Modeling and Simulation of a Hybrid Piezoelectric–Electromagnetic System for Vibration Energy Harvesting

Dejan Shishkovski [1], Zlatko Petreski [1], Simona Domazetovska Markovska [1], Damjan Pecioski [1], Maja Anachkova [1], Anastasija Angjusheva Ignjatovska [1]

[1] Faculty of Mechanical Engineering,
Ss Cyril and Methodius University, Skopje, N.Macedonia
dejan.shishkovski@mf.edu.mk

Abstract. Vibration energy harvesting is a promising approach for enabling energy autonomy in low-power electronic systems operating under ambient mechanical excitations, particularly in the low-frequency range. This paper presents a coupled analytical and numerical investigation of a hybrid vibration energy harvesting system that integrates piezoelectric and electromagnetic transduction mechanisms within a single cantilever-based structure. A reduced-order analytical formulation is introduced to provide physical insight into the strain and velocity-dominated energy conversion mechanisms associated with different vibration modes. In parallel, a Multiphysics finite element model is developed in COMSOL Multiphysics to evaluate the dynamic behavior of the structure and the electrical output of both transducers under harmonic excitation. Numerical simulations show that the piezoelectric transducer achieves its highest performance near the first bending mode at approximately 15 Hz, delivering a maximum output power of 1.7 mW under optimal load conditions. In contrast, the electromagnetic transducer exhibits improved performance at the second resonance frequency near 64 Hz, generating a peak output power of 2.8 mW at a low electrical load resistance. The results demonstrate that the hybrid configuration enables effective energy harvesting across multiple vibration modes and enhances adaptability to varying excitation conditions compared to single-mode harvesters. The proposed modeling framework provides a consistent basis for the design and optimization of hybrid vibration energy harvesting systems intended for low-power autonomous applications.

Keywords: Vibration energy harvesting, Hybrid energy harvester, Piezoelectric transducer, Electromagnetic generator, Multiphysics modeling, Finite element simulation.

1 Introduction

Vibration energy harvesting has attracted significant attention as a means of enabling energy-autonomous operation of low-power electronic devices, particularly in applications related to wireless sensor networks, structural health monitoring, and Internet of Things systems. In such applications, ambient mechanical vibrations represent a widely available energy source; however, their low amplitude and low-frequency nature pose substantial challenges for efficient energy conversion. Conventional vibration energy harvesting systems based on piezoelectric or electromagnetic transduction have been widely investigated in recent years, particularly for autonomous and condition monitoring systems [1–4]. More recent studies have explored hybrid configurations to improve adaptability and power output under low-frequency excitation conditions [5]. Piezoelectric harvesters are well suited for compact designs and high-voltage output but often exhibit limited performance at low frequencies and require high electrical load impedances. Electromagnetic harvesters, on the other hand, are more effective at low-frequency excitation and low electrical load resistances but usually provide lower voltage levels and larger device sizes. As a result, single-mode energy harvesting systems frequently suffer from narrow operational bandwidth and reduced adaptability to varying vibration conditions. To overcome these limitations, hybrid vibration energy harvesting systems that combine multiple transduction mechanisms within a single mechanical structure have been proposed. By exploiting the complementary characteristics of piezoelectric and electromagnetic conversion, hybrid configurations offer the potential for improved robustness, enhanced power output, and broader applicability under low-frequency excitation. The objective of this paper is to develop and investigate a coupled analytical and numerical modeling framework for a hybrid piezoelectric–electromagnetic vibration energy harvesting system operating under low-frequency excitation. The proposed system is based on a cantilever beam structure integrating a surface-bonded piezoelectric patch and an electromagnetic generator. The study focuses on the dynamic behavior of the system under low-frequency excitation and on the numerical evaluation of the electrical power generated by each transduction mechanism. The presented study provides insight into the performance and potential advantages of hybrid vibration energy harvesting systems and establishes a consistent modeling basis for their further development.

2 Methodology

2.1 System Overview

The proposed hybrid vibration energy harvesting system is based on a cantilever beam structure designed for low-frequency mechanical excitation. The system integrates two energy transduction mechanisms, piezoelectric and electromagnetic, within a single mechanical configuration to exploit their complementary characteristics. The mechanical structure consists of a clamped free cantilever beam that serves as the primary vibration-sensitive element. A piezoelectric patch made of lead zirconate titanate (PZT) is surface-bonded near the clamped end of the beam, where the bending-induced strain is maximal, enabling efficient conversion of mechanical deformation into electrical energy. In parallel, an electromagnetic generator is implemented using a permanent magnet attached to the cantilever as an added inertial mass and a stationary coil positioned in its vicinity. Unlike a conventional tip-mounted configuration, the magnet is placed at an optimized position along the beam length, selected to maximize the relative motion (velocity and displacement) associated with the second vibration mode. The resulting relative motion between the magnet and coil induces an electromotive force according to Faraday's law of electromagnetic induction. The hybrid configuration allows simultaneous harvesting of electrical energy from strain-based (piezoelectric) and motion-based (electromagnetic) mechanisms. The piezoelectric subsystem typically exhibits higher voltage levels and higher optimal electrical load resistance, whereas the electromagnetic subsystem is better suited for low-resistance electrical loads and benefits from regions of high vibration velocity. By combining both mechanisms within the same structure and by optimally placing the inertial magnet, the system aims to improve adaptability to varying vibration conditions and enhance the overall harvesting capability compared to single-mode configurations. A schematic representation of the proposed hybrid energy harvesting system is shown on Fig. 1.

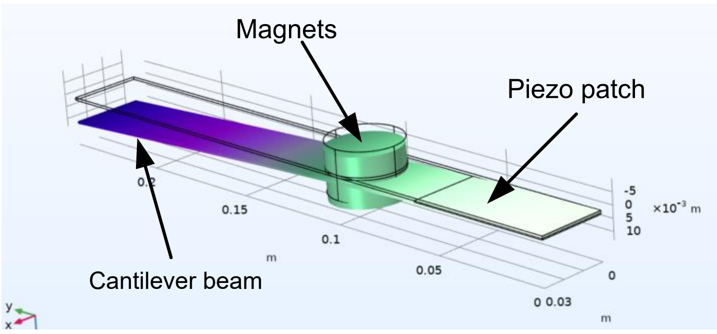

Fig. 1. Schematic representation of the proposed hybrid piezoelectric–electromagnetic vibration energy harvesting system based on a cantilever beam.

2.2 Reduced-Order Analytical Representation

To support numerical analysis and provide physical insight into the hybrid energy harvesting mechanism, a reduced-order analytical representation of the system is introduced. Rather than aiming to fully capture the complex multimodal behavior of the structure, the analytical formulation serves to illustrate the dominant modal contributions associated with the piezoelectric and electromagnetic transduction mechanisms. The mechanical response of the cantilever beam can be expressed using a modal superposition approach, where the generalized displacement is written as

$$x(t) = \sum_{n} \emptyset_n q_n \ (t) \tag{1}$$

whit $\emptyset_n$ denoting the mode shapes and q_n(t) the corresponding generalized coordinates. In the present configuration, the piezoelectric transducer primarily interacts with the strain field of the first vibration mode, while the electromagnetic transducer is designed to exploit regions of high velocity associated with the second vibration mode. For each contributing mode, the electromechanical interaction can be represented in a lumped-parameter form, linking the generalized modal coordinates to the electrical variables of the piezoelectric and electromagnetic subsystems. This reduced-order formulation provides qualitative insight into the complementary harvesting mechanisms without restricting the analysis to a single-degree-of-freedom assumption.

2.2.1 Piezoelectric transducer model

The piezoelectric transducer is modeled using a lumped-parameter representation that relates the mechanical deformation of the cantilever beam to the generated electrical response. When subjected to bending, the piezoelectric patch experiences strain proportional to the modal displacement, resulting in charge generation through the direct piezoelectric effect. In the reduced-order formulation, the electrical behavior of the piezoelectric transducer can be expressed as

$$C_p \dot{v}_p(t) + \frac{v_p(t)}{R_p} = \theta_p \dot{q}_1(t) \tag{2}$$

where v_p is the piezoelectric output voltage, C_p is the intrinsic capacitance of the piezoelectric patch, and R_p is the external electrical load resistance. The coupling coefficient θ_p represents the electromechanical interaction between the first vibration mode and the piezoelectric transducer, while $\dot{q}_1(t)$ denotes the generalized coordinate associated with the dominant strain-based mode. This formulation captures the essential behavior of the piezoelectric energy conversion process and highlights the dependence of the electrical output on both the mechanical response and the electrical load conditions.

2.2.2 Electromagnetic transducer model

The electromagnetic energy harvesting mechanism is based on the relative motion between a permanent magnet and a stationary coil, resulting in an induced electromotive force according to Faraday's law of electromagnetic induction. In the proposed configuration, the electromagnetic transducer is positioned to exploit regions of high vibration velocity associated with the second vibration mode of the cantilever beam. The induced voltage in the coil is proportional to the relative velocity and can be expressed as

$$v_e(t) = \theta_e \dot{q}_2(t) \tag{3}$$

where $v_e(t)$ is the induced electromagnetic voltage, θ_e is the electromagnetic coupling coefficient, and $q_2(t)$ denotes the generalized coordinate associated with the velocity-dominant vibration mode. The electrical dynamics of the electromagnetic transducer connected to a resistive load are described by

$$L_e i_e(t) + R_e i_e = \theta_e \dot{q}_2(t) \tag{4}$$

where $i_e(t)$ is the coil current, and L_e and R_e represent the inductance and electrical load resistance of the coil, respectively. This reduced order electromagnetic model emphasizes the velocity-dependent nature of electromagnetic energy harvesting and its suitability for low-frequency excitation and low-resistance electrical loads.

2.3 Numerical Model

A Multiphysics numerical model of the proposed hybrid energy harvesting system is developed in COMSOL Multiphysics to investigate the dynamic behavior of the structure and to evaluate the electrical output of the piezoelectric and electromagnetic transducers. The finite element model provides a consistent framework for capturing the coupled mechanical and electrical interactions that cannot be fully resolved using reduced-order analytical formulations. The cantilever beam is modeled as a three-dimensional elastic structure with clamped free boundary conditions. The piezoelectric patch is bonded to the beam surface near the clamped end and is modeled using the piezoelectric devices interface, enabling direct coupling between mechanical strain and electrical potential. The electromagnetic transducer is represented by a permanent magnet attached to the beam at an optimized position along its length and a stationary coil located in its vicinity. The electromagnetic interaction is modeled by coupling the mechanical motion of the magnet to an external electrical circuit representing the coil and load resistance. Eigenfrequency analysis is first performed to identify the natural frequencies and corresponding mode shapes of the system. Attention is given to the first two vibration modes, which are associated with dominant strain and velocity responses relevant to the piezoelectric and electromagnetic energy harvesting mechanisms, respectively. The position of the inertial magnet is selected based on this modal analysis to coincide with regions of maximum velocity in the second vibration mode. The first vibration mode, occurring at approximately 15.1 Hz, is illustrated in Fig. 2.

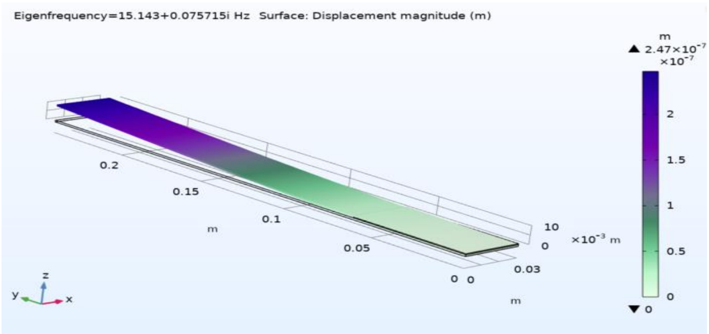

Fig. 2. First bending mode shape of the cantilever beam obtained from eigenfrequency analysis.

Subsequently, frequency-domain simulations are conducted to analyze the frequency response of the piezoelectric and electromagnetic transducers under harmonic excitation.

3 Results

This section presents the numerical results obtained from the multiphysics simulations of the proposed hybrid vibration energy harvesting system. The results are reported separately for the piezoelectric and electromagnetic energy harvesters, considering the first and second resonance frequencies of the cantilever structure. All results are presented in terms of output voltage, harvested power, and optimal electrical load conditions.

3.1 Results of the Piezoelectric Energy Harvester

The performance of the piezoelectric energy harvester was evaluated at the first and second natural frequencies of the cantilever beam without additional tip masses. The numerical results obtained from the simulations are summarized in Table 1.

Table 1. Electrical output characteristics of the piezoelectric energy harvester at resonant frequencies

Mode order	Quantity	Symbol	Value	Unit	Description
First	Resonant frequency	f_1	15.25	Hz	
	RMS voltage	Vrms	18	V	RMS output voltage at the first resonance

Mode order	Quantity	Symbol	Value	Unit	Description
	Output power	P_1	1.7	mW	Electrical output power at the first resonance
	Load resistance	Rload	100	kΩ	Optimal electrical load at the first resonance
Second	Resonant frequency	f_2	89.3	Hz	
	RMS voltage	Vrms	4	V	RMS output voltage at the second resonance
	Output power	P_2	0.48	mW	Electrical output power at the second resonance
	Load resistance	Rload	17.3	kΩ	Optimal electrical load at the second resonance

At the first natural frequency of 15.25 Hz, the piezoelectric transducer achieved an RMS output voltage of 18 V, corresponding to a maximum harvested power of 1.7 mW at an optimal load resistance of 100 kΩ. The voltage response at this resonance frequency is shown in Fig. 3, while the variation of output power with respect to the electrical load resistance is illustrated in Fig. 4.

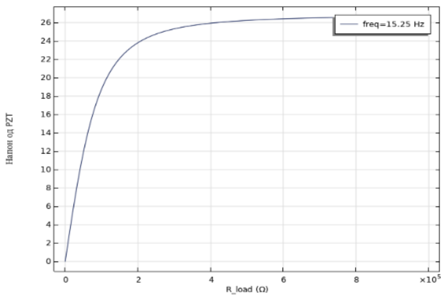

Fig. 3. The voltage response with respect to the electrical load resistance f=15.2Hz

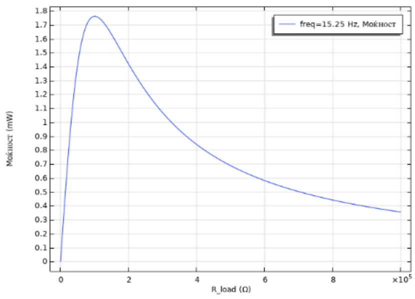

Fig. 4. Output power with respect to the electrical load resistance f=15.2Hz

At the second natural frequency of 89.3 Hz, the piezoelectric energy harvester generated an RMS voltage of 4 V, resulting in a maximum output power of 0.48 mW at an optimal load resistance of 17.3 kΩ. The voltage response at this resonance is presented in Fig. 5, whereas the corresponding power–load characteristic is depicted in Fig. 6.

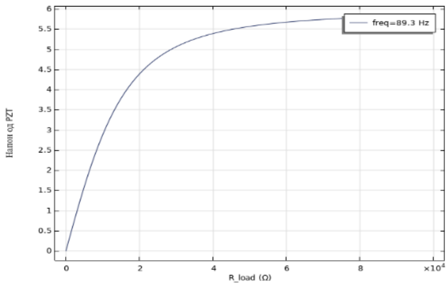

Fig. 5. The voltage response with respect to the electrical load resistance f=89.3Hz

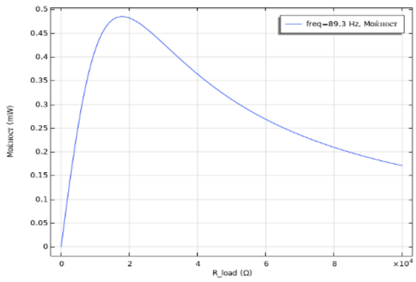

Fig. 6. Output power with respect to the electrical load resistance f=89.3Hz

The obtained results demonstrate a clear dependence of the harvested power on both the excitation frequency and the electrical load, with higher output power observed at the first resonance frequency due to the larger mechanical deformation of the cantilever structure.

3.2 Results of the Electromagnetic Energy Harvester

The parameters summarized in Table 2 were used to configure the electromagnetic transducer within the COMSOL Multiphysics environment and to perform a detailed finite element analysis of the magnet–coil interaction. The model accounts for the three-dimensional distribution of the magnetic field generated by the permanent magnet and its interaction with the conductive coil domain. In addition to evaluating the induced electromagnetic response, the finite element formulation enables the assessment of magnetic flux density distribution and associated electromagnetic losses within the system. The coupled mechanical–electromagnetic formulation ensures that the relative motion between the magnet and the coil, originating from the cantilever vibration, is consistently transferred to the electromagnetic domain. The resulting FEM model of the electromagnetic energy harvester, including the magnet–coil configuration used for the magnetic field analysis, is shown in Fig. 7.

Table 2. Parameters used for configuring the electromagnetic transducer in COMSOL Multiphysics

Parameter name	Symbol	Value	Unit	Description
Magnet displacement	d_m	0.001	m	Axial distance between the magnet and the coil
Magnetic flux density	B	0.8	T	Magnetic flux density of the permanent magnet

Parameter name	Symbol	Value	Unit	Description
Number of turns	N	4500	–	Number of turns of the electromagnetic coil
Load resistance	Rload	3000	Ω	External electrical load resistance
Excitation amplitude	A	0.01	m	Mechanical excitation displacement amplitude

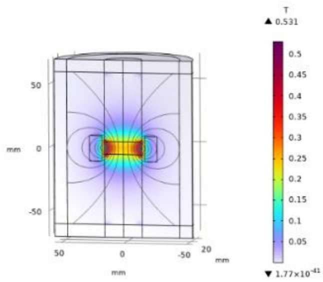

Fig. 7. Finite element model of the electromagnetic energy harvester used for magnetic field distribution and loss analysis.

The electromagnetic energy harvester was analyzed under the same resonance conditions as the piezoelectric transducer. The numerical results obtained from the simulations are reported in Table 3.

Table 3. Electrical output power of the electromagnetic energy harvester at resonant frequencies

Mode order	Resonant frequency	Symbol	Value	Unit	Description
First	Resonant frequency	f_1	14.9	Hz	
	Output power	P_1	0.28	mW	Electrical output power at the first resonance
	Load resistance	Rload	3	kΩ	Optimal electrical load at the first resonance
Second	Resonant frequency	f_2	63.8	Hz	
	Output power	P_2	2.8	mW	Electrical output power at the second resonance
	Load resistance	Rload	3	kΩ	Optimal electrical load at the second resonance

At the first natural frequency of 14.9 Hz, the electromagnetic harvester delivered a maximum output power of 0.28 mW when connected to a load resistance of 3 kΩ. The corresponding power response is shown in Fig. 8. At the second natural frequency of 63.8 Hz, a significantly higher output power of 2.8 mW was obtained at the same load resistance of 3 kΩ. The power response at this frequency is illustrated in Fig. 9.

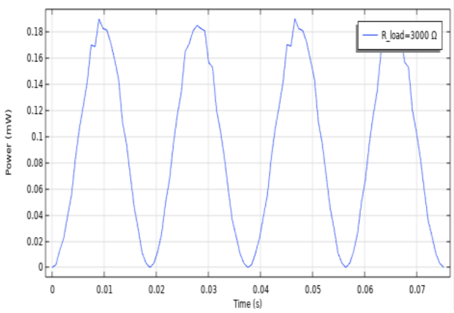

Fig.8. Output power of the electromagnetic energy harvester at the first resonance frequency (f = 14.9 Hz).

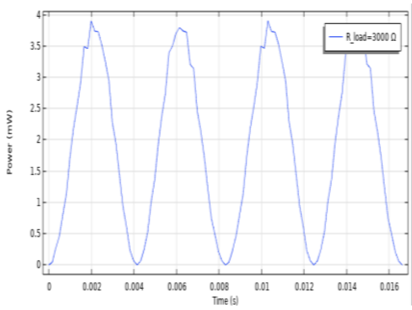

Fig. 9. Output power of the electromagnetic energy harvester at the second resonance frequency (f = 63.8 Hz).

These results indicate that the electromagnetic energy harvester exhibits improved performance at the higher resonance frequency, where increased relative motion between the magnetic and coil components leads to enhanced electromagnetic induction.

4 Conclusions

This paper presented a coupled analytical and numerical investigation of a hybrid piezoelectric–electromagnetic vibration energy harvesting system designed for low-frequency excitation. A reduced-order analytical formulation was introduced to provide physical insight into the dominant strain and velocity-driven energy conversion mechanisms, while a multiphysics finite element model implemented in COMSOL Multiphysics was used to evaluate the dynamic response and electrical performance of the system. The numerical results demonstrated that the proposed hybrid configuration enables efficient energy harvesting across multiple vibration modes of the cantilever structure. The piezoelectric transducer was shown to be most effective in the low-frequency regime associated with the first bending mode, where bending-induced strain is dominant, whereas the electromagnetic transducer achieved higher power output at the higher resonance frequency associated with increased vibration velocity. Overall, the combined multiphysics response confirms that integrating piezoelectric and electromagnetic transduction mechanisms within a single mechanical structure enhances adaptability to varying excitation conditions compared to single-mode energy harvesters. The presented modeling framework provides a consistent basis for future experimental validation and for the design optimization of hybrid vibration energy harvesting systems intended for low-power autonomous applications operating under broadband and low-frequency ambient vibrations.

References

1. Erturk, A., Inman, D.J. (2011) Piezoelectric energy harvesting. John Wiley & Sons, Chichester
2. Ahmad, I., Hee, L.M., Abdelrhman, A.M., Imam, S.A., Leong, M.S. (2022) Hybrid vibro-acoustic energy harvesting using electromagnetic transduction for autonomous condition monitoring systems. Energy Conversion and Management, Volume 258, id.115443
3. Cuji, J., Mendoza, L., Brito, G., Gordon, C. (2021) Portable electromagnetic energy harvesting system. In: Proceedings of the 1st Congress on Sustainability, Energy and City (CSECity), pp. 1–6. Ambato, Ecuador
4. Anton, S.R., Sodano, H.A. (2007) A review of power harvesting using piezoelectric materials (2003–2006). Smart Materials and Structures 16(3), R1–R21
5. Liu, Z.Q., Li, S.Y., Lin, S.Q., Shi, Y.X., Yang, P., Chen, X.Y., Wang, Z.L. (2022) Crystallization-induced shift in a triboelectric series and even polarity reversal for elastic triboelectric materials. Nano Letters 22(10), 4074–4082