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ANALYTICAL CALCULATION OF THE POSITION LOOP GAIN FOR LINEAR MOTOR SERVO DRIVES FOR CNC MACHINE TOOLS

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ABSTRACT: *One of the most important factors which influence on the dynamical behavior of the linear motor servo drives for CNC machine tools is position loop gain or Kv factor. From the magnitude of the Kv-factor depends tracking or following error. In multi-axis contouring the following errors along the different axes may cause form deviations of the machined contours. Generally position loop gain Kv should be high for faster system response and higher accuracy, but the maximum gains allowable are limited due to undesirable oscillatory responses at high gains and low damping factor. Usually Kv factor is experimentally tuned on the already assembled machine tool. This paper presents a simple method for analytically calculation of the position loop gain Kv. The influence of nonlinearities was taken with the correction factor. Our investigations have proven that experimentally tuned Kv factor differs from analytically calculated Kv factor less than 10%, which is completely acceptable.*

KEYWORDS: *analytical calculation, CNC machine tools, linear motor, servo drives, position loop gain*

1 INTRODUCTION

The most important variable, which describes the behaviour of a position control loop for CNC machine tool linear motor servo drives, is position loop gain or Kv-factor. This is the ratio of the command velocity (feed rate) v to the position control deviation (following error, tracking error, lag) Δx (Gordon & Hillery, 2005; Brecher & Weck, 2021; Gross et all., 2006; Luo et all., 2005; Wang, 2004; Altintas et all., 2011; Zirn, 2008; López de Lacalle & Lamikiz, 2009; Soucek, 2004).

$$\mathrm{Kv}[\mathrm{s}^{-1}] = \frac{\mathrm{v}[\mathrm{mm/s}]}{\Delta \mathrm{x}[\mathrm{mm}]}$$

or

$$\mathrm{Kv}\left[\frac{\mathrm{m/min}}{\mathrm{mm}}\right] = \frac{\mathrm{v}[\mathrm{m/min}]}{\Delta \mathrm{x}[\mathrm{mm}]} \tag{1}$$

$$\mathrm{Kv}[\mathrm{s}^{-1}] = \frac{1000}{60}\cdot \mathrm{Kv}\left[\frac{\mathrm{m/min}}{\mathrm{mm}}\right] \tag{2}$$

From the magnitude of the Kv-factor depends tracking or following error. In multi-axis contouring the following errors along the different axes may cause form deviations of the machined contours. Generally position loop gain Kv should be high for faster system response and higher accuracy, but the maximum gains allowable are limited due to undesirable oscillatory responses at high gains and low damping factor. Usually Kv factor is experimentally tuned on the already assembled machine tool (Sanchis & Peñarrocha-Alós, 2022; Brecher & Weck, 2021; Haugen, 2012; Gross et all., 2006)

This paper presents approach for analytically calculation of the position loop gain Kv. A combined 4-th order digital-analog model of the position loop is presented. In order to ease the calculation, the 4-th order system is simplified with a second order model. With this approach it is very easy to calculate the Kv factor for necessary position loop damping. The difference of the replacement of the 4-th order system with second order system is presented with the simulation program MATLAB. Analytically calculated Kv factor is function of the nominal angular frequency ω and damping D of the feed drive electrical parts (motor and regulator), as well as sampling period T.

2 THE COMBINED DIGITAL-ANALOG MODEL OF THE LINEAR MOTOR SERVO DRIVE POSITION CONTROL LOOP AND ANALYTICAL CALCULATION OF THE KV FACTOR

Figure 1 presents digital-analog model of the CNC machine tool linear motor servo drive position control loop, where s represents Laplace operator.

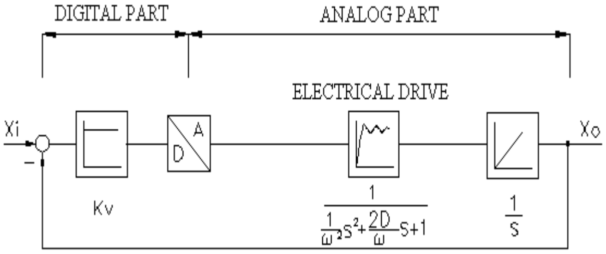

Fig.1 Combined digital-analog model of the linear motor servo drive position control loop

Similar models are presented in (Gross et all., 2006; Brecher & Weck, 2021; Altintas et all., 2011; Zirn, 2008; Soucek, 2004; Smith, 1999; Younkin, 1996).

Because of the existence of the digital part in the presented model we must use z-transformation for analysis. With some approximations and substitutions it is possible to analyze presented model in s-domain (with Laplace transformation). Digital-analog converter is substituted with zero order holder (z.o.h.) and sampler (Brecher & Weck, 2021; Gross et all., 2006; Altintas et all., 2011; Zirn, 2008; Soucek, 2004; Harder & Isaksson, 1995).

The new model is presented in Figure 2.

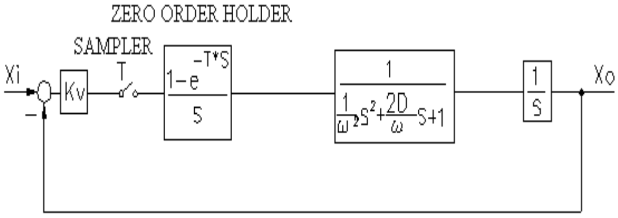

Fig. 2 Modified model of the linear motor servo drive position control loop presented in fig.1

According to (Brecher & Weck, 2021; Gross et all., 2006; Altintas et all., 2011; Zirn, 2008; Soucek, 2004; Harder & Isaksson, 1995).we can approximate sampler and zero order holder (z.o.h.) in Laplace domain with the following transfer function:

$$G(s) = \frac{1-e^{-T\cdot s}}{T\cdot s} \tag{3}$$

With the Padè approximation of the first order for the $e^{-T\cdot s}$ we get:

$$e^{-T\cdot s} \approx \frac{1-\frac{T}{2}\cdot s}{1+\frac{T}{2}\cdot s} \tag{4}$$

where T is sampling time (period).
In that case G(s) becomes:

$$G(s) = \frac{1-e^{-T\cdot s}}{T\cdot s} \approx \frac{1}{1+\frac{T}{2}\cdot s} \tag{5}$$

With these simplifications linear motor servo drive position control loop may be presented with following model (Figure3).

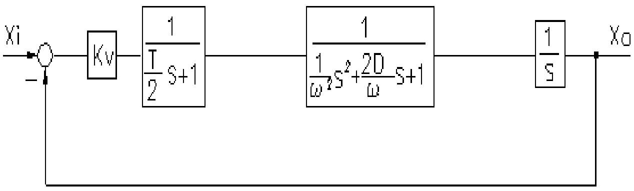

Fig. 3 Analog model of the linear motor servo drive position control loop

The model in Figure 3 may be analyzed in s-domain with Laplace transformation. The transfer function of the linear motor servo drive position control loop presented in Figure 3 is:

$$\frac{Xo(s)}{Xi(s)} = \frac{Kv\cdot\frac{1}{\frac{T}{2}\cdot s+1}\cdot\frac{1}{\frac{1}{\omega^2}\cdot s^2+\frac{2D}{\omega}\cdot s+1}\cdot\frac{1}{s}}{1+Kv\cdot\frac{1}{\frac{T}{2}\cdot s+1}\cdot\frac{1}{\frac{1}{\omega^2}\cdot s^2+\frac{2D}{\omega}\cdot s+1}\cdot\frac{1}{s}} \tag{6}$$

$$\frac{Xo(s)}{Xi(s)} = \frac{Kv}{\left(\frac{T}{2}\cdot s+1\right)\cdot\left(\frac{1}{\omega^2}\cdot s^2+\frac{2D}{\omega}\cdot s+1\right)\cdot\left(\frac{1}{s}\right)+Kv} \tag{7}$$

$$\frac{Xo(s)}{Xi(s)}=\frac{b_0}{a_4\cdot s^4+a_3\cdot s^3+a_2\cdot s^2+a_1\cdot s+a_0} \qquad (8)$$

$$a_4=\frac{T}{2\omega^2},\quad a_3=\frac{1}{\omega^2}+\frac{D}{\omega}\cdot T,\quad a_2=\frac{2D}{\omega}+\frac{T}{2},$$
$a_1=1,\ a_0=Kv$ and $b_0=Kv$.

Having information's about the magnitude of the variables ω, D, and T in real linear motor servo drive position control loops, we can conclude that a_4, a_3 tends towards zero ($a_4, a_3 \to 0$). So in that case we can simplify 4-th order system with the second order system (Kelly et all., 2021; Brecher & Weck, 2021; Gross et all., 2006; Altintas et all., 2011; Zirn, 2008; Soucek, 2004; Gao, 1999).

We will present linear motor servo drive position control loop with the simplified transfer function:

$$\frac{Xo(s)}{Xi(s)}=\frac{b_0}{a_2\cdot s^2+a_1\cdot s+a_0} \qquad (9)$$

where $a_2=\frac{2D}{\omega}+\frac{T}{2},\ a_1=1,\ a_0=Kv$ and $b_0=Kv$.

In that case

$$\frac{Xo(s)}{Xi(s)}=\frac{Kv}{\left(\frac{2D}{\omega}+\frac{T}{2}\right)\cdot s^2+s+Kv} \qquad (10)$$

To check if it is correct to substitute 4-th order with second order system, we will simulate the system transfer function response on step function with simulation program MATLAB. Numerical values of the parameters of the examined system are:

ω=1000 s^{-1}, D=0.7, T=0.001 s and Kv=166.67 s^{-1}.

Figure 4 gives responses of the position control loop transfer function of 4-th and 2-nd order on step function.

From Figure 4 it is obvious that the differences caused by substitution are minimal. It makes substitution completely acceptable. For the second order system it is possible very easy and fast to calculate Kv-factor for necessary position control loop damping.

We can write the second order system transfer function in the following form:

$$\frac{Xo(s)}{Xi(s)}=\frac{1}{\frac{1}{\omega_n^2}\cdot s^2+\frac{2\zeta}{\omega_n}\cdot s+1} \qquad (11)$$

wh

where ζ is position control loop damping (0< ζ <1), and ω_n is nominal angular frequency of the position control loop. We will transform Equation 10 in the form of Equation 11.

$$\frac{Xo(s)}{Xi(s)}=\frac{Kv}{\left(\frac{2D}{\omega}+\frac{T}{2}\right)\cdot s^2+s+Kv}=$$
$$=\frac{1}{\frac{\left(\frac{2D}{\omega}+\frac{T}{2}\right)}{Kv}\cdot s^2+\frac{1}{Kv}\cdot s+1} \qquad (12)$$

Comparing Equation 11 and Equation 12 we can obtain:

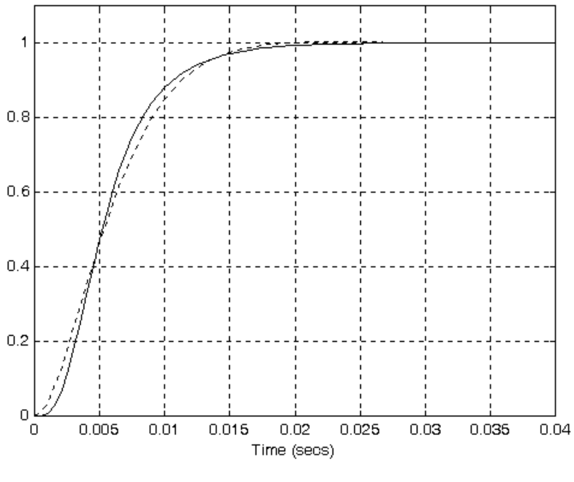

Fig. 4. (--------) Time response of the 4-th order system

(- - - - -) Time response of the 2-nd order system

$$\omega_n=\sqrt{\frac{Kv}{\frac{2D}{\omega}+\frac{T}{2}}} \quad \text{and}$$

$$\zeta=\frac{1}{2}\cdot\sqrt{\frac{1}{Kv\left(\frac{2D}{\omega}+\frac{T}{2}\right)}} \qquad (13)$$

In order to have required position control loop damping ζ, Kv-factor should be calculated with the following equation:

$$Kv = \frac{1}{4\zeta^2\left(\frac{2D}{\omega}+\frac{T}{2}\right)} \tag{14}$$

Due to the presence of nonlinearities in reality the value of Kv must be decreased up to 40% (Gross et all., 2006; Brecher & Weck, 2021; Altintas et all., 2011; Zirn, 2008; Soucek, 2004).

$$Kv = \frac{0.6}{4\zeta^2\left(\frac{2D}{\omega}+\frac{T}{2}\right)} \tag{15}$$

Equation 15 gives direct analytical relationship between Kv-factor and ω, D, T and ζ, which are already known variables, or can be calculated very easy.

With the Equation 15 it is possible to estimate CNC machine tool linear motor servo drive position loop gain Kv without performing experiments.

We checked the correctness of the Equation 15 on real linear motor servo drive position control loop of high speed cutting CNC linear motor machine (HSC 11) (Figure 5) developed on the Institute of Production Management, Technology and Machine Tools (PTW) at Technical University Darmstadt, Germany.

Fig. 5 High Speed Cutting CNC linear motor machine (HSC 11) at the Institute of Production Management, Technology and Machine Tools (PTW) at Technical University Darmstadt, Germany

Position loop damping ζ=0.7 is preferable according (Brecher & Weck, 2021; Gross et all., 2006; Altintas et all., 2011; Zirn, 2008; Soucek, 2004; Lyu et all, 2020; Zhen-Yuan Jia et all. 2018; Pandilov et all., 2015; Pandilov, 2022; Bullock & Younkin, 1995; Lei and Hsu, 2003; Younkin,1996; Weck et all., 2001)

That is the value, which gives minimal contouring errors. Other numerical values of the examined system are: $\omega=1000\,s^{-1}$, D=0.7, and T=0.001 s. With the substitution in the Eq.15 the position loop gain value Kv=157.89 s^{-1} is calculated. Experimentally tuned value of Kv-factor on examined machine tool axis was Kv=166.67 s^{-1}. The difference between analytically calculated and experimentally obtained value of Kv-factor is around 5.5%, which is completely acceptable.

3 CONCLUSION

The Eqation 15 enables very fast, simple and precise analytical calculation of position loop gain Kv as a function of already known position control loop parameters (ω-nominal angular frequency of the feed drive electrical parts, D-damping of the feed drive electrical parts, and T-sampling time). In that way we can avoid long-time experimental tuning of the Kv-factor on machine tool. And of course analytical calculation of the Kv factor gives possibility to estimate the accuracy of the system in the design phase.

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