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Review paper

Graphic statics in the digital age: a critical review of current methods and trends

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ABSTRACT

Graphic statics is a method for analyzing and designing structures based on the geometric representation of equilibrium conditions, where both forces and structural forms are depicted through reciprocal diagrams. This paper presents a comprehensive review of the development, application, and current state of graphic statics, with particular emphasis on its computational and three-dimensional extensions. Although the fundamentals of graphic statics date back to the 18th century, there has been a resurgence in interest in the field throughout the last three decades, primarily due to improvements in digital modeling and visualization tools. The review is structured into three main sections: the historical evolution of graphic statics and its entry into computational domains; an overview of form-finding methods and their integration into design workflows; and the role of graphic statics as a form-finding method in generating spatial, three-dimensional funicular forms. The paper concludes by identifying key research gaps and arguing for further development of graphic statics as a powerful tool for both architectural exploration and structural optimization.

1 Introduction

Graphic statics is a method for structural analysis and design based on the geometric representation of equilibrium, where both forces and forms are expressed through reciprocal diagrams. Using principles of projective geometry, it enables the construction of a force diagram in equilibrium paired with the form diagram, facilitating intuitive and visual determination of internal force distributions.

The graph reveals a marked increase in publications over the past three decades, underscoring a resurgence of interest in graphic statics largely driven by computational innovations, based on a chronological overview derived from the analysis of over 100 publications, Figure 1. For clarity, the major contributions have been grouped into three key periods: the foundational era (1725–1990), early computational developments (2000–2010), and contemporary research (2010–present), which includes recent advances in three-dimensional graphic statics. While the origins of the method can be traced back to the 18th century, the surge in publications in recent years highlights the ongoing evolution and research potential of graphic statics as a dynamic and expanding area within structural design and analysis.

This paper presents a comprehensive review of the development, application, and current state of graphic statics, with particular emphasis on its computational and three-dimensional extensions. The analysis of the selected works is structured into three main sections: the first section provides a summary of the evolution of graphic statics and the recent development of three-dimensional computational graphic statics; the second section reviews form-finding methods, encompassing their historical development in relation to graphic statics and contemporary use; and the third section discusses the role of graphic statics as a method for form-finding in the design of spatial, three-dimensional funicular forms.

2 Historical development of graphic statics

The literature that provides a historical overview of the development of graphic statics is chronologically divided into three segments. The first segment focuses on works that explore the fundamental principles and methods of graphic statics, spanning from its origins in the 16th century to the end of the 20th century. The second segment examines literature from the late 20th century that marks a renewed interest in the field, particularly through its integration with

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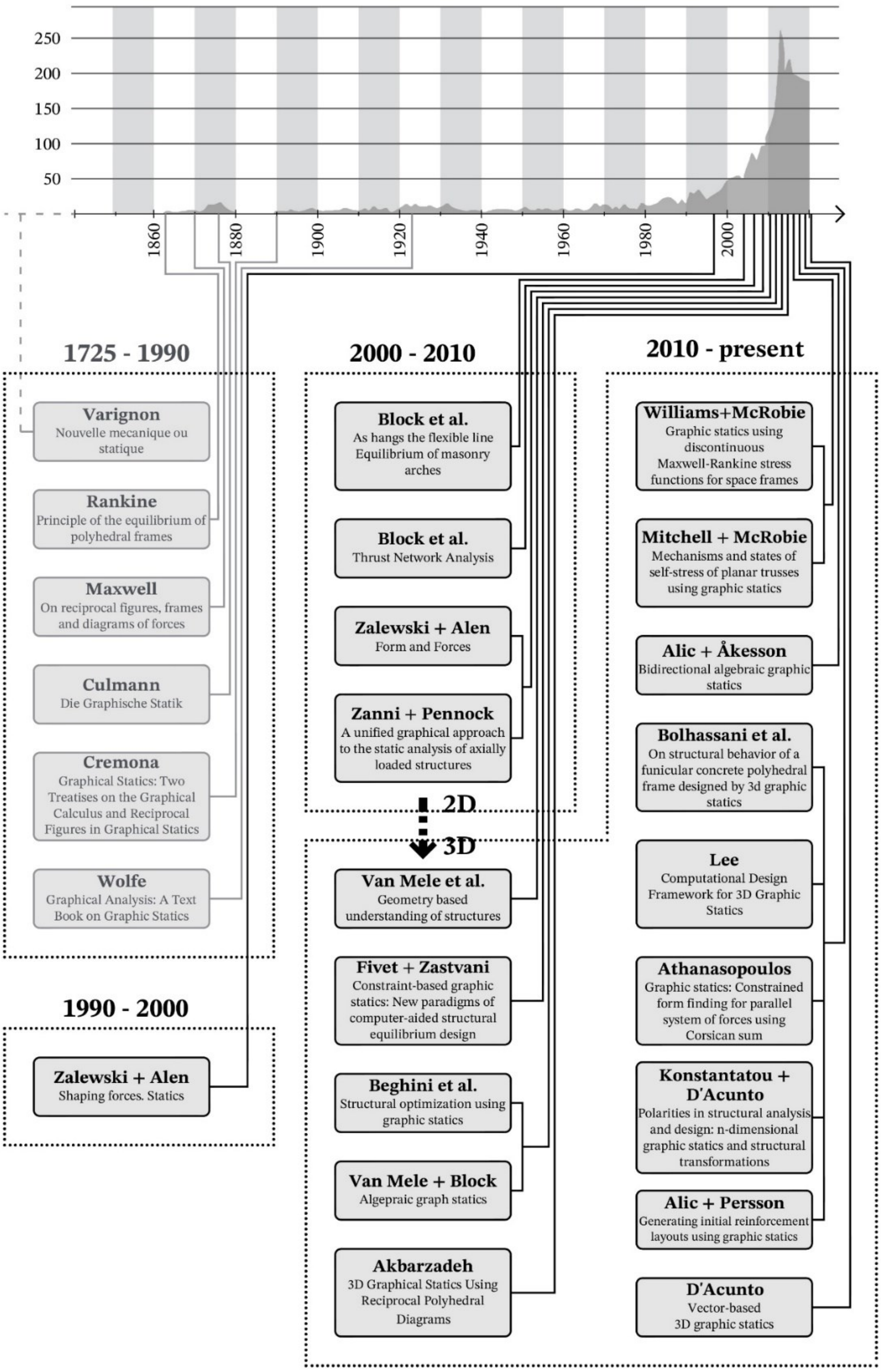


Figure 1. Chronological Overview of Research on Graphic Statics

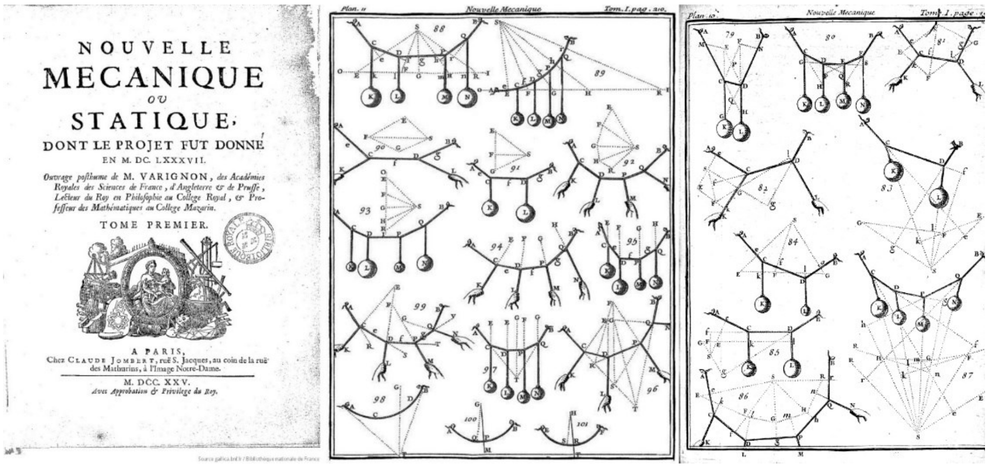
computational technologies, advancements that not only enabled more efficient application of classical principles but also extended their use in the analysis and design of complex structures. The third segment encompasses the most recent developments in the field of three-dimensional graphic statics, from the late 20th century to the present, a period during which, research has utilized three-dimensional form and force diagrams for the analysis and design of complex structures, supported by advanced digital tools.

2.1 Fundamental principles and methods of graphic statics

In the detailed historical overview of graphic statics presented in *The History of the Theory of Structures* by Kurrer [1] and *Symmetrie Gruppe Dualität* by Scholz [2], it is stated that the origins of graphic force analysis date back to the 16th century. The first significant contribution is attributed to Simon Stevin, who in 1586 [3] graphically represented the balance of forces on inclined planes using weighted rope diagrams, concluding that the forces on either side of the inclined planes are in equilibrium if the magnitude of the force on each side is proportional to the length of that side. In one of the earliest examples related to graphic statics, *Nouvelle Mécanique ou Statique* [4], Varignon introduced the key terms: *funicular polygon* and *force polygon*. By employing a polygon, or more precisely, a force diagram, Varignon described the static equilibrium of internal forces in simple structures, as well as in systems consisting of a suspended, inextensible rope with attached weights. The shape assumed by the rope under the action of the applied forces represents the funicular polygon, which visually illustrates the interaction between external and internal forces, Figure 2.

By the beginning of the 19th century, the concept of the force polygon had been employed by several researchers to visualize, analyze, and explain the equilibrium of force systems. In 1822, Poncelet [5] used the force polygon to determine the center of gravity, while Lamé and Clapeyron [6] applied the same method in 1828 to analyze the dome of Saint Isaac's Cathedral. In 1858, Rankine [7] demonstrated how force polygons could be used to calculate internal forces in statically determinate trusses. Graphic methods for analyzing static equilibrium were formally introduced into engineering as graphic statics through Culmann's *Die Graphische Statik* [8], which established that the funicular polygon and the corresponding force polygon are interchangeable as reciprocal diagrams [3], [4]. The theory of reciprocal relationships between the funicular form and force diagram was further developed in the publications of Maxwell [9], [10]. Building on Culmann and Maxwell, Cremona introduced a method for constructing reciprocal diagrams [11], thereby expanding the application of graphic methods for analyzing force distribution in trusses [12], Figure 3.

During the late 19th century, graphic statics gained further popularity due to its application as a complementary technique for the analysis of cast-iron structures[1]. In contrast to traditional structures built from combinations of stone and timber, cast-iron constructions were composed of a series of linear elements subjected to axial forces, either in compression or tension. The equilibrium of forces, as well as the design and analysis of such complex load-bearing forms, could be graphically determined using graphic statics, without resorting to complicated numerical methods and calculations. A comprehensive presentation of the known methods at the time, illustrated through practical examples, was provided by Wolfe in Graphical Analysis: A Text Book on Graphic Statics [13], which represents the final reference from the reviewed period.



NOUVELLE
MECANIQUE
OU
STATIQUE,
DONT LE PROJET FUT DONNÉ
EN M. DC. LXXXVII.

Ouvrage posthume de M. VARIGNON, des Académies Royales des Sciences de France, d'Angleterre & de Prusse, Lecteur du Roy en Philosophie au College Royal, & Professeur des Mathématiques au College Mazarin.

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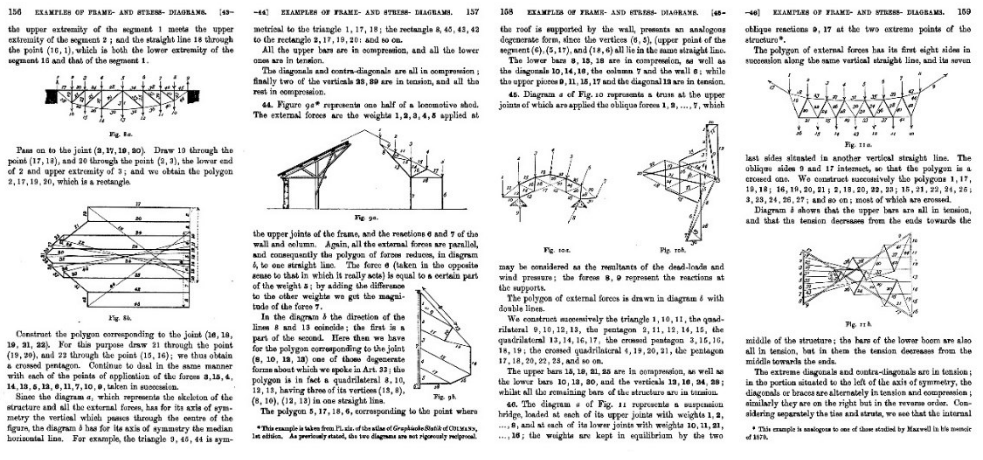
Figure 2. Drawings of funicular polygons in Nouvelle Mécanique ou Statique [4] that graphically represent the static equilibrium of a system of ropes



156 EXAMPLES OF FRAME- AND STRESS- DIAGRAMS. [43-

the upper extremity of the segment 1 meets the upper extremity of the segment 2; and the straight line 18 through the point (16, 1), which is both the lower extremity of the segment 16 and that of the segment 1.

Fig. 8a.

Pass on to the joint (2, 17, 19, 20). Draw 19 through the point (17, 18), and 20 through the point (2, 3), the lower end of 2 and upper extremity of 3; and we obtain the polygon 2, 17, 19, 20, which is a rectangle.

Fig. 8b.

Construct the polygon corresponding to the joint (16, 18, 19, 21, 22). For this purpose draw 21 through the point (19, 20), and 22 through the point (15, 16); we thus obtain a crossed pentagon. Continue to deal in the same manner with each of the points of application of the forces 3, 15, 4, 14, 13, 5, 12, 6, 11, 7, 10, 9, taken in succession.

Since the diagram *a*, which represents the skeleton of the structure and all the external forces, has for its axis of symmetry the vertical which passes through the centre of the figure, the diagram *b* has for its axis of symmetry the median horizontal line. For example, the triangle 9, 46, 44 is sym-

-44] EXAMPLES OF FRAME- AND STRESS- DIAGRAMS. 157

metrical to the triangle 1, 17, 18; the rectangle 8, 45, 43, 42 to the rectangle 2, 17, 19, 20: and so on.

All the upper bars are in compression, and all the lower ones are in tension.

The diagonals and contra-diagonals are all in compression; finally two of the verticals 23, 39 are in tension, and all the rest in compression.

44. Figure 9a* represents one half of a locomotive shed. The external forces are the weights 1, 2, 3, 4, 5 applied at

Fig. 9a.

the upper joints of the frame, and the reactions 6 and 7 of the wall and column. Again, all the external forces are parallel, and consequently the polygon of forces reduces, in diagram *b*, to one straight line. The force 6 (taken in the opposite sense to that in which it really acts) is equal to a certain part of the weight 5; by adding the difference to the other weights we get the magnitude of the force 7.

In the diagram *b* the direction of the lines 8 and 13 coincide; the first is a part of the second. Here then we have for the polygon corresponding to the joint (8, 10, 12, 13) one of those degenerate forms about which we spoke in Art. 33; the polygon is in fact a quadrilateral 8, 10, 12, 13, having three of its vertices (13, 8), (8, 10), (12, 13) in one straight line.

Fig. 9b.

The polygon 5, 17, 18, 6, corresponding to the point where

* This example is taken from Pl. xix. of the atlas of *Graphische Statik* of CULMANN, 1st edition. As previously stated, the two diagrams are not rigorously reciprocal.

158 EXAMPLES OF FRAME- AND STRESS- DIAGRAMS. [45-

the roof is supported by the wall, presents an analogous degenerate form, since the vertices (6, 5), (upper point of the segment (6)), (5, 17), and (18, 6) all lie in the same straight line.

The lower bars 8, 13, 18 are in compression, as well as the diagonals 10, 14, 16, the column 7 and the wall 6; while the upper pieces 9, 11, 15, 17 and the diagonal 12 are in tension.

45. Diagram *a* of Fig. 10 represents a truss at the upper joints of which are applied the oblique forces 1, 2, ..., 7, which

Fig. 10a. Fig. 10b.

may be considered as the resultants of the dead-loads and wind pressure; the forces 8, 9 represent the reactions at the supports.

The polygon of external forces is drawn in diagram *b* with double lines.

We construct successively the triangle 1, 10, 11, the quadrilateral 9, 10, 12, 13, the pentagon 2, 11, 12, 14, 15, the quadrilateral 13, 14, 16, 17, the crossed pentagon 3, 15, 16, 18, 19; the crossed quadrilateral 4, 19, 20, 21, the pentagon 17, 18, 20, 22, 23, and so on.

The upper bars 15, 19, 21, 25 are in compression, as well as the lower bars 10, 13, 30, and the verticals 12, 16, 24, 28; whilst all the remaining bars of the structure are in tension.

46. The diagram *a* of Fig. 11 represents a suspension bridge, loaded at each of its upper joints with weights 1, 2, ..., 8, and at each of its lower joints with weights 10, 11, 21, ..., 16; the weights are kept in equilibrium by the two

-46] EXAMPLES OF FRAME- AND STRESS- DIAGRAMS. 159

oblique reactions 9, 17 at the two extreme points of the structure*.

The polygon of external forces has its first eight sides in succession along the same vertical straight line, and its seven

Fig. 11a.

last sides situated in another vertical straight line. The oblique sides 9 and 17 intersect, so that the polygon is a crossed one. We construct successively the polygons 1, 17, 19, 18; 16, 19, 20, 21; 2, 18, 20, 22, 23; 15, 21, 22, 24, 25; 3, 23, 24, 26, 27; and so on; most of which are crossed.

Diagram *b* shows that the upper bars are all in tension, and that the tension decreases from the ends towards the

Fig. 11b.

middle of the structure; the bars of the lower boom are also all in tension, but in them the tension decreases from the middle towards the ends.

The extreme diagonals and contra-diagonals are in tension; in the portion situated to the left of the axis of symmetry, the diagonals or braces are alternately in tension and compression; similarly they are on the right but in the reverse order. Considering separately the ties and struts, we see that the internal

* This example is analogous to one of those studied by Maxwell in his memoir of 1870.

Figure 3. Pages from Le figure reciproche nella statica grafica by Cremona [12]

By the end of the 20th century, the use of reinforced concrete had become widespread in structural construction, introducing load-bearing elements subjected to complex stress states. Since the method of graphic statics is particularly well-suited for analyzing structures composed of linear elements carrying only axial forces, the design and analysis of reinforced concrete structures increasingly shifted toward numerical methods based on linear algebra.

2.2 Computer implementation of graphic statics

The growing interest in research within the field of graphic statics has been driven primarily by new methods of “research through design,” as well as by the possibilities that emerge when graphic statics is combined with advanced software for algorithmic and computer-aided design. Three-dimensional modeling tools used in architectural design have enabled the exploration of structural design through three-dimensional graphic statics in ways that are not possible with two-dimensional manual or computer-based diagram drawing.

The inherently parametric nature of graphic statics was first implemented through the development of two-dimensional software applications such as *ActiveStatics* [14], *InteractiveTHRUST* [15], and *eQuilibrium* [16], which use parametric software to construct interactive drawings. These implementations allow real-time interaction and visualization but require prior programming and a good understanding of graphic statics. Moreover, modifying the software topology of an existing diagram is complex, which limits the exploration of new architectural forms and often necessitates a complete reconstruction of the interactive diagram.

The transition from two-dimensional to three-dimensional implementation of graphic statics began with RhinoVAULT [17], which is based on thrust network analysis and implemented as a plug-in for the parametric modeling software Rhinoceros3D [18]. This tool enables the interactive design of freeform shells using reciprocal diagrams, by applying a thrust network that represents the spatial equilibrium of compressive forces, Figure 4. RhinoVAULT manipulates the three-dimensional form by using horizontal projections of discretized shells to construct interactive two-dimensional form and force diagrams in planar view, and in this sense does not constitute a true implementation of three-dimensional graphic statics.

The software implementations based on two-dimensional graphic statics provided a critical foundation for the conceptual and computational transition toward true three-dimensional graphic statics. By addressing their limitations and extending their capabilities, later developments introduced methods that operate entirely in three dimensions, both in geometric representation and in static equilibrium.

2.3 Three-dimensional graphic statics

The modeling capabilities integrated into today’s parametric software have enabled contemporary research in graphic statics to focus on its extension into three dimensions, employing a range of approaches and methods. Within this context, the methodologies are commonly categorized into three main approaches: projective, composite, and fully three-dimensional [20]. Two principal methods emerge within the fully three-dimensional category: *the vector-based method* and *the polyhedral-based method*.

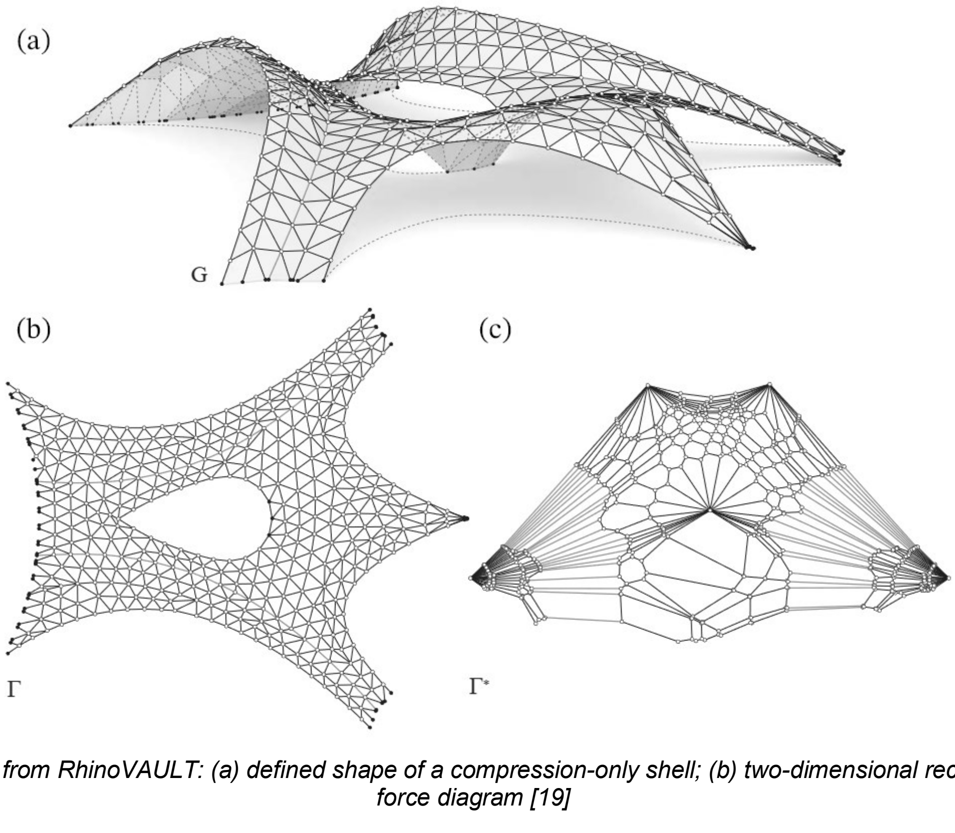


Figure 4. Output from RhinoVAULT: (a) defined shape of a compression-only shell; (b) two-dimensional reciprocal form; (c) force diagram [19]

Vector-based three-dimensional graphic statics resolves spatial force equilibrium using weighted vectors [21], particularly through so-called *“Cremona reciprocal diagrams*” [22] or *“three-dimensional Cremona diagrams”* [23]. While determining equilibrium at a single node using vectors is relatively straightforward, constructing a complete force diagram for a three-dimensional structure often results in ambiguous, overlapping vectors and a force diagram that is no longer reciprocal to the form diagram. To establish a parallel relationship between the spatial form and the force diagram [24], additional numerical computations are required, thereby diminishing the intuitive and transparent qualities traditionally associated with graphic statics, Figure 5.

The vector-based approach to three-dimensional graphic statics is founded on the argument that it remains consistent with the original techniques of two-dimensional graphic statics by using vector lengths to represent force magnitudes; however, since vectors in three-dimensional space do not lie in a single plane, their true lengths often become difficult to interpret visually, which undermines the clarity characteristic of planar graphic statics.

Polyhedral-based three-dimensional graphic statics, which represents force equilibrium through polyhedral geometries, builds upon the foundational work of Rankine [26] and Maxwell [10]. As early as 1864, in his paper titled *Principle of the Equilibrium of Polyhedral Frames* [26], Rankine proposed the possibility of a reciprocal relationship between structural form and force transmission in three-dimensional space. Maxwell subsequently developed a geometric procedure for constructing three-dimensional reciprocal diagrams to express this idea. Although both Rankine and Maxwell use the term frame in their writings, McRobie [23] clarifies that their analyses pertain to truss-like structures in which elements carry only axial forces. This definition differs significantly from the contemporary use of the term frame, which refers to rigid structures capable of transmitting all internal forces, including bending moments.

In two-dimensional graphic statics, the axial force in a truss element is represented by the length of the corresponding edge in the force diagram. In contrast, Maxwell and Rankine’s method uses the areas and orientations of faces in a polyhedral force diagram to represent the directions and magnitudes of forces in a three-dimensional truss. This approach enables the construction of a reciprocal 3D force diagram for a polyhedral truss in static equilibrium, composed of closed polyhedral cells, each representing force equilibrium at a specific truss node, Figure 6.

Akbarzadeh [27], [28] clarified the proposals of Maxwell and Rankine through three-dimensional diagrams and visualizations, combining their foundational ideas with the capabilities and advantages of contemporary parametric software. Their more concrete implementation has been realized through additional plug-ins for Rhinoceros3D [18], such as PolyFrame [29] and 3DGS [30], which provide practical implementations of reciprocal force and form diagrams. The most comprehensive computational framework based on three-dimensional graphic statics using polyhedral force diagrams was developed by Lee [31], who

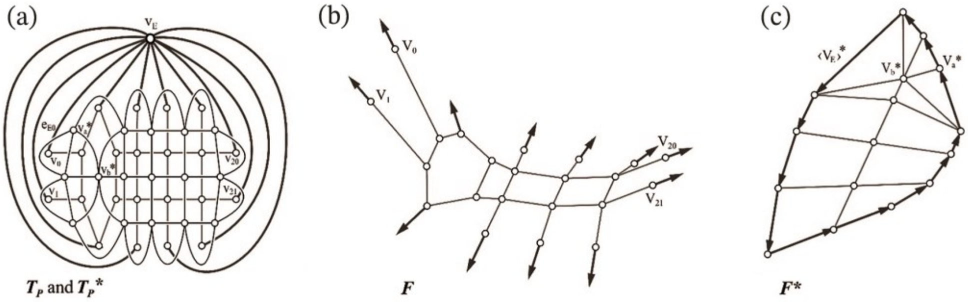


Figure 5. Vector-based three-dimensional graphic statics: (a) planar force diagram and its projection (superimposed); (b) form diagram; (c) three-dimensional force diagram [25]

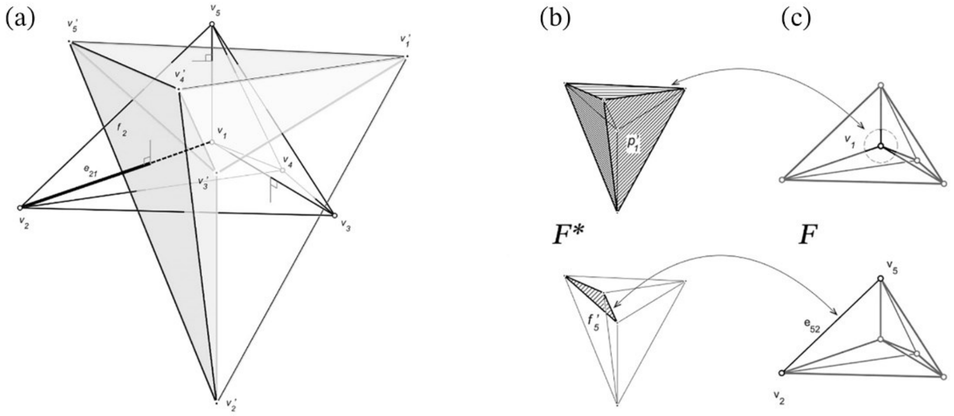


Figure 6. Polyhedral three-dimensional graphic statics: (a) force diagram and form diagram (superimposed); (b) force diagram; (c) form diagram [27]

established a generalized theoretical foundation supported by a data structure formulation to address a wide range of equilibrium problems, including spatial force systems.

In recent years, numerous applications have been developed for design using three-dimensional graphic statics. These include methods based on iterative subdivision of force polyhedral [32], [33], testing of spatial models designed to carry only compressive forces [34], and the full materialization of structures created with 3D graphic statics [35]. In addition to design and form-focused research, McRobie and Konstantatou [36] established a generalized theoretical framework for three-dimensional graphic statics by linking it to fundamental principles of structural engineering. Their research introduces n-dimensional reciprocal construction methods through projective geometry [37], applies Minkowski sum diagrams to interpret Maxwell's load-path theorem [23], addresses the limitations of Rankine's reciprocity, and discusses the generalization of three-dimensional graphic statics for arbitrary 3D trusses [23], [36].

3 Form-finding methods

Form-finding methods represent an advanced process in which parameters are explicitly controlled to define an optimal geometry of a structure that is in static equilibrium under the loads anticipated during the design process [38]. In this context, the *optimal geometry* of the structure refers to the definition of a shape or configuration that most effectively receives and redistributes the applied loads, maximizing the capacity of the cross-section of the structural elements while minimizing material usage. This concept includes the design of structures that follow the natural path of axial force transmission — either in compression or tension — while avoiding the occurrence of bending moments and shear forces. Structural forms that align with the forces they are intended to carry and transmit are called funicular forms. In funicular forms, the stress across the height of the cross-section is evenly distributed, reducing the likelihood of stress concentrations and the formation of weak points that could lead to structural failure.

The earliest form-finding methods are based on Hooke's hanging chain principle [39], where by mapping the shape of a hanging chain, by definition subjected only to tensile forces, an equivalent arch shape under compressive forces is defined. The resulting shape of the chain illustrates the transmission of compressive forces in the arch, effectively defining the thrust line [40]. Hooke's analogy is fundamentally linked to the origins of graphic statics through the investigations of Varignon, in which the term funicular polygon was introduced, derived from the Latin word funiculus, meaning cable or rope. Varignon proposed a graphical method for constructing the shape of a suspended, inextensible rope with attached weights, thereby laying the foundation for the use of geometric methods in the analysis of static equilibrium, Figure 7.

Hooke's hanging chain principle defines the shape of constant-thickness arches subjected solely to their self-weight. Varignon expanded upon this principle by introducing the influence of varying loads on the suspended rope. In doing so, he connected this shape to the concept of the force diagram, demonstrating that the geometry of the funicular polygon is directly dependent on the load distribution [4].

For a rope fixed at its endpoints and loaded with vertical forces of varying intensity applied at discrete points, a corresponding funicular polygon is formed, for which a force diagram is constructed to represent the equilibrium at each node. Specifically, in the funicular polygon, each of the individual triangles represents the force equilibrium of the corresponding node, while the length of each side of the triangle corresponds to the magnitude of the forces acting at that node. Each funicular polygon thus achieves a unique state of static equilibrium for the given boundary conditions, geometry, and loading configuration.

3.1 Computationally implemented funicular form-finding

The definition of a funicular form, which in the design process may be interpreted as a geometric constraint limiting the forms to those that carry only compressive forces, does not necessarily represent a limiting factor in the creative design process, but can instead serve as a driver that focuses the exploration of the design space [41]. However, the interdependence between different domains of form-related constraints is difficult to perceive, considering that the designer must understand and relate formal, external, and practical constraints [42]. Recognizing the relationship among mutually competing constraints, as well as finding solutions within their framework, constitutes a complex methodological problem, the resolution of which requires the

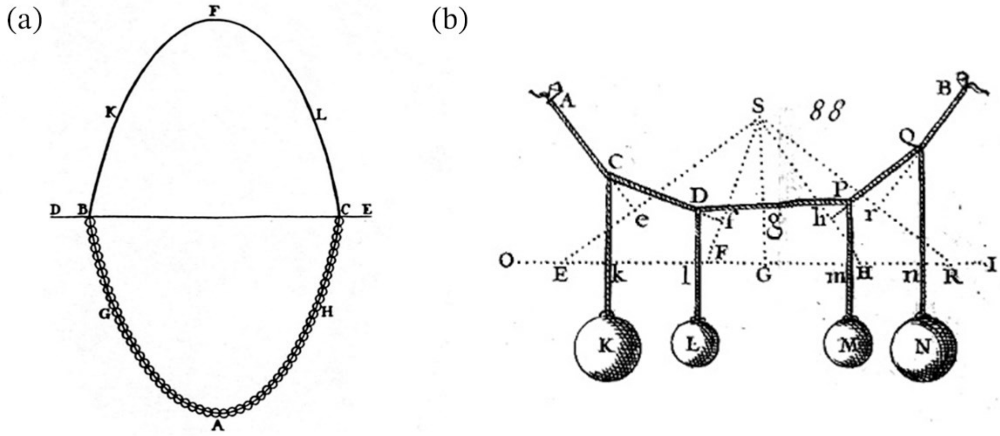


Figure 7. Form-finding of arches: (a) The analogy between an arch and a hanging chain by Hooke (drawing by Poleni [40]); (b) Illustrations by Varignon of funicular polygons and their corresponding force diagrams [4]

application of computationally implemented form-finding methods. Namely, without a computer-implemented method, the architectural designer must possess excellent technical knowledge and understanding of the structural constraints of funicular forms in order to approach the resolution of the design problem.

Modern computationally implemented methods for defining funicular forms are generally classified into three groups [38]:

(1) Stiffness matrix methods, used for structural analysis and applied within the framework of the finite element method (FEM). These are used in the design of various types of structures, where calculations include different geometric and material properties. The application of stiffness matrices allows for the accurate calculation of deformations and stresses, which are then used for further structural optimization.

(2) Dynamic equilibrium methods, which solve the problem of dynamic equilibrium by defining a stable state equivalent to static equilibrium. These methods include dynamic relaxation as well as spring–particle systems.

(3) Geometric stiffness methods, which are independent of the material and include three-dimensional graphic statics and the force density method, in which the ratio between force and element length represents the fundamental unit upon which the calculations are based. All subsequent methods are, in fact, generalizations or extensions of the force density method in combination with three-dimensional graphic statics, most notably reflected through the method of Thrust Network Analysis.

Given that contemporary 3D modeling software enables more precise and faster generation of forms, as well as easy and rapid modification of the model, today the digital implementation of form-finding methods results in increased control over the design process. This allows designers to experiment with different geometric configurations and to optimize the structure to minimize material costs and improve structural efficiency. Simultaneously, digital simulation tools allow for quick adaptation of the project to new conditions or requirements, thereby accelerating the entire design process. Digital implementation not only increases efficiency but also enables greater innovation and creativity by revealing new formal possibilities within the design process.

The application of the finite element method for form-finding is enabled through the use of genetic algorithms, which rely on Darwinian theory of natural selection in an iterative process to select optimal solutions from a random set of potential solutions, combining them to achieve a better solution in the next iteration [43], [44]. The role of the designer in such a process is limited, considering that the final solution results from a large number of steps of numerical iteration.

Dynamic equilibrium methods, such as dynamic relaxation and spring–particle systems, reformulate the problem as a dynamic simulation of physical behavior through different approaches. Dynamic relaxation does not explicitly use particles or springs, but achieves equilibrium by simulating energy dissipation over discrete time steps, while spring–particle systems model interactions through springs connecting discrete particles, solving equilibrium by evaluating internal forces. While both are effective for form-finding, they offer limited control and transparency for the designer, as the internal computations are often unclear and the method primarily yields the final equilibrium form without insight into intermediate force distributions or structural logic.

Geometric stiffness methods offer a fundamentally different approach by focusing on equilibrium as a geometric rather than material-dependent problem. These methods, including three-dimensional graphic statics and the force density method, allow for form-finding that is independent of predefined material properties, making them particularly suitable in the early stages of design. By treating force as a geometric quantity—often defined through the ratio of force magnitude to element length—they enable direct control over the structural logic and allow designers to intuitively manipulate equilibrium configurations. Their visual and reciprocal nature provides greater transparency in the design process, fostering an informed exploration of structurally expressive forms. As such, they support a more interactive and informed role for the designer, bridging conceptual exploration and structural feasibility.

4 Graphic statics as a method for funicular form-finding

Graphic statics, as one of the geometric stiffness methods, stands out from other methods in this category by offering exceptional transparency in the relationship between form and force diagram. The magnitude of a force, represented simply by the length of the corresponding line in the force diagram, is parallel to the line in the form diagram, allowing the designer to observe, through their interdependence, the effects of changes in the form diagram resulting from modifications in force, and vice versa. A key advantage of three-dimensional graphic statics is that it operates fully in three dimensions and the designer is able to impose geometric constraints by defining specific parts of the force diagram, thereby controlling the distribution of internal forces. With these inherent properties, three-dimensional graphic statics proves to be a particularly useful method for form-finding in the early stages of the design process.

The digital implementation of form-finding methods in general, and the implementation of three-dimensional graphic statics specifically, emerges within a broader landscape of computational design technologies and stands in contrast to earlier digital approaches that prioritized surface complexity over structural clarity. The early wave of digital architecture, particularly in the first decade of the 2000s, was marked by a fascination with smooth, continuous forms made possible through parametric design tools and NURBS-based modeling environments. This formal language, driven by algorithms and scripting, enabled highly expressive geometries but often did so at the expense of material rationality and structural clarity. While these approaches were technologically sophisticated, they frequently resulted in high-cost architectural solutions that were difficult to fabricate, energy-intensive to construct, and accessible only to a limited number of well-resourced practices. The emphasis on visual complexity and stylistic innovation often obscured the physical realities of construction, producing buildings that, although formally ambitious, were disconnected from broader economic, social, and environmental considerations.

In contrast, graphic statics, particularly as it has been revitalized through digital tools, offers a fundamentally different paradigm. Rooted in equilibrium, geometry, and transparency, graphic statics allows for a direct and intelligible relationship between form and structural performance. It enables designers to work with forces as tangible design parameters, supporting intuitive and materially grounded exploration. Unlike the opacity of many parametric systems, graphic statics encourages an accessible and visually legible design process that foregrounds structural efficiency, clarity, and constructability. In doing so, it provides not only an alternative aesthetic sensibility but also a more inclusive and responsible approach to digital design, that reasserts the value of constraint, physical logic, and comprehension in the shaping of architectural form.

4.1 Comparison of graphic statics with other form-finding methods

Stiffness matrix methods, such as finite element modeling combined with genetic algorithms, treat form-finding as an optimization problem, where a structural objective, such as minimizing material use or displacement, is achieved through iterative simulation and evolutionary techniques. While powerful and applicable to a wide range of structural types and constraints, these methods often require significant computational resources and abstract the relationship between form and force, which is a central and transparent feature of graphic statics. Additionally, in the early stages of design, the material to be used for the construction is most often unknown, which means that computational methods for form-finding that depend on the properties of the material used, such as FEM-based methods and optimization techniques, are not viable approaches.

Unlike computational techniques such as dynamic relaxation or spring-particle systems, graphic statics offers an explicit and visually intuitive correspondence between geometry and static forces, making it particularly suitable for design-oriented exploration. In contrast, dynamic relaxation and spring-particle systems are iterative numerical methods that compute equilibrium configurations by simulating physical systems with simplified mechanics: dynamic relaxation treats structures as systems of masses and springs subject to iterative force balancing. Although both are computationally efficient and compatible with complex geometries, they often lack the visual clarity and immediacy of graphic statics, which enables a direct reading of internal forces through scaled vector diagrams.

The force density method is a numerical form-finding technique that simplifies the equilibrium equations of a network by expressing them in terms of force densities: defined as the ratio between axial force and member length. Compared to graphic statics, which relies on reciprocal geometric constructions to represent force equilibrium, the force density method abstracts the form-finding process into a matrix-based algebraic system, making it highly suitable for implementation in digital environments, however, it lacks the explicit visual correspondence between force and geometry that characterizes graphic statics. Additionally, because the force density method assumes fixed nodal connectivity and predefined force directions, it offers limited flexibility in exploring new topologies or compression-dominant forms, which are central to the application of graphic statics in the design of funicular structures.

Thrust network analysis combines geometric and numerical approaches by extending the principles of graphic statics into three dimensions using force and form diagrams to generate funicular networks, typically for compression-only structures such as masonry shells. However, unlike traditional graphic statics, thrust network analysis is implemented computationally and often relies on planar projections and dual graphs, which can obscure the full spatial behavior of the system.

4.2 Advantages of computationally implemented graphic statics

The recent interest and progress in the field of graphic statics are largely due to its computational implementation, which opens up a range of possibilities for its application as a method for funicular form-finding. The implementation of form-finding methods in a usable and repeatable manner through software that allows for their independent use has been discussed by Witt [45], who notes that such implementations are most often realized through the concept of the black box, where the internal guiding process remains unknown to the user. In computer engineering, the black box concept refers to a system of input and output data without the user's ability to control the computational process, assuming that the function is appropriate for the intended purpose and does not require examination [46]. To a certain extent, the black box concept can be made more transparent by allowing control over the input and output data [47], which largely depends on the transparency of the implemented method.

The computational implementation of form-finding methods, such as the force density method, dynamic relaxation, or the spring–particle system, based on user feedback regarding the relationship between given input requirements and resulting outputs, creates a component equivalent to the black box concept, limiting the ability to intuitively expand and develop structural knowledge. By not providing an adequate level of control and insight into the underlying form-finding methodology, primarily due to the nature of their implementation, these methods have limited applicability as design tools. In particular, the absence of an explicit link between the structural system's form and its internal forces hinders the identification of effective parameters needed during the form-finding process, generating complex mathematical concepts without enabling intuitive application by architects and engineers in the design process. Although tools developed through the computational implementation of these methods are flexible and versatile depending on the design problem at hand, they do not provide clear guidance to the designer, who must instead rely on a trial-and-error approach. Despite offering possibilities for generating different form definitions, the underlying structural logic remains difficult to grasp, thereby limiting the design process to controlling predefined shapes and conducting their subsequent analysis.

Given that the method of three-dimensional graphic statics is a geometric one, its implementation in computational software enables a highly transparent design process. This facilitates a deeper understanding of the relationship between the structural form and the distribution of internal forces, reducing or avoiding the shortcomings associated with the black box concept. Unlike other form-finding methods, the algorithms in three-dimensional graphic statics establish a direct connection between the force diagram and the form diagram, automating the complex process of manual diagram construction without relying on numerical calculations. In doing so, they expose fundamental aspects of the underlying methodology through the use of three-dimensional models in virtual space. As a result, a designer who understands the form-generation procedure does not rely on opaque numerical routines, thereby reducing the risk of uncritical acceptance of numerical models and outputs. The application of graphic statics is thus closely linked to the understanding of its core principles, which are inherently connected to understanding the design process and its constraints as a whole, an essential factor for developing innovative design solutions.

5 Research outlook and identified gaps

The analysis of existing research indicates that graphic statics, no longer limited to two-dimensional analysis, can be extended into three-dimensional space through its implementation in computer-aided design software. This expansion has opened new possibilities for designing complex spatial structures, enabling designers to visualize and manipulate both form and force in three dimensions. However, moving into spatial modeling increases the method's inherent complexity and the complexity of the generated diagrams, while also reducing intuitiveness in the design process. The geometric relationships between polyhedral cells and spatial boundaries become increasingly difficult to manage, especially when the diagrams grow in scale or deviate from idealized configurations. Although certain repetitive parts of the diagram construction process, in both force and form diagrams, are intended to be automated by combining graphical algorithms with computational modeling and solution strategies, one of the main limitations of three-dimensional graphic statics remains the lack of concrete, generalizable methods for modeling polyhedral force diagrams. In particular, there is a lack of consensus on how to algorithmically define the geometries of reciprocal diagrams under complex load cases.

Despite the establishment of a significant portion of the theoretical foundation for polyhedral 3D graphic statics, there is still a shortage of research focused on defining the specific geometry of the force diagram for various loading conditions, as well as clear strategies for its modeling, control, and interpretation. Much of the current work remains focused on highly symmetrical or idealized structures, often assuming vertical loading and homogeneous boundary conditions. The most recent advancements in the field of three-dimensional graphic statics, particularly when combined with sophisticated optimization strategies, are primarily focused on the design and analysis of compression-only structures, specifically on defining their optimal funicular form. However, these optimization-driven approaches often prioritize output generation over spatial legibility, limiting the designer's ability to critically engage with the internal force flows. Given the large number of possible variations of the force diagram—each generating a different three-dimensional solution for a given set of loads and boundary conditions—its usefulness becomes limited if the designer lacks a clear strategy for its interpretation and construction. In such cases, the design process risks becoming an exercise in formal rationalization, applying geometric concepts without genuine exploration based on an understanding and intentional shaping of the structural form and force distribution. Moreover, as diagrams become more abstract and computationally generated, their legibility to non-expert users diminishes, reducing the pedagogical and intuitive potential that originally defined graphic statics as a design tool. Even when integrated within an interactive design environment, such approaches often fail to fully exploit the potential of the intuitive, graphical nature of the method through the use of form and force diagrams.

The review of the available literature also leads to the conclusion that previous research has not been focused on determining the three-dimensional form of force diagrams under specific loading scenarios that include the influence of horizontal forces, or combinations of horizontal and vertical forces, and their graphical interpretation. This omission is significant, as horizontal forces—particularly those arising from wind or seismic activity—play a critical role in the design of compression-dominant structures such as vaults, shells, and spatial frameworks. Without methods to accurately model and manipulate force diagrams under these conditions, the full potential of three-dimensional graphic statics as a generative design tool remains underutilized. Therefore, in order to expand the application of polyhedral force diagrams beyond their current use as tools for visualization or verification of spatial equilibrium, it is necessary to develop methods that allow for appropriate modeling and manipulation of such diagrams under varying load conditions. This includes not only defining geometrically consistent force diagrams but also exploring how changes in loading, boundary conditions, or form constraints influence their reciprocal counterparts.

It can be concluded that there is a need to explore the potential for modeling three-dimensional force diagrams by linking them to established methods for funicular form-finding, where the form is understood as the optimal geometry for force transmission. Such integration would not only fill existing gaps in the methodology but also unlock new opportunities for intuitive, performance-driven design in complex spatial structures—pushing the boundaries of current computational form-finding techniques toward a more informed and transparent design logic.

CRediT authorship contribution statement

According to the Contributor Role Taxonomy (CRediT) guidelines, the authors contributed to this work as follows:

- Teodora Mihajlovska: Writing – Original Draft Preparation; Conceptualization; Methodology; Formal Analysis; Data curation; Visualization
- Ana Trombeva-Gavriloska: Writing – Review & Editing; Methodology; Supervision
- Meri Cvetkovska: Supervision
- Liljana Dimevska Sofronievska: Data curation; Visualization

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] K.-E. Kurrer, *The history of the theory of structures: from arch analysis to computational mechanics*. Berlin: Ernst & Sohn, 2008.

[2] E. Scholz, *Symmetrie, Gruppe, Dualität*. Basel: Birkhäuser Basel, 1989. doi: 10.1007/978-3-0348-9267-4.

[3] S. Stevin, *De beghinselen der weeghconst*. Raphelengius, Franciscus, 1586. Accessed: Jul. 16, 2024. [Online]. Available: http://archive.org/details/ned-kbn-all-00011058-001

[4] P. Varignon, *Nouvelle mecanique ou statique*. in Nouvelle mecanique ou statique: Dont le projet fut donné en M. DC. LXXXVII. chez Claude Jombert, 1725.

[Online]. Available: https://books.google.mk/books?id=VOoamm3BKs8C

[5] J. V. Poncelet, *Traité des propriétés projectives des figures : ouvrage utile à ceux qui s'occupent des applications de la géométrie descriptive et d'opérations géométriques sur le terrain*. Bachelier, Libraire, 1822. doi: 10.3931/E-RARA-55268.

[6] G. Lamé and E. Clapeyron, "Mémoire sur la construction des polygones funiculaires," *J. Génie Civ.*, vol. I, pp. 496-504., 1828.

[7] W. J. M. Rankine, *A manual of applied mechanics*. London: Griffin, 1858.

[8] K. Culmann, *Die graphische Statik*. Meyer und Zeller, 1864. [Online]. Available: https://books.google.mk/books?id=-HiztAEACAAJ

[9] J. C. Maxwell, "XLV. On reciprocal figures and diagrams of forces," *Lond. Edinb. Dublin Philos. Mag. J. Sci.*, vol. 27, no. 182, pp. 250–261, Apr. 1864, doi: 10.1080/14786446408643663.

[10] J. C. Maxwell, "On Reciprocal Figures, Frames, and Diagrams of Forces," *Trans. R. Soc. Edinb.*, vol. 26, no. 1, pp. 1–40, 1870, doi: 10.1017/S0080456800026351.

[11] L. Cremona, *Graphical Statics: Two Treatises on the Graphical Calculus and Reciprocal Figures in Graphical ... - Scholar's Choice Edition*. Oxford: Clarendon Press., 1890.

[12] L. Cremona, *Le figure reciproche nella statica grafica del prof. L. Cremona*. Tipografia di Giuseppe Bernardoni, 1872. [Online]. Available: https://books.google.mk/books?id=L07MKlwNulsC

[13] W. S. Wolfe, *Graphical Analysis: A Text Book on Graphic Statics*. New York: McGraw-Hill Book Co. Inc., 1921.

[14] S. Greenwold and E. Allen, "Active Statics." Accessed: May 07, 2024. [Online]. Available: https://acg.media.mit.edu/people/simong/statics/Start.html

[15] P. Block, "Equilibrium systems Studies in Masonry Structure," Master's, Massachusetts Institute of Technology., 2005.

[16] Block Research Group, "eQUILIBRIUM." Accessed: May 07, 2024. [Online]. Available: https://block.arch.ethz.ch/eq/

[17] M. Rippmann, L. Lachauer, and P. Block, "Interactive Vault Design," *Int. J. Space Struct.*, vol. 27, no. 4, pp. 219–230, Dec. 2012, doi: 10.1260/0266-3511.27.4.219.

[18] R. McNeel, *Rhinoceros 3D, Version 7.0*. (2010).

[19] M. Rippmann, "Funicular Shell Design: Geometric approaches to form finding and fabrication of discrete funicular structures," ETH Zurich, 2016. doi: 10.3929/ETHZ-A-010656780.

[20] J.-P. Jasienski, C. Fivet, and D. Zastavni, *Various perspectives on the extension of graphic statics to the third dimension*. 2014.

[21] A. Föppl, *Das fachwerk im raum*. DOGMA. in Europäischer Hochschulverlag GmbH KG, 1982. [Online]. Available: https://books.google.mk/books?id=v7GNKBfn9_0C

[22] H. Crapo, "Structural rigidity," 1979, [Online]. Available: http://hdl.handle.net/2099/521

[23] A. McRobie, "Maxwell and Rankine reciprocal diagrams via Minkowski sums for two-dimensional and three-dimensional trusses under load," *Int. J. Space Struct.*, vol. 31, no. 2–4, pp. 203–216, Jun. 2016, doi: 10.1177/0266351116660800.

[24] P. D'Acunto, J.-P. Jasienski, P. O. Ohlbrock, C. Fivet, J. Schwartz, and D. Zastavni, "Vector-based 3D graphic statics: A framework for the design of spatial structures based on the relation between form and forces," *Int. J. Solids Struct.*, vol. 167, pp. 58–70, Aug. 2019, doi: 10.1016/j.ijsolstr.2019.02.008.

[25] P. D'Acunto, "Structural Folding for Architectural Design: A New Approach to the Design of Folded Plate Structures in Architecture," ETH Zurich, 2018. doi: 10.3929/ETHZ-B-000337742.

[26] W. J. M. Rankine, "XVII. Principle of the equilibrium of polyhedral frames," *Philos. Mag. Ser. 1*, vol. 27, pp. 92–92, 1864.

[27] M. Akbarzadeh, "3D Graphical Statics Using Reciprocal Polyhedral Diagrams," ETH Zurich, 2016. doi: 10.3929/ETHZ-A-010867338.

[28] M. Akbarzadeh, T. Mele, and P. Block, *3D Graphic Statics: Geometric Construction of Global Equilibrium*. 2015.

[29] A. Nejur and M. Akbarzadeh, "PolyFrame, Efficient Computation for 3D Graphic Statics," *Comput.-Aided Des.*, vol. 134, p. 103003, May 2021, doi: 10.1016/j.cad.2021.103003.

[30] J. Milošević and O. Graovac, "An Approach to Designing Architectural Structures Using 3D Graphic Statics," in *Advances in Architectural Geometry 2023*, K. Dörfler, J. Knippers, A. Menges, S. Parascho, H. Pottmann, and T. Wortmann, Eds., De Gruyter, 2023, pp. 427–440. doi: 10.1515/9783111162683-032.

[31] J. Lee, "Computational Design Framework for 3D Graphic Statics," ETH Zurich, 2018. doi: 10.3929/ETHZ-B-000331210.

[32] M. Akbarzadeh, T. V. Mele, and P. Block, "Compression-only Form finding through Finite Subdivision of the Force Polygon," 2014.

[33] A. Ghomi, M. Bolhassani, A. Nejur, and M. Akbarzadeh, "The Effect of Subdivision of Force Diagrams on the Local Buckling, Load-Path and Material Use of Founded Forms," Jun. 2018.

[34] M. Bolhassani, M. Akbarzadeh, M. Mahnia, and R. Taherian, "On Structural Behavior of a Funicular Concrete Polyhedral Frame Designed by 3D Graphic Statics," *Structures*, vol. 14, pp. 56–68, Jun. 2018, doi: 10.1016/j.istruc.2018.02.002.

[35] M. Bolhassani *et al.*, "Structural Behavior of a Cast-in-Place Funicular Polyhedral Concrete : Applied 3 D Graphic Statics," 2018.

[36] A. McRobie, "The geometry of structural equilibrium," *R. Soc. Open Sci.*, vol. 4, no. 3, p. 160759, Mar. 2017, doi: 10.1098/rsos.160759.

[37] M. Konstantatou, P. D'Acunto, and A. McRobie, "Polarities in structural analysis and design: n-dimensional graphic statics and structural transformations," *Int. J. Solids Struct.*, vol. 152–153, pp. 272–293, Nov. 2018, doi: 10.1016/j.ijsolstr.2018.07.003.

[38] S. Adriaenssens, P. Block, D. Veenendaal, and C. Williams, Eds., *Shell structures for architecture: form finding and optimization*. London ; New York: Routledge/ Taylor & Francis Group, 2014.

[39] R. Hooke, *A description of helioscopes, and some other instruments*. London: printed by T.R. for John Martyn, 1676. doi: 10.3931/e-rara-2171.

[40] G. Poleni, *Memorie istoriche della gran cupola del tempio vaticano, e de'danni di essa, e de'ristoramenti loro, divise in libri cinque ...* Stamperia del Seminario, 1748. doi: 10.3931/E-RARA-13403.

[41] A. Kilian, “Design exploration through bidirectional modeling of constraints,” Thesis, Massachusetts Institute of Technology, 2006. Accessed: Jul. 30, 2024. [Online]. Available: https://dspace.mit.edu/handle/1721.1/33803

[42] B. Lawson, *How designers think: the design process demystified*, Reprint. Amsterdam Heidelberg: Elsevier Architectural Press, 2010.

[43] K.-U. Bletzinger and E. Ramm, “Form finding of shells by structural optimization,” *Eng. Comput.*, vol. 9, no. 1, pp. 27–35, Mar. 1993, doi: 10.1007/BF01198251.

[44] K.-U. Bletzinger, R. Wüchner, F. Daoud, and N. Camprubí, “Computational methods for form finding and optimization of shells and membranes,” *Comput. Methods Appl. Mech. Eng.*, vol. 194, no. 30–33, pp. 3438–3452, Aug. 2005, doi: 10.1016/j.cma.2004.12.026.

[45] A. Witt, “A Machine Epistemology in Architecture, Encapsulated Knowledge and the Instrumentation of Design,” *Candide J. Archit. Knowl.*, vol. 3, no. 12, pp. 37–88, 2010.

[46] R. Glanville, “Black boxes,” *Cybern. Hum. Knowing*, vol. 16, no. 1–2, pp. 153–167, 2009.

[47] R. Glanville, “Inside every white box there are two black boxes trying to get out,” *Behav. Sci.*, vol. 27, no. 1, pp. 1–11, Jan. 1982, doi: 10.1002/bs.3830270102.