t, V is the average velocity of flow, D is the pipe diameter, λ is the friction factor in the Darcy-Weisbach formula, x is the distance along the centerline of the pipe, α is the angle between the horizontal and the centreline of the pipe, taken as positive for the pipe sloping downwards in the direction of positive x, g is the gravitational constant; and a is the celerity of the pressure surge, i.e. the velocity with which the surge is propagated relative to the liquid. The positive direction for V coincides with that for x.

3.1 Method for Solving Partial Differential Equations

The method of characteristics exceptionally solved both positive and negative pressure waves and has remained one of the widely applied methods [1,2–6]. Therefore, the method of characteristics has been proven in the research so far as a method of exceptional compatibility with numerical solutions and the same one is applied in this research.

By the method of characteristics, the basic partial differential equations which are not integrable in closed form are transformed into ordinary differential equations which have solution in closed form [1,7–11]. Basic equations, continuity equation and dynamic equation can be marked with L_1 и L_2:

$$L_1=\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+g\frac{\partial P}{\partial x}+\frac{\lambda}{2D}V|V|=0 \tag{3}$$

$$L_2=\frac{\partial P}{\partial t}+V\frac{\partial P}{\partial x}-Vsin\alpha+V\frac{a^2}{g}\frac{\partial V}{\partial x}=0 \tag{4}$$

From the previously stated equations, it can be concluded that it is a question of two families of curves, which are practically straight lines, where the propagation speed is constant and variously larger than the basic flow speed, and thus the system of two partial differential equations transforms into a system of four ordinary differential equations marked with C+ и C– which determine straight lines:

$$\left.\begin{array}{c}\dfrac{dP}{dt}+\dfrac{a}{g}\dfrac{dV}{dt}+\dfrac{\lambda}{2D}v|v|=0\\ \dfrac{dx}{dt}=+a\end{array}\right\}C^{+} \tag{5}$$

$$\left.\begin{array}{c}\dfrac{dP}{dt}-\dfrac{a}{g}\dfrac{dV}{dt}+\dfrac{\lambda}{2D}v|v|=0\\ \dfrac{dx}{dt}=-a\end{array}\right\}C^{-} \tag{6}$$

3.2 Numerical Model

Figure 1 shows discretization of physical system in numerical network with calculating steps Δx and Δt where the solutions are obtained in the section of positive and negative lines of characteristics [3,7].

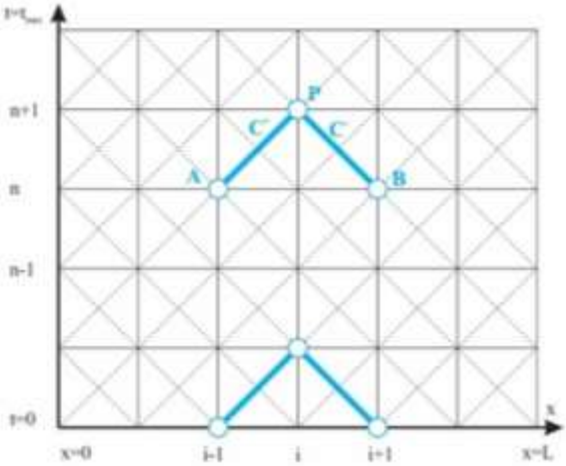


Figure 1. Numerical network for the method of characteristics

According to the given numerical method, the equations (5) and (6) may be written in the following manner:

$$\frac{d}{dt}\left(P\pm\frac{a}{g}V\right)+\lambda\frac{a}{D}\frac{V|V|}{2g}=0 \tag{7}$$

The previously stated equation can be integrated lengthwise of the positive and negative characteristic, i.e. lengthwise along the lines AP and BP, as follows and after the integration, the equations of positive and negative characteristics are written as follows:

$$\frac{P_P-P_A}{\Delta t}+\frac{a}{g}\frac{V_P-V_A}{\Delta t}+\frac{\lambda a}{2gD}V_A|V_A|=0 \tag{8}$$

$$\frac{\mathrm{P_P-P_B}}{\Delta \mathrm{t}}-\frac{\mathrm{a}}{\mathrm{g}}\frac{\mathrm{V_P-V_B}}{\Delta \mathrm{t}}+\frac{\lambda \mathrm{a}}{\mathrm{2gD}}\mathrm{V_B}|\mathrm{V_B}|=0 \tag{9}$$

If it is known that in the hydraulic analysis it is important to determine the flow change and height position of the hydrodynamic line in any section along the pipe, and in a certain time interval, additional approximation is introduced that the cross section of the pipe along its entire length is constant, and if it is known that median speed can be determined by the equation V=Q/A, the previously stated equations, knowing the numerical network, for the pressure, can be written in the following form:

$$P_i^{n+1}=P_{i-1}^{n}-B(Q_i^{n+1}+Q_{i-1}^{n})-MQ_{i-1}^{n}|Q_{i-1}^{n}|=0 \tag{10}$$

$$P_i^{n+1} = P_{i-1}^n + B(Q_i^{n+1} - Q_{i-1}^n) + MQ_{i+1}^n|Q_{i+1}^n| = 0 \tag{11}$$

If:

$$B = \frac{a}{gA} \quad \text{and} \quad M = \frac{\lambda \Delta x}{2gDA^2} = 0 \tag{12}$$

If the flow parameters in the time interval (n) are known, then the following is obtained:

$$P_i^{n+1} = CP - BQ_i^{n+1} \tag{13}$$

$$P_i^{n+1} = CM + BQ_i^{n+1} \tag{14}$$

Where:

$$CP = P_{i-1}^n + BQ_{i-1}^n - MQ_{i-1}^n|Q_{i-1}^n| \tag{15}$$

$$CM = P_{i+1}^n + BQ_{i+1}^n + MQ_{i+1}^n|Q_{i+1}^n| = 0 \tag{16}$$

From the equations (13) and (14), the basic equation of characteristics is obtained for determining the peak elevation of the hydrodynamic line:

$$P_i^{n+1} = \frac{CP + CM}{2} \tag{17}$$

3.3 Time Interval Selection for Analysis of Borderline Conditions

In real water supply systems, the problem with analysis of water hammer is always reduced to analysis of more complex systems, whereas initial and borderline conditions there are several possible ones that occur – characteristic forms of equations for borderline conditions. It is important to be mentioned here that the selection of the time calculation step has a big influence on the solution for each individual part of the complex system.

In order to determine borderline conditions in connecting one or more pipes of different geometrical and hydraulic characteristics, it is assumed that directions of positive C+ and negative C– characteristic cut in one point. This assumption in complex systems, such as water supply systems, is very rarely accurate, since the inclination of each line of characteristics depends on the propagation wave, flow speed in the pipe, horizontal position of the pipes connected in one joint and the number of sections in which the pipe is divided [1].

According to the previously stated, it can be said that in certain period, the lines of positive and negative characteristics of the connected pipes will not cut in one point. It can be said that this is a basic problem in making the mathematical model for the water hammer analysis. Due to it, in making of the mathematical model, certain assumptions about the time interval should be made in order to overcome this problem. Namely, time step should be selected in order to fulfill the Courant–Friedrichs–Lewy (CFL) "$\Delta x \geq a\Delta t$" condition of stability [7], i.e.:

$$\Delta t = \frac{\Delta x}{max|v + a|} = \frac{\Delta x}{V + a} = \frac{L}{N(V + a)} \tag{18}$$

One approach by which satisfactory results can be obtained is to reduce the time step – this procedure is used in creating the mathematical-numerical model. Namely, the time step is reduced for all pipes up to the value which enables the characteristic lines of all pipes to cut into one point. The pipe with the smallest value of time step is called "control pipe" of the model. This approach in the model of the joint position itself where pipes are connected, will "force" the characteristic lines in some way to cut into one point. However, in the initial and final borderline conditions, cutting of characteristic lines in rectangular grid will not be provided. In order to overcome this problem, additional interpolations should be made which make the mathematical model more complex, by which the condition for the characteristic lines to cut in one point, and initial and final conditions to cut in a rectangular grid will not be disturbed. This means that pipe sections should increase in all pipes that have primarily had time step larger than the "control pipe" by which the need for interpolation becomes smaller. Finally, as presented in figure 2, in the state obtained, the characteristic lines for all pipes are cut in rectangular grid and they all have the same time step.

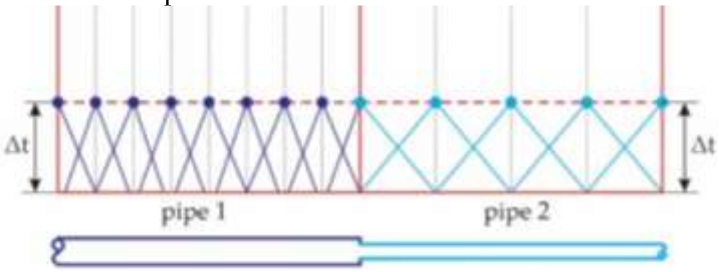


Figure 2. Connecting characteristic lines with identical time step

3.3.1 Borderline Conditions

Serial connection of two pipes in one joint:

$$Pressure: \quad P_{1,N}^{n+1} = P_{2,1}^{n+1} = P \tag{19}$$

$$Flow: \quad Q_1^{n+1} = \frac{(P_1^{n+1} - CM)}{B} \tag{20}$$

Connection of several pipes in one joint:

$$Pressure: \quad P^{n+1} = \frac{\frac{CP_1}{B_1} + \frac{CM_2}{B_2} + \frac{CM_3}{B_3} + \frac{CM_4}{B_4}}{\frac{1}{B_1} + \frac{1}{B_2} + \frac{1}{B_3} + \frac{1}{B_4}} \tag{21}$$

$$Flow: \quad -Q_{1,N}^{n+1} = \frac{P^{n+1}}{B_1} - \frac{CP_1}{B_1}; \quad -Q_{2,N}^{n+1} = \frac{P^{n+1}}{B_2} - \frac{CP_2}{B_2}; \quad -Q_{3,N}^{n+1} = \frac{P^{n+1}}{B_3} - \frac{CP_3}{B_3} \tag{22}$$

Tank at the end of pipeline:

$$Pressure: \quad P^{n+1} = P_R \tag{23}$$

$$Flow: \quad Q_1^{n+1} = \frac{(P_1^{n+1} - CM)}{B} \tag{24}$$

Pump as borderline condition:

There are several methods of presenting pumps as borderline conditions in the mathematical models for water hammer analysis in water supply networks. For preliminary analyses of the water hammer in water supply networks, as a characteristic of the pump, the parabola as a pump performance curve can be used [8]:

$$H_P = H_0 + AQ^2 + BQ \tag{25}$$

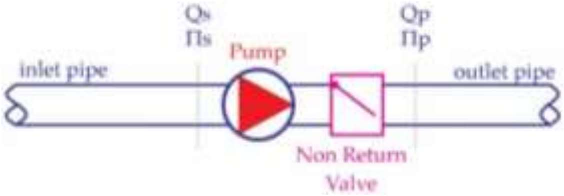


Figure 3. Pump as Borderline Condition

At the ends of the suction and pressing pipes which are connected to the pump, figure 3, the flow and height of the hydrodynamic line are unknown. From the continuity equation and the assumption that both cuts before and after the pump are at a small distance, it can be understood that Qs = Qp = Q. On the other hand, for determining the heights of the hydrodynamic line, the equations of positive and negative characteristic lines are used:

$$H_P = P_P - P_S + \frac{(V_P^2 - V_S^2)}{2g} \tag{26}$$

4 Development of HTM Mathematical Model

The mathematical model HTM is created in a way that it can analyze water supply system only in unsteady regime. The steady regime which dominates in the system before the occurrence of the unsteady state is taken from already made up-to-date software packages for analysis of systems under pressure, such as EPANET.

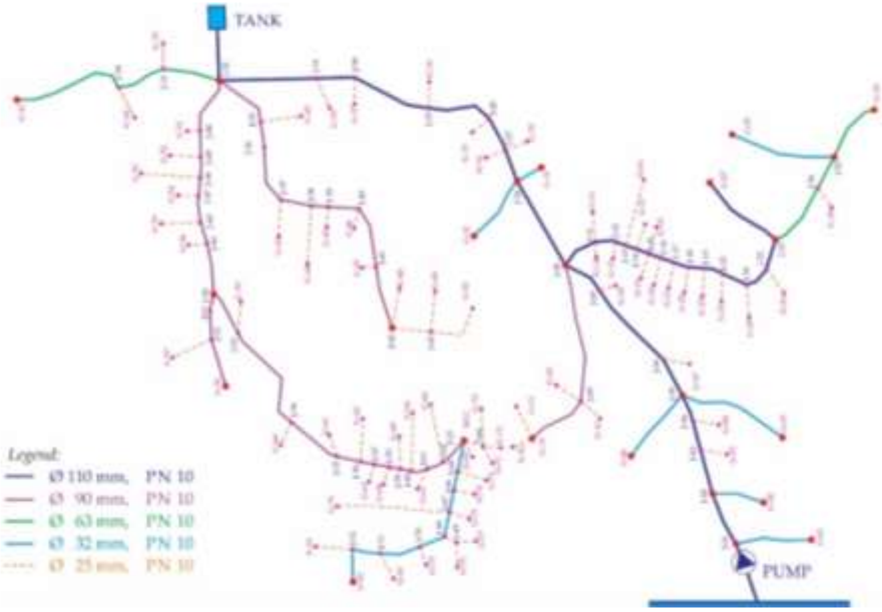


Figure 4. Schematic presentation of the real water supply system (Ø-pipe diameter, PN-Pipe's nominal pressure)

Such created mathematical model HTM is applied on real – existing water supply system, figure 4. The analyzed real water supply system is a pumping water supply system with reservoir beyond an inhabited place, while the water supply network is of branching system. Regarding the fact that it is a pumping water supply system, the initiator of the unsteady state is the pump station, i.e. in this system, the water hammer occurrence will be analyzed during pump switching on and off in the pump station.

In defining time step of the analysis, all recommendations previously stated are taken into consideration, and in this case the time step is $\Delta t = 0.02$ sec.

5 Experimental Model

For the calibration of the mathematical model HTM in this research, suitable measuring equipment has been made and built-in at certain critical places of the real water supply system, which is actually physical model in ratio 1:1. A characteristic of this water supply system is that the distribution of the consumers – households is at a large difference in height, i.e. from the minimum peak elevation of 724 m up to the maximum of 810 m, while the maximum water level in the tank is 824 m, i.e. such water supply system has a hydrostatic pressure from 14 to 100 m.

In the planning phase of this water supply system, all recommendations for dimensioning of water supply systems with assumed quasi-steady flow regime along the lines are observed. According to the calculations, it can be concluded that in none of the parts of the water supply network there are pressures larger than the maximum ones and smaller than the minimum ones - there are no negative pressures. However, in phase of certain part exploitation, frequent defects started occurring during time. Certainly, the reasons for occurrence of these defects cannot be explained by the operator/user of the system – i.e. frequent excuses refer to the quality of the material from which the pipes are made – they have no capacity to endure the pressure of 90 m although they are dimensioned by the manufacturer for such pressure. Is that the fact?

5.1 Program and Dynamics of Experimental Measurements

From the analysis of water supply network, terrain configuration, distribution of consumers and geometrical characteristics of the lines, the need for selection of a total of twenty measuring places emerged, by which entire coverage of network was provided. Figure 5 presents locations of all measuring places.

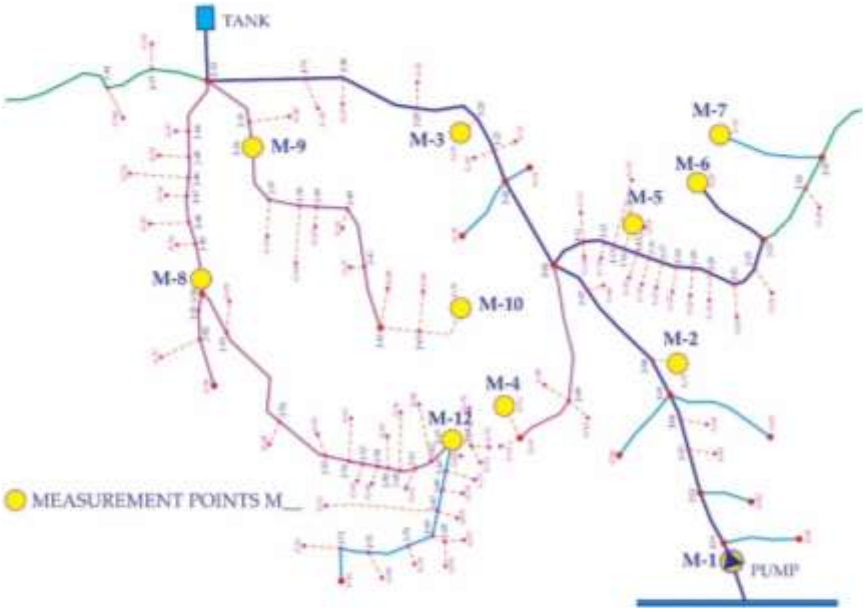


Figure 5. Arrangement of measuring places

5.2 Calibration of the Mathematical Model HTM with the Physical (in-situ) Model

For the calibration of the water supply systems in unsteady regime – in water hammer occurrence, there are certain recommendations which are used in steady regime, i.e.: calibration of pressure in joint positions, calibration of flow along lines, calibration of hydraulic gradient.

However, in the analysis of the obtained results, it has been shown that previous calibrations in steady regime are not enough for successful calibration of the mathematical model HTM with the physical (in-situ) model to be made. This conclusion owes to the fact that the material from which the pipelines are made, and their thickness have the biggest influence on the speed of pressure wave propagation through one water supply system. At this point it can be easily said that for the calibration of mathematical models for water hammer analysis, in addition to the parameters during stationary mode, it is essential to further determine – calibrate the speed of propagation of the pressure wave. Considering that the analyzed system is a branched water supply network as well as the limitation in available equipment, the measured pressures in the analyzed points were used as parameters for calibration.

The statistical operation error variance (σ^2) was used for data analysis. The error variance is proportional to the sum of the square of the differences between the measured and modelled responses (i.e., proportional to the objective function) and represents the unbiased sample variance of the model error after calibration (i.e., the objective function divided by the number of data points minus the number of model parameters) as shown in Equation 27 [3]:

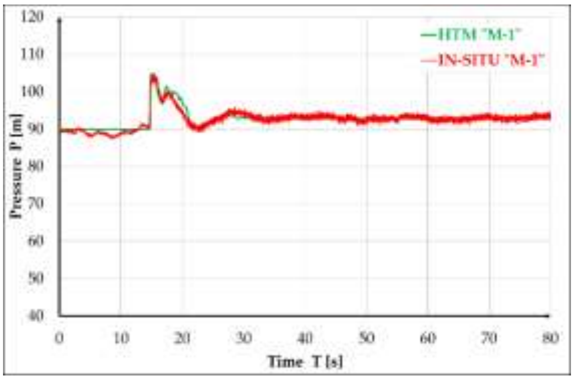
$$\sigma^2 = \frac{1}{M-N}\sum_{i=1}^{M}(H_i^m - H_i)^2 \tag{27}$$

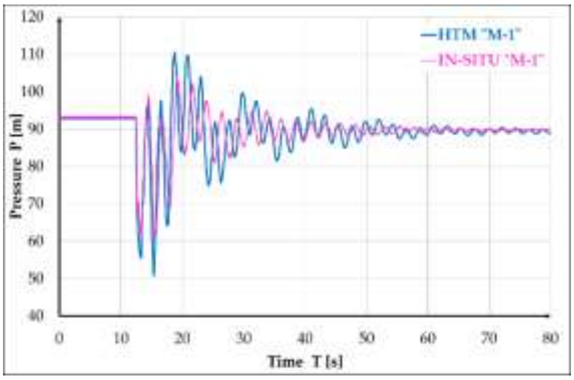
where M is the number of measured data points, N is the number of model parameters, H_i^m is the measured pressure response and H_i is the predicted pressure response.

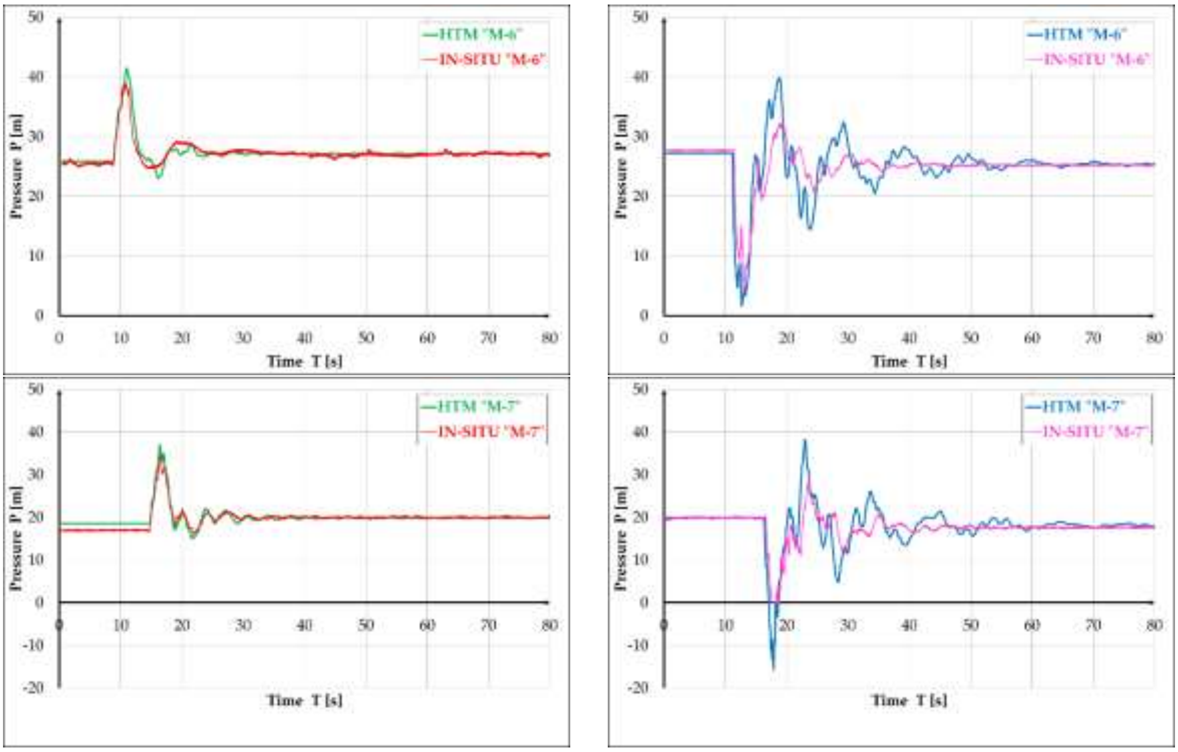
6 Results and Discussion

As it was previously mentioned in this paper, the need for this research was imposed by the fact to find out reasons for occurrence of defects in real water supply system which is subject to analysis. Namely, in the analyzed system from the very beginning of its exploitation of sections with small pressures, frequent occurrence of defects has been noticed, and defects of the pumping system pipeline near the pump station have started appearing later.

In addition, in the following figures there are output results of the performed analysis presented, i.e. for the characteristic states - during pump switching on and off.







(**a**) (**b**)

Figure 10. Characteristics of water hammer, experimental from performed in-situ measurements and analytical from HTM model during pump (a) switching on and (b) switching off in the pump station

If we analyze the graphs of the pressure change in the pump station (measuring place M-1), where in the immediate vicinity frequent defects can be noticed, it can be noted that in both switching on and off of the pump, the maximum pressure in water hammer occurrence is larger than 100 m (10 bar), whereas in steady regime of operation, the same one is smaller than 100 m (10 bar), i.e. it does not exceed the value of 90 m (9 bar), and if it is known that on that line to the pump station the pipeline is from HDPE PE100 NP 10 bar, it can be immediately concluded that water hammer occurrence is the reason for occurrence of defects along that line. It is understandable that in the initial exploitation period these defects have not been noticed, which primarily owes to the age of the pipe material which, during time, due to constant exposure to pressure above the maximum permitted one has led to material wear and tear – the pipeline has lost its elasticity and defects have increased occurring.

While analyzing the graph of pressure change at the measuring position M-7, it can be noticed that in state of switching off the pump, there are negative pressures occurring which due to limitation of capability of measuring equipment, which can measure only positive values of the pressure, they are not detected on the experimental model (in-situ). However, on the graph it can be noticed that in the period of the negative pressure occurrence, the measuring device shows zero value. Precisely the occurrence of negative pressure – vacuum in the pipeline, in the very beginning of the system exploitation, and occurrence of frequent defects has been noticed in the immediate vicinity of the measuring place M-7. It is important to be mentioned here that water hammer occurrence in real water supply system takes place during each pump switching on and off, i.e. it happens several times during the day, and exactly besides the occurrence of increased/decreased pressures in the system, the main reason for occurrence of defects is the frequency – repetition of increased, i.e. decreased pressures.

7 Conclusions

From the aspect of hydraulic analysis, water supply systems where occurrence of water hammer is expected should be analyzed and dimensioned in such manner that they can be "adjusted" to both steady and unsteady flow regime. This recommendation should especially be respected in water supply systems in which in the exploitation phase there is even slightest possibility of water hammer occurrence, such as pumping water supply systems with reservoirs beyond inhabited place, i.e. as the analyzed system in this research. Actually, as it can be seen from the results in this research of the subject, it is significant to analyze the locations with increased pressure, but also the locations with minimum pressures in order to avoid occurrence of vacuum in these sections.

References:

[1] Taseski, G.; Popovska, C. (2015). Water hammer analysis-impact of the pipe material in water supply system"; WMHE2015, 103-110
[2] Loh. Lohrasbi, A. & Attarnejad, R. (2008). Water Hammer Analysis by Characteristic Method. American Journal of Engineering and Applied Sciences, 1(4), 287- 294
[3] Stephens, M.L., Lambert, M.F., Simpson, A.R. and Vitkovsky, J. (2011). Calibrating the water hammer response of field pipe networks using a mechanical damping model. Journal of Hydraulic Engineering, Vol. 137, No. 10, Oct., 1225-1237
[4] Goldberg, D.E.; Wylie, E.B. Characteristic method using timeline interpolations. Journal of Hydraulic Engineering 1983, 670–683.
[5] Chaudhry, M.H. (2013). Applied hydraulic transients; 3rd ed.; Springer; ISBN 13: 978-1461485377.
[6] Parmakian, J. (1995). Water hammer Analysis; 1st ed.; Prentice Hall, Inc; ISBN B0000CJ5U0.
[7] Mohamed, S.; Ming, Z.; Duncan, A.M. (2005). A Review of Water Hammer Theory and Practice. Applied Mechanics Reviews, Vol. 58.
[8] Kishore, S. (2007). Transient analysis in pipe networks. Master thesis,
[9] A.R.D. Fluid Transients in Pipeline Systems, Second Edition (Pipelines and Pressure Vessels); The American Society of Mechanical Engineers; ISBN 13: 978-0791802106, 2004.
[10] Stephen L., W. (2011) Modeling of Pipeline Transients: Modified Method of Characteristics. http://digitalcommons.fiu.edu/etd.
[11] Vitkovsky, J.P.; Liggett, J.A.; Simpson, A.R.; Lambert, M.F. [2003]. Optimal Measurement Site Locations for Inverse Transient Analysis in Pipe Networks. Journal of Water Resource Planning and Management.