Modeling and Simulation of an Electromagnetic Energy Harvesting System

Damjan Pecioski, Dejan Shishkovski, Anastasija Angjusheva Ignajatovska, Simona Domazetovska Markovska, Viktor Gavriloski
Faculty of Mechanical Engineering - Skopje
Ss. Cyril and Methodius University in Skopje,
Skopje, N. Macedonia
damjan.pecioski@mf.edu.mk

Abstract—**Low-power electronic devices can be powered using the concepts of micro-scale energy harvesting techniques to ensure a near-infinite lifetime work cycle. Nonetheless, this conversion of energy is not ideal and as a result, there is always a certain percentage of wasted energy. One must strive that the modeling and simulation of these systems is as close as possible to the real systems and conditions. Within this paper, an electromagnetic energy harvesting system that is attached to an elastic mechanical structure in the form of a cantilever beam has been developed. A mathematical model for the structure and the linear electromagnetic generator has been derived. The simulations concerning the obtained model have been conducted using the Matlab/Simulink software package. Finally, an experimental model has been created to validate the prior acquired results from the simulation.**

Keywords – energy harvesting, cantilever beam, electromagnetic generator.

I. Introduction

The exploration of different physical phenomena such as ambient energy sources, machine vibration, dissipated heat, etc. to harness useful electrical energy has been the interest of researchers for many decades [1]. Energy harvesting on a macro scale aims to generate power of hundreds of Wats while on a micro scale, the aim is to generate hundreds of μW to a couple of mW. The possible application of such harvesters is in powering low-power micro-electromechanical system (MEMS). The proposed energy harvesters for such applications convert kinetic energy into electrical energy using electrostatic, electromagnetic, or piezoelectric energy transduction mechanisms [2-8].

The generated power from these systems is proportional to the value of the vibration frequency, which is usually in the resonant range [9]. This means that there is a narrow bandwidth in which the resonant frequency of the scavenger is matched with the ambient frequency and leads to optimal results [10].

Faraday's laws of electromagnetic induction establish the basis of electromagnetic conversion methods [11]. Electromagnetic energy harvesting techniques usually use a simple mechanical resonator structure with lower frequencies which achieves high energy conversion efficiency [12-15]. Huicong et al. have researched a novel electromagnetic energy harvester that has multiple modes of vibration in [12]. Within [13], an electromagnetic MEMS energy harvester is presented, which generates energy from low-frequency vibrations with low displacement amplitude even in non-resonant vibrations. Von T, and G Troster [15] designed a linear electromagnetic generator able to generate 2-20 μW which is suitable for power supply of body-worn sensor nodes.

II. Modeling of an Electromagnetic System

A. Modeling of the cantilever beam

The cantilever beam as a system is often used to model and analyze robotic arms, vibrating beam gyroscopes, bio-chemical sensors, and energy harvesting devices. In order to model the dynamics of cantilever systems, certain approximations in the system must be adopted. In this paper, those approximations include material homogeneity and an ideally constant width of the beam along its entire length. The differential equation of motion for a continuous beam system, using the Euler-Bernoulli method can be expressed as:

$$EI_y \frac{\partial^4 y(x,t)}{\partial x^4} = f(x,t) - \rho A \frac{\partial^2 y(x,t)}{\partial t^2} \quad (1)$$

The legend of all the parameters and their values for the cantilever beam are given in Table I.

TABLE I.

Parameter	Value
Length – l	0.3 m
Width – b	0.026 m
Hight – h	0.001 m
Area of cross section – A	$0.26*10^{-4}$ m^2
Moment of Inertia – I	$2.16*10^{-12}$ m^2
Young's modulus – E	$2*10^{11}$ N/m^2
Density - ρ	7850 kg/m^3
Mass – m	0.061 kg

Research has shown that the dominant behavior of the cantilever dynamics appears at the first mode of vibration. For these reasons, a reduced model is derived considering a concentrated mass at the end of the cantilever. In order to generate a reduced model, certain parameter such as equivalent mass, equivalent stiffness, and equivalent damping would have to be calculated.

The equivalent stiffness can be calculated as:

$$K_{ek} = \frac{F}{y} \quad (2)$$

where $y_{max} = \frac{Fl^3}{3EI_x}$ is the maximum displacement of the beam.

The equivalent mass for the first mode of vibration is calculated through the kinetic energy of the cantilever beam for the first mode of vibration:

$$E_k = \frac{1}{2}\frac{m}{l}\left(\frac{\dot{y}_{max}}{2l^3}\right)^2 \int_0^l (3x^2 l - x^3)^2 \, dx \tag{3}$$

$$m_{ekv} = \frac{33}{140}\rho \cdot l \cdot b \cdot h \tag{4}$$

Finally, the coefficient of viscous damping can be calculated using the following expression:

$$c_{ekv} = 2\xi\omega_n m_{ekv} \tag{5}$$

By imputing all the parameter values from Table I into the equations (2-5), the values for the equivalent mass, stiffness, damping are obtained and given in Table II:

Table II.

Parameter	Value
m_{ekv}	0.0144 kg
K_{ekv}	46.66 N/m
c_{ekv}	0.006458 Ns/m
ω_n	56.06 rad^{-1}
ξ	0.004

B. Modeling of the coil generator

The moving parts of the harvester are the coil, made of cupper and the coil carrier, as shown in Fig. 1 (a). The coil is attached to the coil carrier, which is in the form of a piston made of plastic polyacetal material. The fixed parts of the harvester are the permanent magnets and the soft iron core, shown in Fig. 1 (b). They are placed in a configuration that provides constant magnetic field in the space in which the coil is located, Fig. 1 (c). The mass of the coil and the coil carrier has an approximate value of 1 gram, which is an important parameter as it does not make significant changes to the dynamics of the cantilever beam. By using this configuration, additional structural components for the harvester are avoided, such as: linear bearings, guide, and spring. The advantages of this concept are: fewer structural parts of the harvester, elimination of the influence of the stiffness and damping of the harvester on the structure, and no frictional energy losses.

The specifications for the coil are given in Table III:

Table III.

Parameter	Value
Specific resistance - ρ	1.6*10^{-8} Ωm
Diameter of wire – d_w	0.0001 m
Diameter of coil – d	0.0145 m
Hight of coil – h	5.3 mm
Length of coil – L	3.3 m
Number of loops (windings) – N	73
Resistance of the coil - R	7.4 Ω

a) b) c)

Figure 1. Physical model of the coil generator

The disadvantages of this concept are: the need to adjust the concentricity between the coil and the permanent magnet and the non-axial movement of the coil in relation to the permanent magnet. This limits the movement of the coil and may physically damage the coil.

C. Mathematical model of the harvester

The mathematical model of the electromagnetic harvester can be expressed through the resistance (R), inductivity (L) and a small electrical input, as shown on Fig. 2.

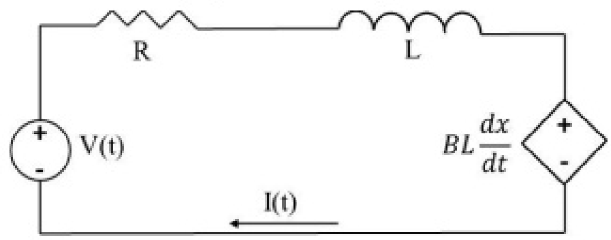

Figure 2. Mathematical model of an electromagnetic harvester

The mathematical representation of the electromagnetic harvester can alternatively be given by the following equation:

$$V_t = RI_t + L\frac{dI}{dt} + Bl\frac{dx}{dt} \tag{6}$$

where V_t is the input from the cantilever beam vibrations, RI_t is the voltage drop due to the resistance R, $L\frac{dI}{dt}$ is the self-induced voltage and $Bl\frac{dx}{dt}$ is the voltage generated in the coil due to movement in the magnetic field.

The induction and the current are calculated while the magnetic inductance where the coil is located is obtained experimentally using a Linear Hall effect sensor, type Honeywell SS495A. The mean value was calculated and adopted as the value for the magnetic inductance $B = 0.26\ T$.

An equivalent mechanical model of the electromagnetic coil harvester is given on Fig. 3.

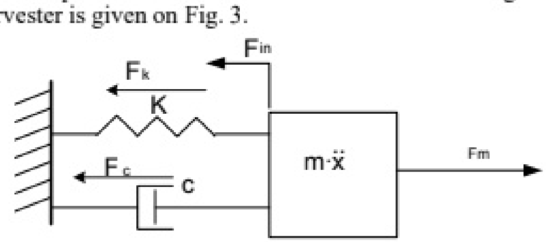

Figure 3. An equivalent mechanical model of the electromagnetic coil harvester

The mathematical model of the equivalent mechanical model of the electromagnetic coil harvester can be expressed as:

$$F_m - F_{in} - F_k - F_c = 0 \tag{7}$$

where,

- $F_m = BlI$ is the electromagnetic (Lorenc) force,
- $F_{in} = m\ddot{x}$ is the inertial force.
- $F_k = kx$ is the stiffness of the spring.
- $F_c = c\dot{x}$ is the damping force.

Therefore, the differential equation of the electromagnetic harvester can be written as

$$m\ddot{x} = BlI - kx - c\dot{x} \tag{8}$$

If the electromagnetic harvester is seen as a single independent component, then the transfer function can be calculated through a Laplace transformation of equations (6) and (8) where the final form is given in equation (9) which shows the dependence between the position of the coil and the voltage of the system:

$$\frac{X_s}{V_s} = \frac{Bl/mL}{s^3 + s^2 + \left(\frac{mR + cL}{mL}\right) + s\left(\frac{cR + kL + Bl^2}{mL}\right) + \frac{KR}{ml}} \tag{9}$$

The position of the electromagnetic harvester is determined by two set conditions. The first requirement is the relationship between the maximum allowable operation range of the coil and the maximum expected displacement of the cantilever at the location where the harvester is positioned. Using the following relationship and the values from Table III the possible positions for the harvester can be calculated.

$$\frac{y_a}{y_{max}} = \frac{x^2(3l - x)}{2l^3} \tag{10}$$

where, $y_a = \pm 2.6\ mm$ is the maximum allowable displacement of the coil within the harvester and $y_{max} = \pm 15\ mm$ is the maximal displacement of the cantilever beam. By inputting these values in expression (16) a cubic equation is obtained and the solutions are:

$$-0.277x^3 + 0.25x^2 - 0.00265 = 0 \tag{11}$$

$$x_1 = -0.09\ m\ ;\ x_2 = 0.11\ m\ ;\ x_3 = \ 0.88\ m$$

The only possible placement for the setup is at the distance of $x_2 = 0.11m$. With this position of the harvester the second requirement is also fulfilled which states that the harvester should not be placed at one of the nodes of vibration. The final experimental configuration is shown at the end of Chapter III.

III. Simulation and Experimental Results of Electromagnetic System

A simulation of the system has been made using the Simulink package for MATLAB which validates the mathematical model as well as allowing for flexibility in the experimental research.

Fig. 4 represents the simulation model for the electromagnetic energy harvesting system. The cantilever beam is modeled using the relations in chapter II.A. for a reduced equivalent model.

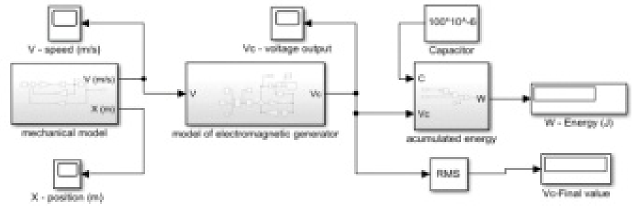

Figure 4. A simulation model of an electromagnetic energy harvesting system

The outputs of the mechanical model are displacement and speed of deformation of the cantilever beam. The speed at which the magnetic field changes affects the voltage in the circuit, hence it is an input to the electromagnetic harvester model. The voltage value along with the values for the length of the coil, the magnetic induction and the capacitor that is used to store the energy are needed within the model of the electromagnetic harvester which is modeled using the relations in chapter II.C. is shown in Fig. 5.

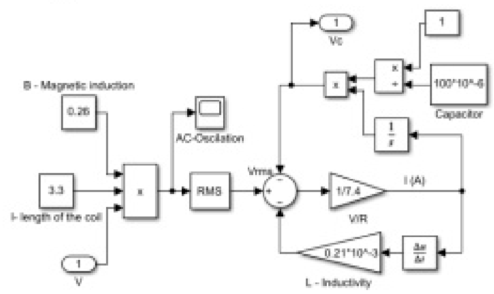

Figure 5. Simulation model of electromagnetic generator

This model outputs a value for the harvested voltage from the system, which is later used to calculate the accumulated energy for the whole system. The simulation is executed in 30 seconds and the obtained results are shown on Fig. 6. For better representation of the results, the figures present the behavior of the system until 14 seconds which is the time when the oscillations start to stabilize.

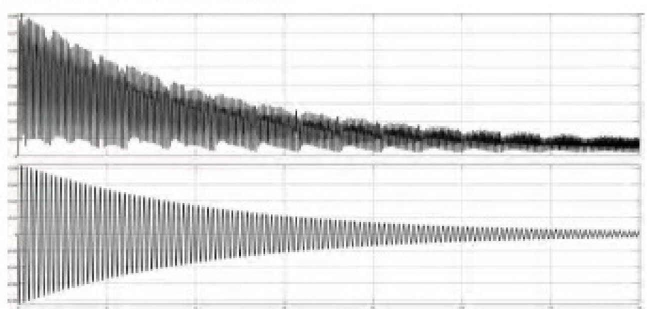

Figure 6. Speed of deformation of the free end of the cantilever beam [m/s] (top), Harvested voltage [V] (bottom)

It can be seen that the free end of the cantilever beam oscillates with a speed of 56 mm/s and it takes around 25 s for the oscillations to completely stabilize. On Fig. 6 (bottom) it can be seen the peak value for the harvested voltage is 0.055V.

Following the simulation an experimental model of the energy harvesting system has been developed. The model is based on the parameters given on Table I, for the cantilever beam and in Table III, for the coil generator. The final setup is shown on Fig. 7.

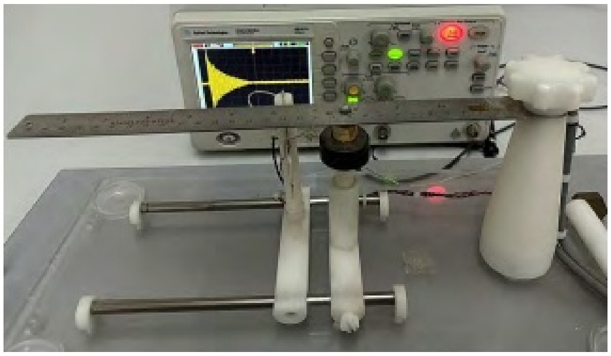

Figure 7. Experimental model of of an electromagnetic energy harvesting system

The experimental setup validates the results achieved by the simulation in MATLAB. Using an oscilloscope the values for the voltage output of the coil generator are measured and they are comparable with the simulation results achieving a real voltage output of 0.06 V.

IV. CONCLUSION

Theoretical analysis and mathematical modeling of an electromagnetic energy harvesting system attached to an elastic mechanical structure cantilever has been performed in this paper. In order to validate the obtained mathematical model, simulation and experiment of the energy harvesting system have been conducted. The proposed system uses the energy from its environment and converts it into usable electrical energy.

The modeling and theoretical analysis of the dynamic behavior of the mechanical system uses Euler-Bernoulli theory of elasticity of linear supports. The explanation of the electromagnetic behavior of the system bases on the Faraday's law, which quantifies the induced electromagnetic force.

The experimental setup consists of a generator which is placed in a configuration that provides a constant magnetic field in the space where the console is located. The main advantage of the system consists in eliminating the need of additional structural components for the actuator, such as: linear bearings, guide and spring. In this way, the potential frictional energy losses of these components and their additional influence on the dynamics of the mechanical construction are removed. The adjustment of the mathematical model of the actuator, which becomes an integral part of the mechanical construction, can be singled out as a disadvantage of this concept.

The simulation and experimental data have been based on the mathematical calculations for the complete electromagnetic energy harvesting system and it was shown that the final results between the two are comparable achieving low fluctuations in the results. The system is designed as a proof of concept for energy harvesting. It is modeled as a simple cantilever beam with larger dimensions which do not affect the electromagnetic harvester. It should be noted that the final output voltage that is achieved by the system needs to be converted to DC using a AC/DC converter. This was tested and the value of this voltage is too low to pass through the diodes on the rectifier hence a different method for storing the voltage needs to be used. This limitation is mainly due to the size of the electromagnetic coil and having a different coil would yield higher voltage gains.

REFERENCES

[1] JA. Paradiso, T. Starner, Energy scavenging for mobile and wireless electronics, Pervasive Comput. IEEE 4 (1) (2005) 18–27,

[2] S Mitcheson P D, Green T C, Yeat E M and Holmes H S 2004 Architectures for vibration-driven micropower generators J. Microelectromech. Syst. 13 429–40

[3] Anton S R and Sodano H A 2007 A review of power harvesting using piezoelectric materials (2003–2006) Smart Mater. Struct. 16 R1

[4] Khaligh A, Zeng P and Zheng C 2010 Kinetic energy harvesting using piezoelectric and electromagnetic technologies—state of the art IEEE Trans. Ind. Electron. 57 850–60

[5] Arnold D 2007 Review of microscale magnetic power generation IEEE Trans. Magn. 43 3940–51

[6] Yang B, Lee C, Kee W L and Lim S P 2010 Hybrid energy harvester based on piezoelectric and electromagnetic mechanisms J. Micro/Nanolith. MEMS MOEMS 9 023002

[7] Tang L and Yang Y 2011 Analysis of synchronized charge extraction for piezoelectric energy harvesting Smart Mater. Struct. 20 085022

[8] Suzuki Y 2011 Recent progress in MEMS electret generator for energy harvesting IEEJ Trans. Electr. Electron. Eng. 6 101–11

[9] S.P. Beeby, M.J. Tudor, N.M. White, Energy harvesting vibration sources for microsystem applications, Measurement Science and Technology 17 (2006) R175–R195

[10] Williams C B and Yates R B 1996 Analysis of a micro-electric generator for microsystems Sensors Actuators A 52 8–11

[11] Alexander, C.K., 2013. Fundamentals of electric circuits. McGraw-Hill,.

[12] Huicong Liu, BoWoon Soon, Nan Wang, C. J. Tay, Chenggen Quan and Chengkuo Lee, "Feasibility study of a 3D vibration-driven electromagnetic MEMS energy harvester with multiple vibration modes," 2012.

[13] Ozge Zorlua and Haluk Kulaha, "AMEMS-based energy harvester for generating energy fromnon-resonant environmentalvibrations," 2011.

[14] Shuo Cheng and David P. Arnold, "A study of a multi-pole magnetic generator for low-frequency vibrational energyharvesting," J. Micromech. Microeng. **20**, 025015 (10pp) (2010).

[15] T. Von B¨uren and G. Tr¨oster, "Design and optimization of a linear vibration-driven electromagnetic micro powergenerator," Sensors Actuators A **135**, 765–75 (2007).