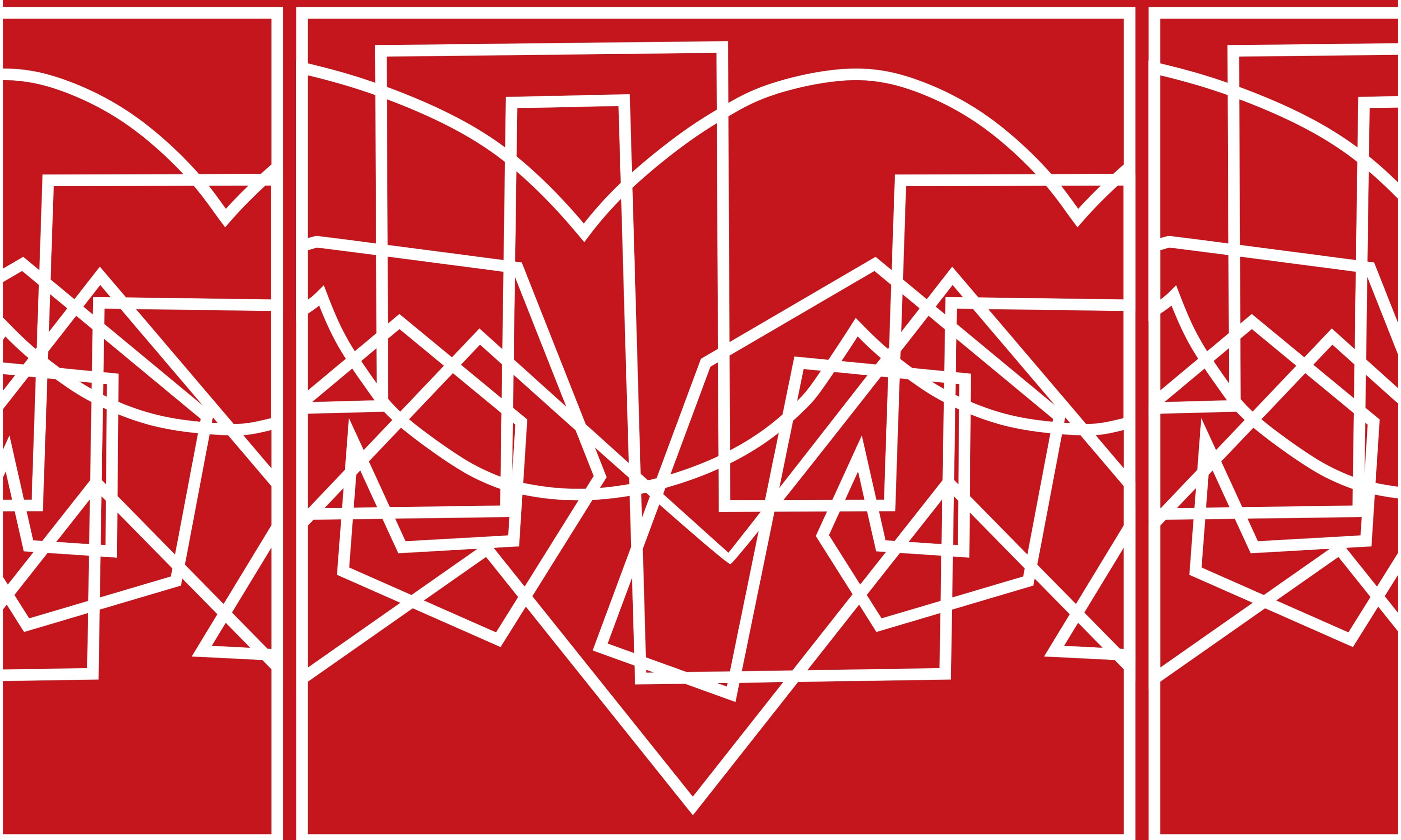

ПРИРОДНИ КАТАСТРОФИ И КОНСТРУКЦИИ
NATURAL HAZARDS AND STRUCTURES

ДГКМ
ДРУШТВО НА ГРАДЕЖНИ КОНСТРУКТОРИ НА МАКЕДОНИЈА

MASE
MACEDONIAN ASSOCIATION OF STRUCTURAL ENGINEERS

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Владимир ВИТАНОВ[1]

МАТЕРИЈАЛЕН МОДЕЛ ЗА АНАЛИЗА НА АБ ЅИДОВИ ЗАЈАКНАТИ СО FRP ПО ДЕЈСТВО НА ЦИКЛИЧЕН ТОВАР

РЕЗИМЕ

Употребата на армиранобетонски ѕидови како примарен систем за обезбедување на сеизмичка отпорност претставува стандарт во современото асеизмичко проектирање на армиранобетонските конструкции. Развиени се, тестирани и користени, повеќе методи за поправка и зајакнување на конструкции кои содржат АБ ѕидови, а постојано се појавуваат и нови. Една од популарните нови техники вклучува употреба на надворешно аплицирани FRP композитни материјали. Студијата прикажана овде предлага нов материјален модел за армиран бетон зајакнат со FRP. Моделот го усвојува постоечкиот пристап со т.н. „распределено" моделирање и го надградува со вклучување на зајакнувањето од FRP. Предложениот модел овозможува симулација на однесување на АБ елементи зајакнати со FRP земајќи ја во предвид пропагацијата на пукнатини заради аплицираните статички и циклични товари. Тој овозможува симулација на однесувањето на АБ елементи во рамнинска состојба на напрегање.

Клучни зборови: Материјален модел, армиран бетон, FRP, ANSYS

Vladimir VITANOV[1]

MATERIAL MODEL FOR ANALYSIS OF FRP STRENGTHENED RC WALLS UNDER CYCLIC LOADING

SUMMARY

The use of shear walls as primary earthquake load resisting system is a standard in the contemporary aseismic design of reinforced concrete structures. Many methods for repair and strengthening of shear wall structures have been devised, tested and used in the last decades and new ones are continuously emerging. One of the most popular new techniques involves using externally bonded FRP composite materials. The study presented here proposes new material model for reinforced concrete strengthened with FRP. The model adopts the "smeared" approach in modeling the nonlinear behavior of reinforced concrete and extends it by inclusion of the FRP strengthening. The proposed model enables simulation of reinforced concrete members strengthened with FRP taking into account the damage crack propagation by the applied static and cyclic loading. It enables simulation of the behavior of reinforced concrete members in plane stress.

Keywords: Material model, reinforced concrete, FRP, ANSYS

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1. INTRODUCTION

The conventional earthquake resistant design of reinforced concrete structures advises use of shear walls as effective way to add earthquake resistance to the reinforced concrete frames. A problem arises with structures erected decades ago following design rules which are by today's standards obsolete, inadequate and inefficient. Major earthquake events from around the world have shown the design deficiencies of these structures by inducing extensive damages in the structural members. Many of the old shear wall buildings are at risk of suffering damages from a major earthquake mostly due to their insufficient in-plane stiffness, flexural and shear strengths and ductility owing to the older design codes which didn't adequately estimate the demands that major earthquakes impose on the structures. This problem is ever increasing as the existing structures are getting older and their members gradually deteriorate.

This raised the need to find practical and efficient repairing, strengthening and retrofitting techniques. These techniques would enable damaged structures to be repaired and the old and seismically deficient structures to be strengthened and retrofitted so that they could meet the contemporary seismic design criteria.

Many different methods of seismic strengthening and repair of shear wall structures have been developed and tested in the last thirty years. Recently, state-of-the-art strengthening and retrofit techniques increasingly utilize externally bonded FRP composites, which offer unique properties in terms of strength, lightness, chemical resistance, and ease of application. Such techniques are most attractive for their fast execution and low labor costs.

These materials were used for the first time in the civil engineering structures in the mid 50's but real expansion of their application is felt only recently. Their application as strengthening materials for the RC structures is mostly seen in cases when the use of conventional strengthening techniques is problematic. For example, one of the most popular strengthening techniques for RC members involves attaching steel plates with epoxy glues onto the elements. This technique is simple, cheap and efficient but still has some disadvantages as: degradation of the bond between the concrete and the steel due to the steel corrosion, difficulties connected with the manipulation of the heavy steel plates on the site, need for scaffolding, limited length of the steel plates etc. Another popular strengthening technique is the so called "jacketing" i.e.increasing the RC element dimensions with additional layers of reinforced concrete or steel. This technique is very efficient in terms of the strength, stiffness and ductility which can be achieved but in the same time it leads to increase of the elements cross-sections thus increasing the dead loads on the structure, requires heavy physical labor, disrupts the functionality during the execution, increases the element stiffness which can have unfavorable influence on the structural behavior etc. These disadvantages of the conventional strengthening techniques of the RC elements can be avoided with the use of FRP strips and plates. Applied on the RC elements, these materials enable significant increase of the strength and ductility of the elements without the unfavorable increase of the stiffness. Due to their specific properties these materials enable fast execution without functionally disruption which leads to low costs. Presently, these advanced composite materials are more often used in the retrofit and repair of columns and beams. Information on the use of advanced composite materials for shear wall repair and retrofit are lacking.

FRP composites are formed by embedding continuous fibers in resin matrix which connects the fibers. The used fibers are usually made of carbon, glass or aramid and the resin is usually epoxy, polyester or vinylester. So, depending onthe fibers used, the FRP composites can be CFRP, GFRP and AFRP. Considering their mechanical properties the three main types of FRP composites have wide range of strength and stiffness values, and their common property is that when in tension they behave linear elastic until the final rupture.

The behavior of reinforced concrete members is normally studied by full-scale experimental investigations. The results are compared to theoretical calculations that estimate deflections and internal stress/strain distributions within the members. Finite element analysis can be used to model the behavior numerically to confirm these calculations, as well as to provide a valuable supplement to the laboratory investigations, particularly in parametric studies.

Modeling the complex behavior of reinforced concrete, which is both nonhomogeneous and anisotropic, is a difficult challenge in the finite element analysis of civil engineering structures. Most early finite element models of reinforced concrete included the effects of cracking based on a pre-defined crack pattern. With this approach, changes in the topology of the models were required as the load increased; therefore, the ease and speed of the analysis were limited.

A smeared cracking approach was introduced using isoparametric formulations to represent the cracked concrete as an orthotropic material. In the smeared cracking approach, cracking of the concrete occurs when the principal tensile stress exceeds the ultimate tensile strength. The elastic modulus of the material is then assumed to be zero in the direction parallel to the principal tensile stress direction.

Only recently have researchers attempted to simulate the behavior of reinforced concrete strengthened with FRP composites using the finite element method. Different element formulations and material laws were implemented in the FEM computer codes in order to facilitate the analysis of such reinforced concrete structures strengthened with FRP composites.

2. NUMERICAL MODEL

The proposed model of reinforced concrete members in plane stress strengthened with FRP is presented in this section. This is certainly not the first attempt to model FRP strengthened RC in plane stress. Many other material models were formulated, used and published. Various approaches with different adopted assumptions were used by the researchers in this field with varying level of success. The model proposed here is an attempt in modeling monotonic and cyclic behavior of FRP strengthened RC members by adopting the principle of simplicity and ease of application. It is based on a older inelastic model of reinforced concrete formulated by Darwin and Pecknold [1] which is also considered to be simple and efficient. The RC model of Darwin and Pecknold [1] uses the "equivalent uniaxial stress" approach with compressive loading curve proposed by Saenz [2] to model the biaxial material loading state, and the concrete failure surface proposed by Kupfer and Gerstle [3] based on the experimental data of Kupfer, Hilsdorf, and Rusch [4]. Although comparably simple, this model is still capable of simulating the cycling behaviour of reinforced concrete members in plane stress state. Because of it's simplicity and computational efficiency it was selected as a basis for the new model proposed in this study. The proposed model further extends the RC model by introducing additional material, the FRP, with it's unique properties.

The stress-strain curves for plain concrete, strongly suggest stress-induced orthotropic material behavior. Since the material model is designed to be used in conjunction with the finite element technique, the constitutive equations must be written in a form applicable to that technique. The material is treated as an incrementally linear, elastic material. That is, during each load increment the material is assumed to behave elastically. Between increments, the material stiffness and stress are corrected to reflect the latest changes in deflection and strain.

The equations relating change in strain to change in stress, for an incrementally linear orthotropic material while not subjected to shear may be written as follows [5]:

$$\begin{aligned} d\varepsilon_1 &= \frac{d\sigma_1}{E_1} - \nu_2 \frac{d\sigma_2}{E_2} \\ d\varepsilon_2 &= \frac{d\sigma_2}{E_2} - \nu_1 \frac{d\sigma_1}{E_1} \end{aligned} \tag{1}$$

Where E_1, E_2, ν_1 and ν_2 are stress-dependent material properties and $d\varepsilon_i$ and $d\sigma_i$ are principal strain and stress increments respectively. The material axes, 1 and 2, coincide with the current principal axes.

Solving these equations for change in stress in terms of change in strain and rewriting the solution in a matrix form gives:

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \end{Bmatrix} = \frac{1}{1-\nu_1\nu_2} \begin{bmatrix} E_1 & \nu_2 E_1 \\ \nu_1 E_2 & E_2 \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \end{Bmatrix} \tag{2}$$

Following the assumption of linear elastic material behavior, it can be said that the *Maxwell's reciprocal theorem* [see 6] holds true, which implicates symmetrical property of the stiffness matrix. Therefore, from eq. (2) directly follows:

$$\nu_1 E_2 = \nu_2 E_1 \tag{3}$$

Simpler form of eq. (2) can be obtained by introducing the concept of 'equivalent' Poisson's ratio which is defined as:

$$\nu^2 = \nu_1 \cdot \nu_2 \tag{4}$$

where ν is the 'equivalent' Poisson's ratio that is stress and strain dependent. Substituting eqs. (3) and (4), into eq. (2) a symmetrical form is obtained:

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \end{Bmatrix} = \frac{1}{1-\nu^2} \begin{bmatrix} E_1 & \nu\sqrt{E_1 E_2} \\ \nu\sqrt{E_1 E_2} & E_2 \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \end{Bmatrix} \tag{5}$$

So far the shear term have been excluded in the derivation of eq. (5). They can now be introduced into the equation which yields:

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\tau_{12} \end{Bmatrix} = \frac{1}{1-\nu^2} \begin{bmatrix} E_1 & \nu\sqrt{E_1 E_2} & 0 \\ & E_2 & 0 \\ \texttt{symm.} & & (1-\nu^2)G \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{Bmatrix} \tag{6}$$

or more simply:

$$d\boldsymbol{\sigma} = \boldsymbol{D}_{\mathrm{C}} d\boldsymbol{\varepsilon} \tag{7}$$

where $\boldsymbol{D}_{\mathrm{C}}$ is the concrete constitutive matrix in material coordinates.

It should be noted that the shear strain component γ_{12} in eq. (6) is the 'engineering' shear strain measure which is two times the tensorial shear strain, $\gamma_{12} = 2\varepsilon_{12}$ [5].

Considering the added shear term, similarly as in the case of the Poisson's ratio, it is desirable that no particular direction is favored with respect to the shear stiffness of the material model. Therefore, it can be assumed that the shear modulus does not change when the coordinate axes rotate by an arbitrary angle θ.

Starting from this assumption, an appropriate expression for the shear modulus can be derived. The concrete constitutive matrix $\boldsymbol{D}'_{\mathrm{C}}$, in the rotated configuration would be:

$$\boldsymbol{D}'_{\mathrm{C}} = \boldsymbol{T}^{\mathrm{T}} \boldsymbol{D}_{\mathrm{C}} \boldsymbol{T} \tag{8}$$

Where $\boldsymbol{T}$ is the matrix that transforms strains between axes [7]:

$$d\varepsilon = \boldsymbol{T} d\varepsilon' \tag{9}$$

$$\boldsymbol{T} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \tag{10}$$

Performing the calculation in eq. (8) for an arbitrary angle θ, an expression for $D'_{3,3}$ is obtained [2]:

$$D'_{3,3} = G(\cos^2\theta - \sin^2\theta)^2 + \frac{\sin^2\theta\cos^2\theta}{1-\nu^2}(E_1 + E_2 - 2\nu\sqrt{E_1 E_2}) \tag{11}$$

Note that $D'_{3,3}$ actually holds the value of the shear modulus in the rotated configuration. It will be independent of the axis rotation only if it's value does not change after the rotation, i.e. $D'_{3,3} = D_{3,3}$

$$G(\cos^2\theta - \sin^2\theta)^2 + \frac{\sin^2\theta\cos^2\theta}{1-\nu^2}(E_1 + E_2 - 2\nu\sqrt{E_1 E_2}) = G \tag{12}$$

Solving this equation for G will give the expression for the calculation of a shear modulus that is independent on the axis rotation:

$$G = \frac{1}{4\,(1-\nu^2)}(E_1 + E_2 - 2\nu\sqrt{E_1 E_2}) \tag{13}$$

For convenience, a the expression of eq. (13) for the shear modulus in rotated configuration can be rearranged as:

$$(1-\nu^2)G = \frac{1}{4}(E_1 + E_2 - 2\nu\sqrt{E_1 E_2}) \tag{14}$$

Substituting eq. (14) into eq. (8) gives the constitutive matrix *at an angle θ with the material coordinates*:

$$\boldsymbol{D}'_{\mathrm{C}} = \frac{1}{1-\nu^2}\begin{bmatrix} E_1 c^2 + E_2 s^2 & \nu\sqrt{E_1 E_2} & \frac{1}{2}(E_1 - E_2)\,s\,c \\ & E_1 s^2 + E_2 c^2 & \frac{1}{2}(E_1 - E_2)\,s\,c \\ \texttt{symm.} & & \frac{1}{4}(E_1 + E_2 - 2\nu\sqrt{E_1 E_2}) \end{bmatrix} \tag{15}$$

with $s = \sin\theta$ and $c = \cos\theta$. It can be seen that besides the shear modulus the off-diagonal terms that contain the Poisson's ratio, ν, are also independent of orientation. Then, by taking $\theta = 0°$ into eq. (15) the constitutive matrix for the plain concrete in the material coordinates is obtained:

$$\boldsymbol{D}_{\mathrm{C}} = \frac{1}{1-\nu^2}\begin{bmatrix} E_1 & \nu\sqrt{E_1 E_2} & 0 \\ & E_2 & 0 \\ \texttt{symm.} & & \frac{1}{4}(E_1 + E_2 - 2\nu\sqrt{E_1 E_2}) \end{bmatrix} \tag{16}$$

Finally, the constitutive equations for plane concrete in material coordinates are [8, 9]:

[2]The expressions for the other components of $\boldsymbol{D}'$ are quite lengthy and not important for the derivation of the expression for the shear modulus, therefore, they are not presented here

$$
\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\tau_{12} \end{Bmatrix} = \boldsymbol{D}_C \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{Bmatrix} \tag{17}
$$

and in global, $x - y$, coordinates:

$$
\begin{Bmatrix} d\sigma_x \\ d\sigma_y \\ d\tau_{xy} \end{Bmatrix} = \boldsymbol{D}'_C \begin{Bmatrix} d\varepsilon_x \\ d\varepsilon_y \\ d\gamma_{xy} \end{Bmatrix} \tag{18}
$$

It is should be noted that the constitutive matrix is defined by only three quantities, E_1, E_2 and ν. They are stress and strain state dependent variables whose values are determined at each individual load increment at each material point considered. These quantities define the constitutive matrix in material coordinates. Therefore, before using it to calculate the element stiffness it must me rotated to the global coordinates.

The concrete model takes into account crack propagation as a main nonlinearity inducing phenomena. The ”smeared” approach is also addopted in the crack modeling as well i.e. the cracks are considered as smeared throughout the conrete volume. The opened cracked is modeled by reducing the elasticuty modulus at the point to 0. The crack width is then calculated in each load step in order to detect if the crack has closed in the case of cyclic loading. Once the crack is closed the bearing capacity of the concrete at that point is restored.

2.1. Steel model

The steel is treated as a uniaxial material that is “smeared” throughout the concrete. A simplified bilinear model for the stress-strain behaviour of steel is used. The model is such that the steel may be either elasto-plastic or strain-hardening. The composite material constitutive matrix is obtained by adding the constitutive matrix for the steel to that of the concrete. In material coordinates, the constitutive matrix for the steel is given by:

$$
\boldsymbol{D}_\mathrm{S} = p_S \begin{bmatrix} E_S & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{19}
$$

where E_S is the tangent stiffness of the steel and p_S is the reinforcing ratio. $\boldsymbol{D}_\mathrm{S}$ is rotated to global coordinates using eq. (8).

The value of the tangent elasticity modulus of the steel can be:

$$
E_S = \begin{cases} E_{S,0} & \text{if } |\sigma_S| \le |f_y| \\ \delta E_{S,0} & \text{otherwise} \end{cases} \tag{20}
$$

where $E_{S,0}$ initial tangent stiffness of the steel, δ is the strain hardening stiffness ratio, σ_S is the current stress in the steel and f_y is the steel yield strength.

Since it is inherent to the “smeared” approach, perfect bond between the concrete and the steel is assumed.

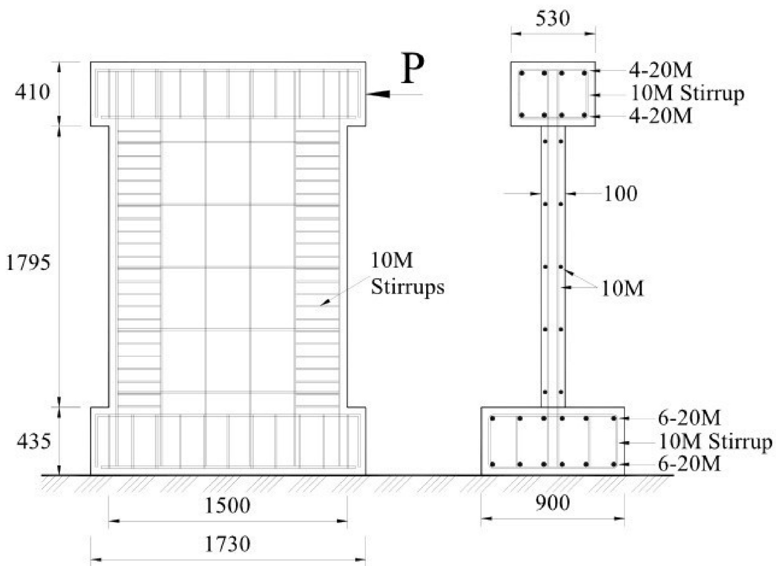

Figure 1: Reinforced Concrete Wall

2.2. FRP Model

The FRP material is modelled as linear elastic until the point of rupture. The material is capable to receive and transfer only tensional stress [10]. In the same manner as the steel, the FRP is treated as "smeared" throughout the concrete. The constitutive matrix of the FRP in material coordinates is given by:

$$\boldsymbol{D}_{\mathrm{F}} = p_F \begin{bmatrix} E_F & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{21}$$

where E_F is the tangent stiffness of the FRP and p_F is the "strengthening" ratio, i.e. the ratios of FRP cross-sectional area to the concrete one. $\boldsymbol{D}_{\mathrm{F}}$ is rotated to global coordinates using eq. (8).

2.3. Composite model

The strengthened-reinforced-concrete constitutive matrix is obtained by adding the constitutive matrix of the FRP (eq. (21)) to the concrete (eq. (16)) and steel (eq. (19)) ones, in global coordinates.

$$\boldsymbol{D}' = \boldsymbol{D}'_{\mathrm{C}} + \sum_{i=1}^{n} \boldsymbol{D}'_{\mathrm{S,i}} + \sum_{i=1}^{m} \boldsymbol{D}'_{\mathrm{F,i}} \tag{22}$$

where $\boldsymbol{D}'$, $\boldsymbol{D}'_{\mathrm{C}}$, $\boldsymbol{D}'_{\mathrm{S,i}}$ and $\boldsymbol{D}'_{\mathrm{F,i}}$ are the constitutive matrices of the composite material, the steel and the FRP in global coordinates, respectively; n is the number of different steel reinforcements and m is the number of different FRPs.

3. MODEL VERIFICATION

Lombard [11] conducted a testing on reinforced concrete shear wall specimens (Figure 1). The walls were constructed using $40\,\mathrm{MPa}$ concrete with identical reinforcement of $400\,\mathrm{MPa}$, $10\,\mathrm{mm}$ reinforcing bars. The height of the walls from the base of the panel to the centre of the cap beam is $2\,\mathrm{m}$, the length is

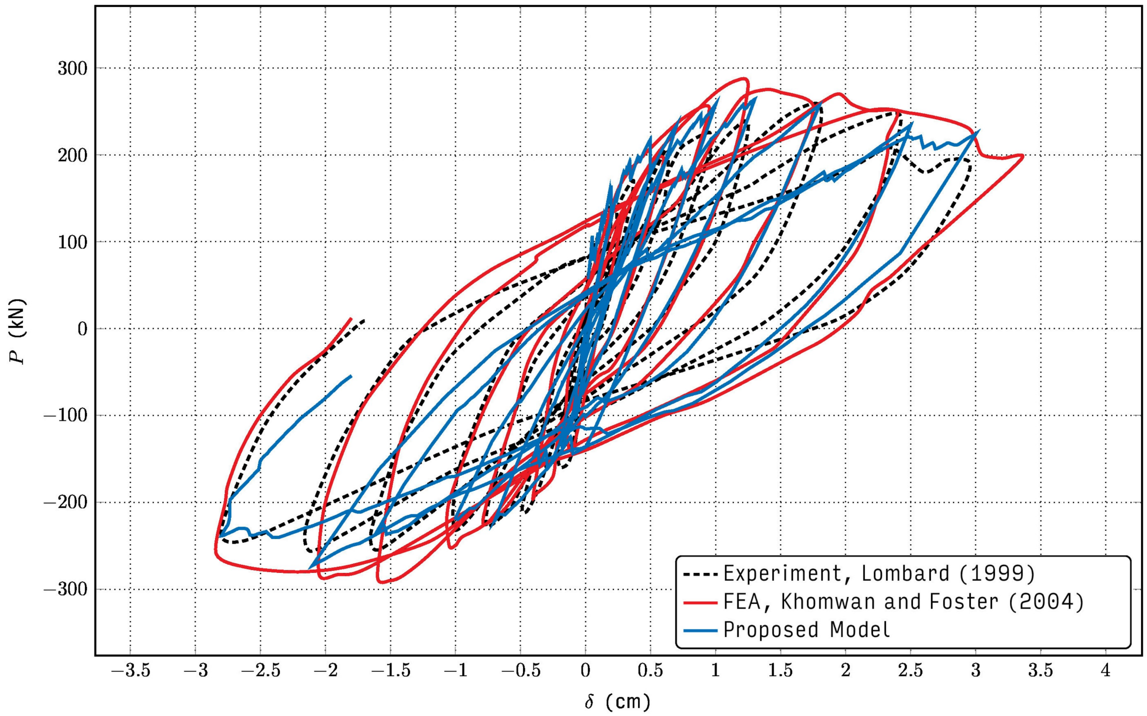

Figure 2: Wall 1

$1.5\,\mathrm{m}$ and the thickness is $10\,\mathrm{cm}$. The vertical reinforcement consist of five pairs of $10\,\mathrm{mm}$ bars, spaced at $40\,\mathrm{cm}$ for a reinforcement ratio of $0.8\,\%$. The horizontal steel consisted of five pairs of $10\,\mathrm{mm}$ bars, spaced at $40\,\mathrm{cm}$ for a reinforcement ratio of $0.5\,\%$.

Three of the test specimens included a control wall and two strengthened walls. The control wall was tested in its original state which provided a baseline for the evaluation of the repair and strengthening techniques. The two strengthened shear walls were strengthened by applying $0.11\,\mathrm{mm}$ carbon fiber sheets to the walls without pre-damage. The carbon fibre sheets had an elastic tensile modulus of $230\,\mathrm{GPa}$ and failure strain of $1.5\,\%$.The first specimen was strengthened with one vertical layer of FRP externally bonded to each face of the wall (Wall 1). The second specimen had one horizontal and two vertical FRP layers on each face of the wall (Wall 2). Both specimens were not loaded until the strengthening was applied. The experimental data was used to calibrate the analytical models of the three walls.

Five different sections of the wall with different properties were defined: top and bottom beam, two side section (‘columns’) and a middle section (‘panel’). Since the top and the bottom beam are significantly stiffer that the wall and their actual purpose is to provide the load transfer and anchorage for the tested wall, they were modelled as linear-elastic with very high elasticity modulus. The confining effect of the stirrups in the ‘columns’ was approximately accounted for by slightly increasing the concrete compressive strength in those regions.

The cyclic load was applied at the middle of the top beam as a series of small displacements. The force and displacement at the same point were taken as results of the performed analyses. These were compared not only with the experimental data, but also from an numerical investigations performed by Khomwan and Foster [12]. They took a more conventional approach into modelling the same shear walls. Their models were created using 4-node concrete membrane elements with the main steel reinforcement modelled as 1D bar overlay elements. The stirrups in the ‘columns’ were modelled as smeared through concrete elements. The FRP elements were overlaid over the concrete elements. The connection between them was established via 2D interface elements. The top and bottom beam were also modelled as linear-elastic.

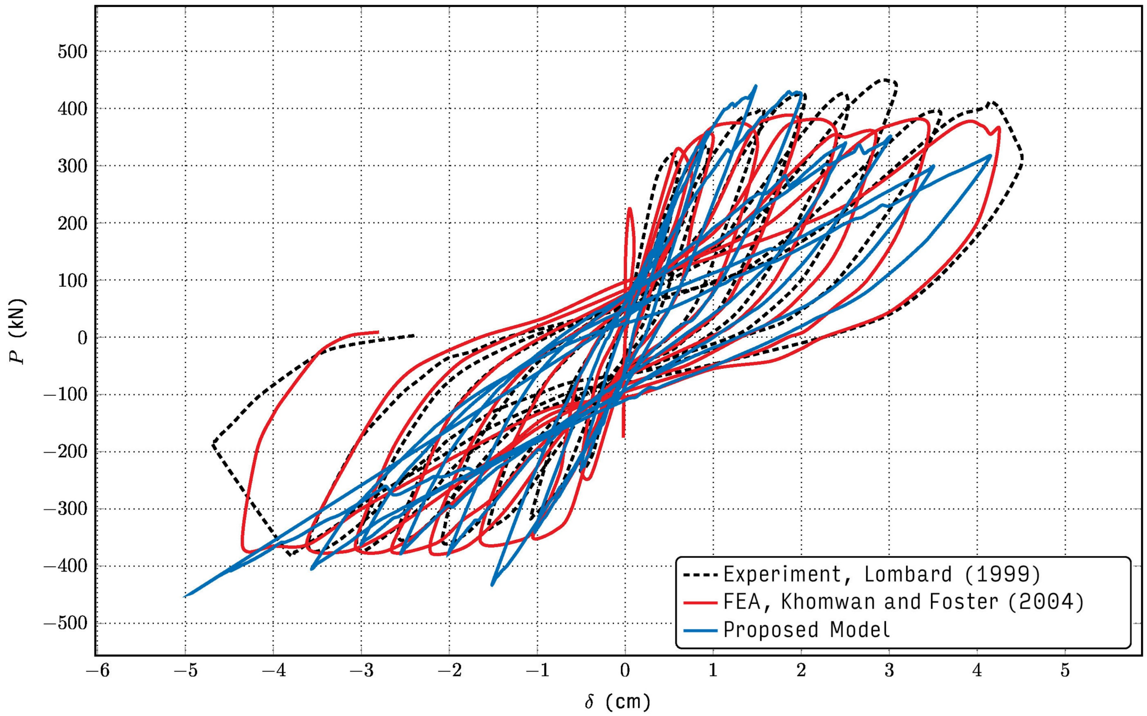

Figure 3: Wall 2

The resulting load-deflection curves are shown in Figure 2 for Wall 1 and Figure 3 for Wall 2. To measure how the numerical results compare to the experimental data the energy dissipated at each cycle (which corresponds to the area of the hysteretic loop) was calculated. The calculated energy dissipation is given in Table 1. The results indicate quite good correspondence with the experimentally acquired data with maximum difference of about 20 % (except in the cases of loop #8 in the Wall 1 and loop #6 in the Wall 2 which show greater differences compared to the experimentally obtained result).

	Loop nr.	Exp.*	FEM*	Ratio
Wall 1	3	62.225	49.481	0.80
	4	141.225	144.294	1.02
	5	226.810	196.496	0.87
	6	460.300	351.148	0.76
	7	696.088	579.795	0.83
	8	866.070	816.154	0.94
Wall 2	3	388.050	347.732	0.90
	4	532.765	519.467	0.98
	5	652.780	600.068	0.92
	6	864.965	762.338	0.88
	7	871.805	768.169	0.88
	8	1752.080	727.918	0.42

* In kN cm

Table 1: Energy Dissipation Comparison

4. CONCLUSIONS

The proposed model for reinforced concrete strengthened with FRP gives a good match with experimental results for monotonic biaxial loading. It also compared well with the experimental data from cycled load tests on FRP strengthened FRP members in plane stress matching the stress-strain behaviour and the energy loss per cycle rather well.

The results of the numerical analyses show that good match with the structural behaviour is obtained by combining the individual constitutive properties of the concrete, the steel and the FRP using the "smearing" approach to obtain the constitutive model of the "composite material". This approach enables very simple FEM structural modeling eliminating the need to dedicate separate elements for each of the material components reducing the size and the complexity of the model.

The performed analyses also indicate that in certain cases the model shows higher sensibility to the input parameters (material properties, element type, shape and size or load-step size). This can be expected to a certain degree considering the highly nonlinear nature of the problem and the complex behaviour of this composite material especially in cyclic loading conditions or high compression states when failure occurs due to concrete crushing. On the other hand, part of the model result sensibility can be attributed to the exclusion of some other present nonlinear effects like bond-slip, tension stiffening or reinforcement dowel action. Therefore, the final set of input parameters which would ensure stable solution be determined by performing several preliminary analyses on the FEM model.

REFERENCES

[1] D. Darwin and D.A. Pecknold. *Inelastic Model for Cyclic Biaxial Loading of Reinforced Concrete*. Civil Engineering Studies SRS-409. Univeristy of Illinois, Urbana-Champaign, Illinois, July 1974.

[2] L.P. Saenz. "Discussion of "Equation for the Stress-Strain Curve of Concrete", by Desayi and Krishman". In: *ACI Journal* 61.9 (Sept. 1964), pp. 1229–1235.

[3] Helmut B. Kupfer and Kurt H. Gerstle. "Behavior of Concrete Under Biaxial Stresses". In: *Journal of the Engineering Mechanics Division, ASCE* 99.4 (July 1973), pp. 852–866.

[4] Helmut Kupfer, Hubert K. Hilsdorf, and Hubert Rusch. "Behavior of Concrete Under Biaxial Stresses". In: *ACI Journal Proceedings*. Vol. 66. ACI. Aug. 1969, pp. 656–666.

[5] Arthur P. Boresi, Richard J. Schmidt, and Omar M. Sidebottom. *Advanced mechanics of materials*. 5th. John Wiley & Sons, 1993.

[6] S. Timoshenko and J.N. Goodier. *Theory of elasticity*. McGraw-Hill classic textbook reissue series. McGraw-Hill, 1951.

[7] Robert Davis Cook. *Concepts and applications of finite element analysis*. New York: John Wiley & Sons, 1974.

[8] C.S. Desai and H.J. Siriwardane. *Constitutive laws for engineering materials, with emphasis on geologic materials*. Prentice-Hall, 1984.

[9] W.F. Chen. *Plasticity in Reinforced Concrete*. New York: McGraw-Hill, 1976.

[10] FIB. *Externally Bonded FRP Reinforcement for RC Structures*. 130 pp. Fédération Internationale du Béton. Bulletin 14, 2001.

[11] J. Lombard. "Seismic Strengthening and Repair of Reinforced Concrete Shear Walls Using Externally Bonded Carbon Fibre Tow Sheets". MA thesis. Ontario, Canada: Carleton University, 1999.

[12] Nantawat Khomwan and Stephen J. Foster. *Finite Element Modelling of FRP Strengthened Beams and Walls*. Tech. rep. UNICIV Report R-432. ISBN: 85841 399 X. Kensington, Sydney 2052 Australia, 68 pp.: The University of New South Wales, School of Civil and Environmental Engineering, Nov. 2004.