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Original scientific paper

DEVELOPMENT OF SYSTEM FOR DISPLACEMENT MEASUREMENT OF A CANTILEVER BEAM WITH STRAIN GAUGE SENSOR

Dejan Šiškovski, Zlatko Petreski, Goce Tasevski

Institute of Mechanics, Faculty of Mechanical Engineering,
"Ss. Cyril and Methodius" University in Skopje,
Karpoš II bb, P.O. box 464, 1001 Skopje, Republic of Macedonia
siskovskidental@yahoo.com

A b s t r a c t: In this paper, strain gauge based experimental system for displacement measurement of a cantilever beam was developed. The system comprises strain gauge sensors, signal conditioning module and data acquisition unit. Two strain gauges installed at the beam surface are connected in Wheatstone half-bridge configuration. Signal conditioning process is described in details along with the procedure for signal processing. Eventually, experimental investigations were conducted to verify the analytical gained results and certain conclusions were drawn.

Key words: cantilever beam; strain gauge; Wheatstone bridge; signal conditioning

РАЗВОЈ НАСИСТЕМ ЗА МЕРЕЊЕ НА ПОМЕСТУВАЊА НА КОНЗОЛНА ГРЕДА СО МЕРНА ЛЕНТА КАКО СЕНЗОР

А п с т р а к т: Во овој труд, развиен е експериментален систем за мерење на поместувања на конзолна греда врз основа на мерна лента. Системот се состои од мерни ленти како сензори, модул за кондиционирање на сигналите и единица за аквизиција на податоците. Две мерни ленти кои се инсталирани на површината на гредата се поврзани во конфигурација на Витстонов полу-мост. Процесот на кондиционирање на сигналот е опишан во детали, заедно со процедурата за процесирање на сигналите. На крајот, извршени се експериментални испитувања за да ги верифицираат аналитички добиените резултати и извлечени се одредени заклучоци.

Клучни зборови: *к*онзолна греда; мерна лента; Витстонов мост; кондиционирање на сигнали

INTRODUCTION

Measurement of mechanical structure parameters is very important issue in engineering research since they are exposed to different loads. If loads exceed their residue limits may cause inadmissible deformation or permanent damage to the structure that might endanger safety exploitation. Therefore, it is necessary to periodically measure and test the parameters, such as stress and strain of the responsible parts from the mechanical structures.

A wide variety of techniques exists for measuring strain or deformation [1,2] but the most frequent method is with a strain gauge which converts force, pressure, tension, weight etc., into a change in electrical resistance [3].

The advantages of using strain gauges are the small size and very low mass, excellent linearity over wide range of strains, low and predictable thermal effects, high stability with time, lack of moving parts and very small hysteresis [4]. Although the strain gauge is inexpensive and relatively easy to use, care must be exercised to ensure it is properly bonded to specimen, aligned in the direction of measurement, less sensitivity to temperature, and more importantly the lead wire resistance, the excitation source and the accuracy of other components used in the signal conditioning circuit [5].

Resistance changes in strain gauge are very small and they need to be measured with a suitable electrical circuit. Because of its outstanding sensitivity, the Wheatstone bridge circuit [6] is the most frequently used circuit designed to convert small changes in resistance to changes in voltage.

This paper presents the designing steps of a system for displacement measurement of a cantilever beam at any point, pressed by an acting force at their end point. In that context, section 2 describes the developed system and all necessary phases for its implementation, supported by theoretical background. Experimental verification of the system is made in section 3, while in the final section 4 certain conclusions are brought regarding the realized system.

MEASUREMENT SYSTEM ARCHITECTURE

The conceptual design of the system architecture for displacements measurement of a cantilever beam is presented in Figure 1, given by block diagram.

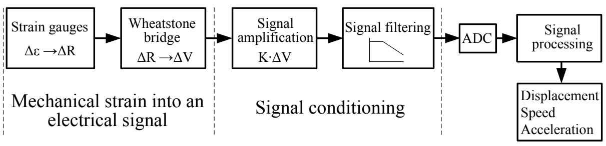

Fig. 1. Block diagram of the measurement system.

Whole structure from the proposed system can be divided into 3 main segments. The first segment consists of a cantilever beam on which two active strain gauges as sensors are installed combined with a Wheatstone half-bridge. This section enables mechanical strain conversion into a proportional electrical signal. The next step that is carried out in part two refers to the signal conditioning matter because the signals produced by the half-bridge are not immediately suitable for data acquisition. This stage is necessary in order to be able to move towards further signals processing, that is done in the third section.

Conversion of mechanical strain into an electrical signal

Figure 2 shows long and thin cantilever beam of uniform rectangular cross section, exposed under an acting force F attached at a distance l from the fixed point.

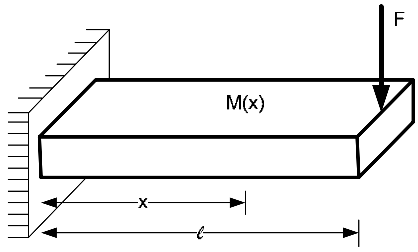

Fig. 2. Cantilever beam exposed under an acting force F

The magnitude of the bending moment at a given point along the cantilever beam can be expressed by the well known equation [7]:

$$M_{(x)} = F\left(l - x\right). \tag{1}$$

Distribution of the normal stresses in a given section depends only upon the value of the bending moment M in that section and the geometry of the section (moment of inertia). Therefore, the sizes of the normal stresses along the beam, at the most distant segments from the elastic curve, where $y = h/2$, shall be derived as follows:

$$\sigma_{(x)} = \frac{6 \cdot F\left(l - x\right)}{b \cdot h^2}. \tag{2}$$

In the area of elasticity of the materials, Hooke's law is valid, thus by its substitution in equation (2), the strain of the cantilever beam at any distance x can be determined as:

$$\varepsilon_{(x)} = \frac{6 \cdot F\left(l - x\right)}{E \cdot b \cdot h^2}. \tag{3}$$

Starting from the basic differential equation of the elastic curve [7], taking into consideration equation (1) and after double integration, the displacements of the cantilever beam for any distance x, resulting from the applied force F attached at their end point are obtained by the equation:

$$y_{(x)} = -\frac{F \cdot x^2\left(3l - x\right)}{6 \cdot E \cdot I}. \tag{4}$$

From equation 3 it could be conclude that major stresses and strains appear at the anchored point of the cantilever beam. Next equation 5 derives from the equations 3 and 4.

$$y_{(x)} = -\varepsilon_{(x_m)} \frac{x^2\left(3l - x\right)}{3h\left(l - x_m\right)}. \tag{5}$$

It determines the displacements for any location x of the cantilever beam, depending on the measured strain at a location x_m.

The conversion of mechanical strain into an electrical signal is performed by using strain gauge sensors. Because of their increased sensitivity in the vicinity of the fixed point, strain gauges were placed at distance x_m = 30 mm from that point, as shown in Figure 3.

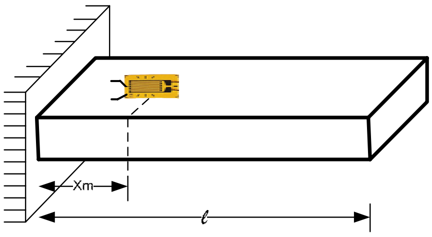

Fig. 3. Installation of the strain gauges

The strain can be tensile or compressive, as distinguished by a positive or negative change in the nominal resistance of the gauge. When used in a Wheatstone bridge configuration, this property of the strain gauge is exploited to convert the change in resistance of the strain gauge to a voltage that corresponds to the strain applied [6].

Wheatstone half bridge shown in Figure 4 is composed of two 100 Ω resistors and two active strain gauges of 6/120LY11 type from HBM.

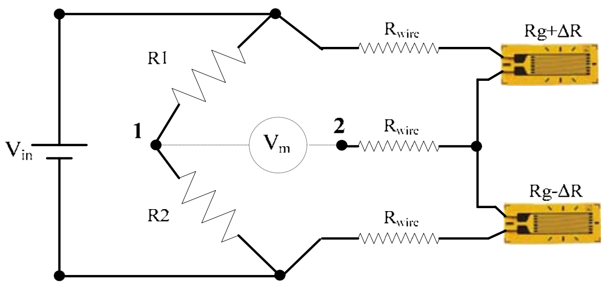

Fig. 4. Half-bridge with three wires

In this configuration the strain gauges are connected with three wires. Two of them are used to carry the excited voltage up to the strain gauges while measurements are performed through the third wire which is popularly called "sense".

Taking into account the effects of the resistance of the wires and the resistance change that occurs in the strain gauges under stress, the relationship between the measured voltage V_m and the strain ε, in the case where $R_1 = R_2 = R_g$, can be expressed as:

$$\frac{V_m}{V_{in}} = -\frac{1}{2}\frac{G_F \cdot \varepsilon}{\left(1+\dfrac{R_{wire}}{R_g}\right)}. \qquad (6)$$

Because the length of the wires used in the experiment are very short, resistance R_{wire} is very small, thus the term $(1 + R_{wire}/R_g)$ from equation 6 can be neglected. Therefore, the dependence between beam displacements and the measured voltage V_m from the bridge is obtained when equation 6 is added in the equation 5, given by:

$$y_{(x)} = -\frac{V_m}{V_{in}}\frac{2x^2(3l-x)}{3h \cdot G_F(l-x_m)}, \qquad (7)$$

where: h and l are height and length of the beam and G_F is the gauge factor.

Signal Conditioning

The expected maximum displacements of the cantilever beam are between the ranges of ±15 mm. According to equation (7), the measured voltage from the bridge V_m will be between the ranges of ±0.55 mV. As data acquisition equipment NIRio9636 is used, containing of 16-bit ADC converter with range of ±10 V. For these reasons, the signal must be amplified more than a thousand times. This way resolution increases and the signal/noise ratio improves. Figure 5 shows the entire process of signal conditioning in the time domain, displayed on an oscilloscope.

Figure 5a, shows the real signal from the bridge when the beam freely oscillates with its first natural frequency of 9 Hz. It may be noted that the noises that appear in the signal are multiple greater than the signal of interest and they are in range between ±10 mV. First step to remove these noises is the differential amplifier, which is suitable for amplification of very small signals in the range of millivolts. Figure 6 shows the amplification of a signal that is contaminated with noise, through a differential amplifier.

Due to the “inverting” input characteristic, the induced voltage that appears in the wires will be annulled after the amplification. Figure 5b shows the amplified signal by the differential amplifier by 47 times so improvement of the signal/noise ration can be noticed. For further amplification of the signal two “inverting” amplifiers are used with amplification factors of 47 and 5. As the last step in the signal conditioning process is the application of low-pass filter. Figure 5c shows the filtered signal with a total amplification by factor K = 11045 times. The values for the resistor and capacitor from the low-pass filter are taken as R = 900 Ω and C = 10 μF hence the cut-off frequency of the filter has been chosen almost twice the value of the first natural frequency of the beam i.e. 17.7 Hz.

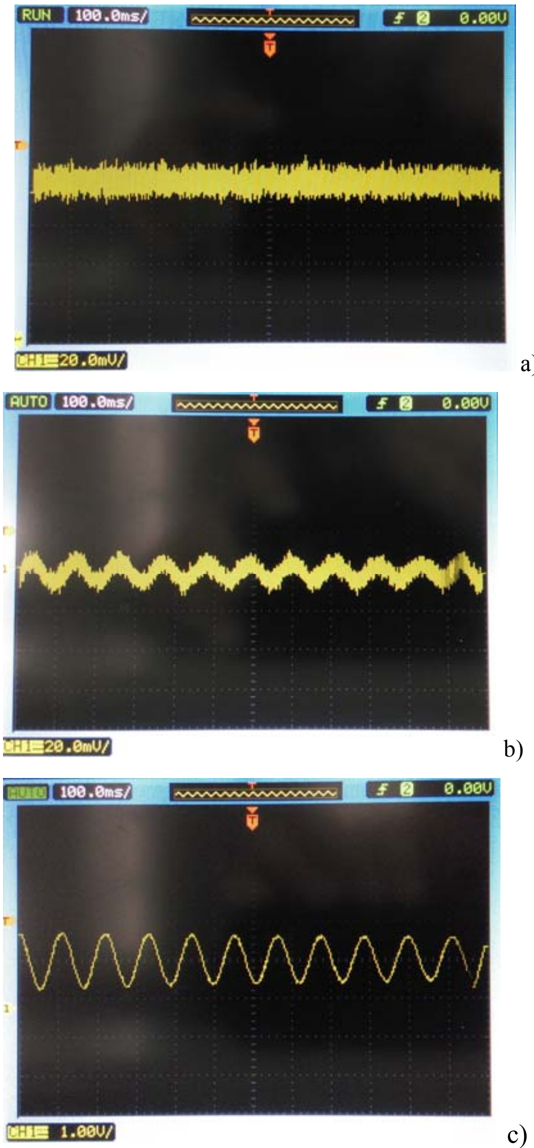

Fig. 5. Signal conditioning

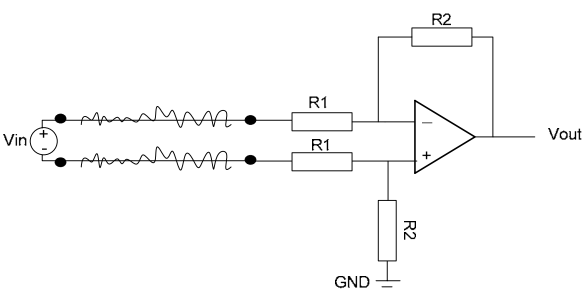

Fig. 6. Differential amplifier

At this frequency the amplitude of the output signal relative to the amplitude of the input signal will be reduced for –3 db, while the phase delay will be –45°. The transfer function of the low-pass filter will be:

$$\frac{V_{out}}{V_{in}} = \frac{1}{0{,}009 \cdot s + 1}. \quad (8)$$

Figure 7 shows the Bode diagram of the transfer function of the filter.

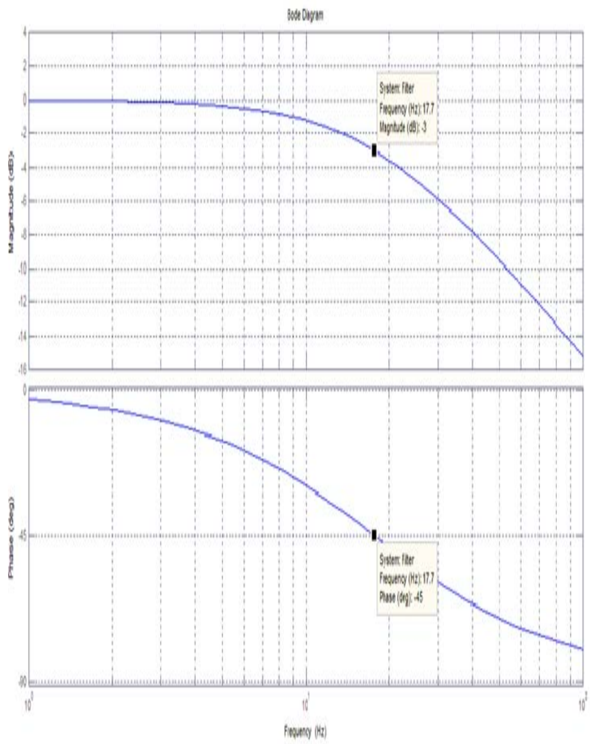

Fig. 7. Bode diagram of the low-pass filter

Signal Processing

Once completed the process of signal conditioning, data acquisition and converting of the signal into physical quantity are the next necessary steps. Knowing that the V_{out} voltage measured by the data acquisition equipment is equal to $V_{out} = K \cdot V_m$, by replacing this expression in equation (7) the dependence between the maximum displacement of the beam and voltage V_{out} is obtained. From there, a new coefficient C_P derives by which the measured voltage is converted into physical size-displacement. This coefficient can be calculated for any location x of the cantilever beam, so in case when measuring the displacement of the end point of the beam (x = l), C_P can be determined as:

$$C_P = \frac{4 \cdot l^3}{3 \cdot V_{in} \cdot K \cdot h \cdot G_F \left(l - x_m\right)} = 2,441 \text{ mm/V}. \quad (9)$$

According to this, the relation of maximum displacement of the beam as a function of voltage V_{out} can be written as:

$$y_{\max} = 2,441 \cdot V_{out} \text{ mm}. \quad (10)$$

Starting from the equation 4, taking into account also the expression 10, another coefficient C_F can be determined as:

$$C_F = 0,002441 \frac{3EI}{l^3} = 0,113 \text{ N/V}. \quad (11)$$

From here, the relation between the force and the output voltage V_{out} can be set as:

$$F = 0,113 \cdot V_{out} \text{ N}. \quad (12)$$

EXPERIMENTAL VERIFICATION

Verification of coefficients C_P and C_F is carried out by experimental measurement with 6 test masses of 5, 10, 15, 20, 25 and 30 g which are placed at the end point of the beam. The whole experimental setup of the developed system for measuring the displacements of a cantilever beam is given by Figure 8.

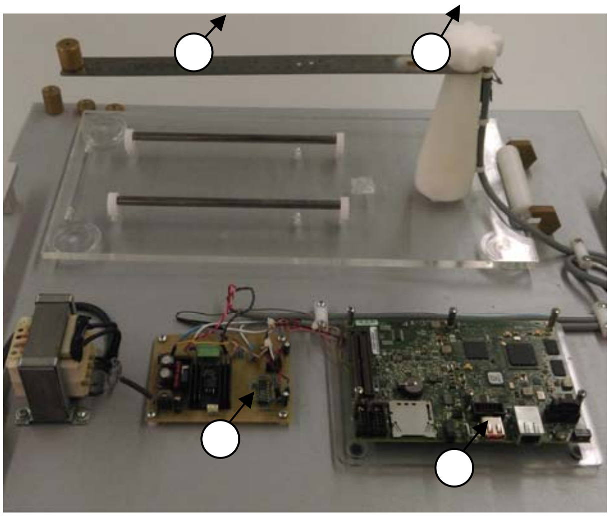

Fig. 8. Experimental setup of the system
1) Cantilever beam, 2) Strain gauge,
3) Signal conditioning module, 4) data acquisition unit

During the measurement, the cantilever beam is set in a horizontal position and the masses are converted as acting forces. Figure 9 shows a diagram where output voltage V_{out} is measured in relation to the displacements of the beam, while Figure 10 present dependence between voltage V_{out} and the acting forces. From the diagrams it may be noted linear relationship among voltage, force and displacements.

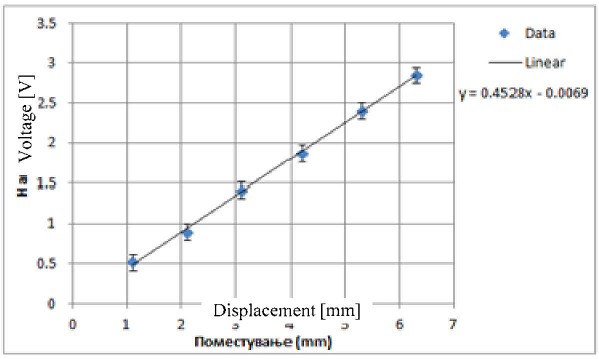

Fig. 9 Relations between voltage and displacements

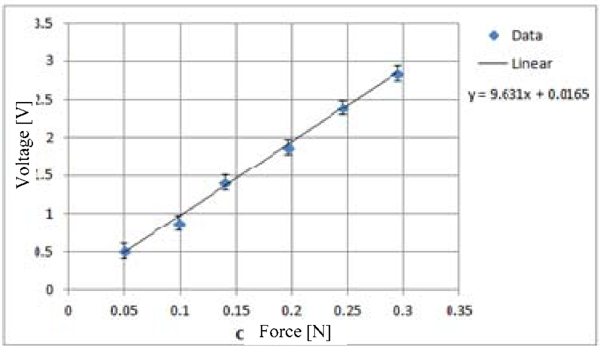

Fig. 10. Relations between voltage and forces

Using the method of least squares, linearization is performed in terms of displacement and force and after certain mathematical operations the following expressions are obtained:

$$y_{\max} = 2.2 \cdot V_{out} \text{ mm}. \quad (13)$$

$$F = 0.103 \cdot V_{out} \text{ N}. \quad (14)$$

Table 1 shows the coefficients C_P and C_F obtained by analytical and experimental way.

Table 1

Analytical and experimental values of the coefficients C_P and C_F

Coefficient	Analytical values	Experimental values
C_P	2,441 mm/V	2,2 mm/V
C_F	0,113 N/V	0,103 N/V

CONCLUSIONS

A strain gauge based system for displacements measurement of a cantilever beam was developed in this paper. The design procedure requires knowledge of mechanical engineering area, electrical engineering in terms of conversion and signals conditioning as well as data acquisition equipment capabilities. The applicability of the developed system can be perceived by inserting the equation 12 in equation 10 where it can directly be determined the displacements value of the cantilever beam depending on the applied forces.

Certain experimental tests were carried out on the cantilever beam in order to verify the obtained analytical results of the proposed system. The results given in Table 1 indicate sufficiently close values for the coefficients C_F and C_P hence with this developed system, deformations of the cantilever beam can be measured quite accurately. Differences in analytical and experimental results owed to certain imperfections in the constituent components of the measurement system.

By differentiating the signals obtained from the displacements, velocity and acceleration of the cantilever beam can be determined. Hence, with certain modifications and adjustments in the designed measurement system, these signals can be used as feedback in a closed loop control system for vibration measurement.

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