COMPARISON OF 4D SURFACES DEFINED BY TWO AND THREE VARIABLES

Risto Tashevski [1]

[1]*Institute of MKMMV, Faculty of Mechanical Engineering,*
"Ss. Cyril and Methodius"University in Skopje,
Karpos II b.b., P.O. Box 464, 1000 Skopje, Macedonia
E-mail: risto.tashevski@mf.edu.mk

Abstract. The paper is a continuation of previous research on 4D surfaces defined by two variables. In this paper, a geometric, a mathematical and computer algorithm of projections of 4D surfaces defining with functions with tree variables are presented. Presentation of 4D surfaces are reduced of presentation of 4D points in 4D geometric space, points are transformed in 3D and 2D space and are previewed on the display. Determined points are connected in mesh of horizontal and vertical isolines. 4D surfaces are analyzed and confirmed analogy between 3D surfaces and 4D surfaces. Graphical presentations of the projections of 4D surfaces composed of three variables are more complex than the projections of the 4D surfaces composed of two variables.

Keywords: *4D geometry, 4D space, 4D surfaces.*

I. INTRODUCTION

A. 4D Point

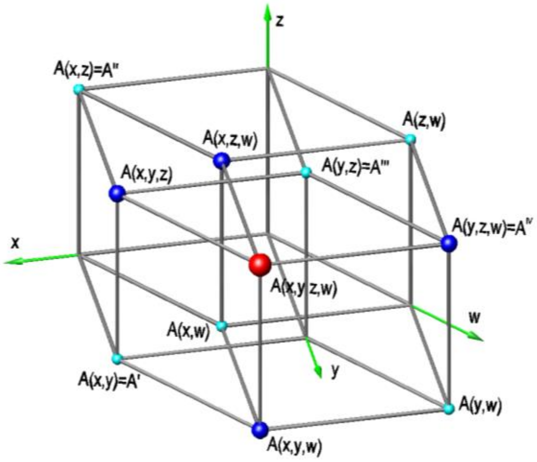

Fig.1 4D point

Point defined with a number of coordinates or parameters of the relationship in that space is. In 1D space determined with one coordinate *A(x)*, in 2D space with two coordinates *A(x,y)*, in 3D space with three coordinates *A(x,y,z)*, in 4D space with four coordinates *A(x,y,z,w)*. If point *A* of 4D space *A(x,y,z,w)* projected in 3D space are receive 4 projections on the coordinate hyperplane *A(x,y,z)*, *A(x,y,w)*, *A(x,z,w)* and *A(y,z,w)* and 6 projections on the coordinate plane *A(x,y)*, *A(x,z)*, *A(y,z)*, *A(x,w)*, *A(y,w)* and *A(z,w)* (Fig.1) (e.g. [1], [2], [3] and [4]).

B. 4D Surfaces and computer algorithm

4D surfaces are surfaces in which the position of each point is determined with four coordinates. Presentation of 4D surfaces are reduced to present points in 4D space. 4D point *T(x,y,z,w)* transformed into 3D and 2D point *T(x,y)* and shown on the screen. To create computer algorithms are used matrices for 4D transformations - scaling, translation and rotation (e.g. [5]).

4D surface defined by 4D function

f(x,y,z,w)=0,

that can be set with 3 variables

x(x,y,z)=0; y(x,y,z)=0; z(x,y,z)=0 and *w(x,y,z)=0,*

after, the coordinates of 4D points *T(x,y,z,w)* will be determinate, they'll be transformed and connect using a simple algorithm:

```
Begin
s1=x2-x1/bragli;
s2=y2-y1/brrad;
        for(i=0; i<s1; i++){
                for(j=0; j<s2; j++){
                function = f(x,y,z,w);
                line(T(x[i],y[j]), T(x[i+1],y[j+1]));
                        }}
end
```

4D points connected to a mesh of horizontal (*bragli*) and vertical (*brrad*) isolines. Through the menu of the designed computer program set the limits of variables $x1<x<x2$, $y1<y<y2$ and $z1<z<z2$ of coordinate hyperplane in which is performed projecting (Fig.2).

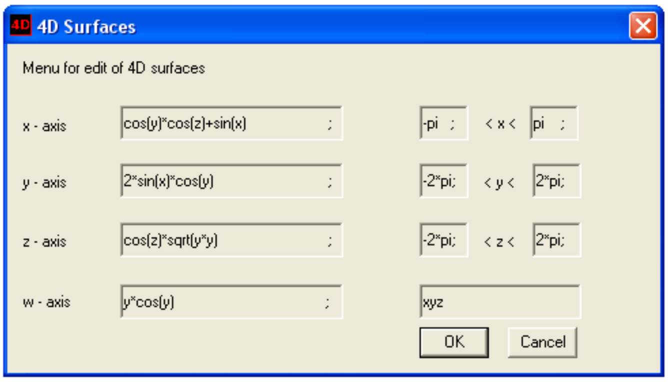

Fig.2 Menu for setting 4D surfaces

II. COMPARISON OF 4D SURFACES DEFINED BY TWO AND THREE VARIABLES

Comparing of 4D surfaces composed of two variables and 4D surfaces composed of three variables can be done only visually, because we have no knowledge of the layout of surfaces. Specifically, we will compare the projections of 4D surfaces composed of two variables for which we have knowledge of the previous examinations with 4D surfaces composed of three variables (e.g. [6]).

We will examine 4D surface composed of 2 variables:

$$f(x,y,z,w)=(\ y\cos(x), y\sin(x), \sin(y)+\cos(y), \sin(y)+\cos(y))$$

within the limits

$$for\ -\pi<x<\pi\ \ and\ -\pi<y<\pi$$

projected on the coordinate hyperplane *xyz* and obtained surface shown in Figure 3.

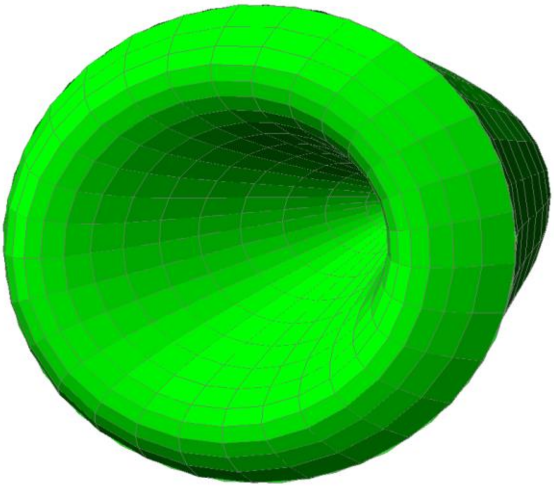

Fig.3 4D surface set with function
$f(x,y,z,w)=(\ y\cos(x), y\sin(x), \sin(y)+\cos(y), \sin(y)+\cos(y))$
$for\ -\pi<x<\pi\ \ and\ -\pi<y<\pi$

If you introduce one more variable in the function that defines the 4D surface will get more complicated surface.

We will examine 4D surface composed of 3 variables:

$$f(x,y,z,w)=(\ y\cos(x), y\sin(x), \sin(z)+\cos(z), \sin(y)+\cos(y))$$

within the limits

$$for\ -\pi<x<\pi\ ,\ -\pi<y<\pi\ \ and\ -\pi<z<\pi$$

projected on the coordinate hyperplane *xyz* and obtained surface shown in Figure 4.

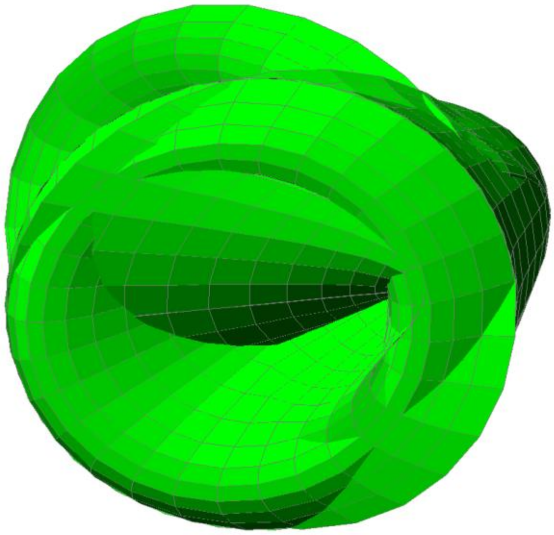

Fig.4 4D surface set with function
$f(x,y,z,w)=(\ y\cos(x), y\sin(x), \sin(z)+\cos(z), \sin(y)+\cos(y))$
$for\ -\pi<x<\pi\ ,\ -\pi<y<\pi\ \ and\ -\pi<z<\pi$

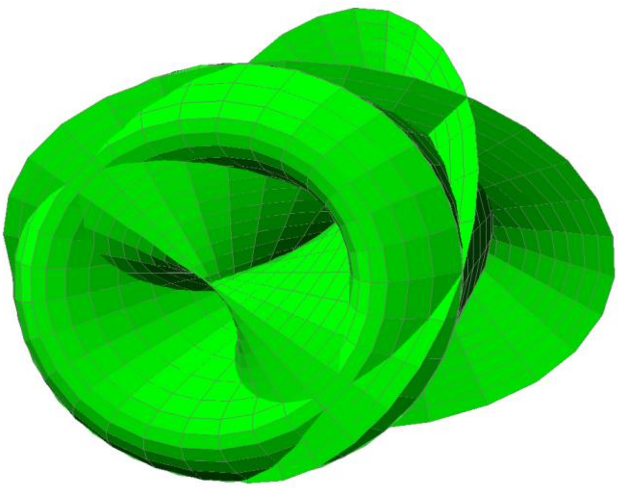

Fig.5 4D surface set with function
$f(x,y,z,w)=(\ y\cos(x), y\sin(x), \sin(z)+\cos(z), \sin(z)+\cos(z))$
$for\ -\pi<x<\pi\ ,\ -\pi<y<\pi\ \ and\ -\pi<z<\pi$

Third variable introduced in the z-axis of the function that defines the 4D surface. If you change the value of the w-axis by introducing a third variable will get:

$$f(x,y,z,w)=(\ y\cos(x), y\sin(x), \sin(z)+\cos(z), \sin(z)+\cos(z))$$

within the limits

$for\ -\pi < x < \pi\ ,\ -\pi < y < \pi\ \ and\ \ -\pi < z < \pi$

projected on the coordinate hyperplane *xyz* and obtained surface shown in Figure 5.

Received 4D surface is more complicated than previous one, although there is a certain similarity between them. The 4D surface is presented of the Figure 5.

III. GEOMETRIC ANALYSIS OF COMPLEXITY OF THE 4D SURFACES

Geometric analysis is consisted of presentation of the projection of complex 4D surfaces composed of 3 variables projected in *xyz* coordinate hyperplane (Fig.6, 7 and 8).

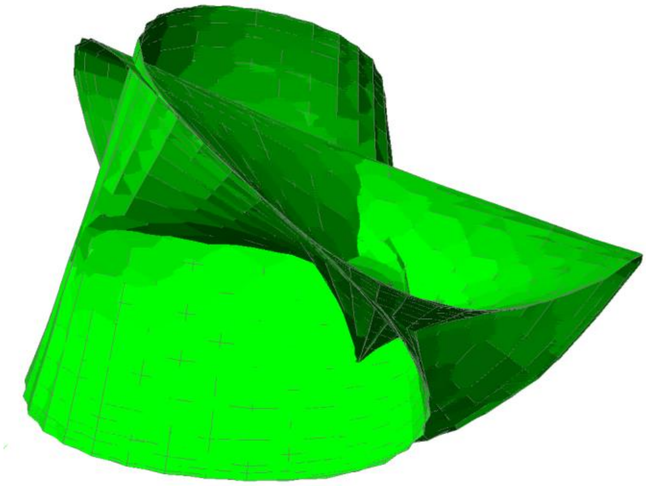

Fig.6 4D surface set with function
$f(x,y,z,w)=(\sin(x)\cos(z), \sin(y)\sin(x), \sin(y)\cos(z), \sin(z)\sin(x))$
$for\ -\pi < x < \pi\ ,\ -\pi < y < \pi\ \ and\ \ -\pi < z < \pi$

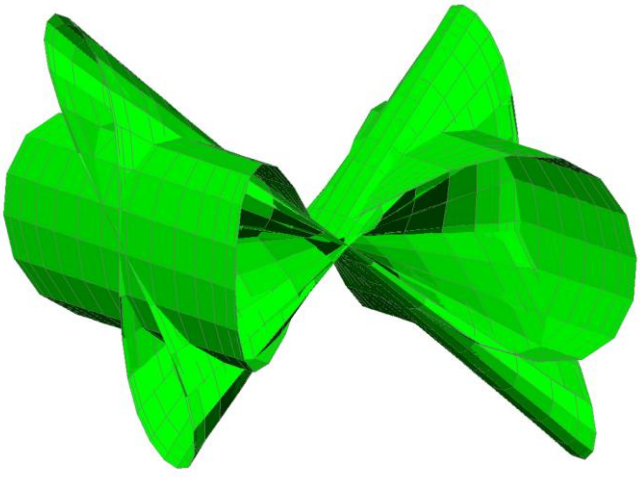

Fig.7 4D surface set with function
$f(x,y,z,w)=(\cos(x)\sin(z), \sin(y), \cos(z)\sin(y), \cos(x)\sin(y))$
$for\ -\pi < x < \pi\ ,\ -\pi < y < \pi\ \ and\ \ -\pi < z < \pi$

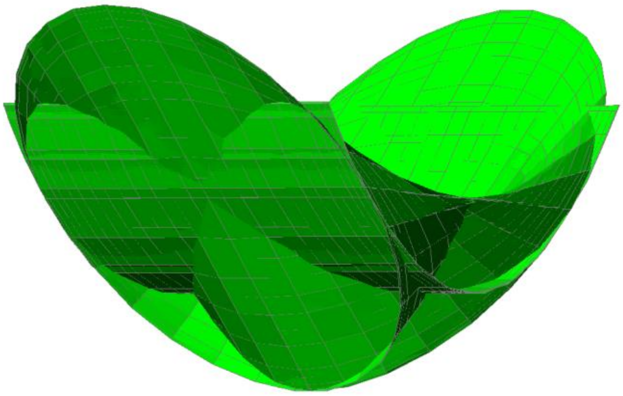

Fig.8 4D surface set with function
$f(x,y,z,w)=(\cos(z), \sin(y), \cos(z)\sin(x), \sin(x))$
$for\ -\pi < x < \pi\ ,\ -\pi < y < \pi\ \ and\ \ -\pi < z < \pi$

IV. CONCLUSION

This paper contribute for perception of 4D space, exactly, with geometric algorithm are processed projections of 4D surfaces set with function with three variables. Computer program is created with that can easily and quickly analyze the projections of 4D surfaces. Graphical presentations of the projections of 4D surfaces composed of three variables are more complex than the projections of the 4D surfaces composed of two variables.

ACKNOWLEDGMENT

In the references are mentioned older researches of the authors (e.g. [4], [1] - V. Filippov and T. Banchoff), because professor Vladimir Filippov first used descriptive geometry in the presentation of four dimensional space, and for the first concrete results in the presentation of 4D surfaces contributed professor Thomas Banchoff.

REFERENCES

[1] T. Banchoff (1994): *Interactive Computer Graphics, Higher Dimensional Geometry*, Proceedings of the North American Serials Group The Haworth Press, USA.

[2] C. Hinton (1997): *Fourth Dimension*, Kessinger Publishing; Washington.

[3] H. Manning (2005): *The Fourth Dimension Simply Explained*, Dover Publications, New York.

[4] В. Филиппов (1979): *Начертательная геометрия многомерного пространства и ЕЕ приложения*, Ленинград.

[5] R. Tashevski (2008): *Projections of 4D Surfaces*, XXIV Conference of Descriptive Geometry and Engineering Graphics "MoNGeometrija 2008", pp.365-373, Vrnjacka Banja, Serbia.

[6] R. Tashevski (2010): *Projections of 4D Surfaces composed of three variables*, XXV Conference of Descriptive Geometry and Engineering Graphics "MoNGeometrija 2010", pp.641-649, Beograd, Serbia