Analytical and experimental model of an energy harvesting system

Dejan Shishkovski, Damjan Pecioski, Anastasija Ignjatovska, Simona Domazetovska, Maja Anachkova
Faculty of Mechanical Engineering department of Mechanics
ss. Cyril and Methodius University (UKIM)
Skopje, N. Macedonia
dejan.shishkovski@mf.edu.mk

2023 International Conference on Control, Automation and Diagnosis (ICCAD) | 979-8-3503-4707-4/23/$31.00 ©2023 IEEE | DOI: 10.1109/ICCAD57653.2023.10152439

***Abstract*—The conversion of energy in real processes is not ideal and as a result, there is always a certain percentage of wasted energy. This energy, such as mechanical energy from vibrations, could be reused by employing energy-harvesting systems. In this paper, a cantilever beam energy harvesting system for low-power consumption devices using piezoelectric materials has been developed. This system uses the byproduct of mechanical vibrations generated from a rotating machine. Piezoelectric materials' characteristic to generate voltage as a result of being deformed, combined with their high robustness and efficiency makes them an ideal energy-harvesting tool. A mathematical model of these materials along with the whole system has been developed in Matlab and a simulation of the model using a Final Element Method (FEM) software has been created. Thereafter, an experimental model has been created in order to test and validate the obtained results from the simulations. A higher voltage output of the piezoelectric materials could be achieved by increasing the strain of the material. Therefore, maximal strain of the beam for the given frequency range of the vibration source would have to be achieved. Using the Euler-Bernoulli method, the beam dimensions have been calculated so that its' natural frequency matches the operating machine frequency. Finally, a consumer has been connected in order to calculate the power output of the designed system, so its practical application could be tested. Within this work it has been demonstrated that the power output of the system is able to power low power devices for a couple of seconds with a down time for recharging showing the possible implementation in systems**

Keywords— energy harvesting from vibrations, piezoelectric transducers, natural frequency, power generation

I. Introduction

Science and researchers are constantly trying to develop sustainable solutions for the energy crisis which is one of the main problems facing humanity today. As a result, macro energy harvesting technologies that use renewable energy sources such as solar energy, kinetic energy, thermal, or bio-energy have noted rapid progress. Nonetheless, continuous development has been made in micro energy harvesting technologies with the goal of decreasing the percentage of wasted energy and low-power electronics. Energy harvesting on a global macro scale is studied with the goal of replacing fossil fuels with clean energy, while on a micro-scale the goal differs. With the recent advances in wireless and microelectromechanical systems (MEMS) technology, the demand for portable electronics and wireless sensors grows rapidly.

At some level, mechanical vibrations are always present in machines and even when they are in the nominal range there is a possibility for harvesting this energy. The current state of the art is mainly based on energy harvesting using a cantilever beam that is excited at its natural frequency coupled with one or more piezoelectric layers in order to achieve maximum electrical output [1]. The frequent use of a cantilever beam compared to other structures is due to its lower resonant frequency which provides higher stress and strain with less ambient vibrational force [2]. Liu et al. [3] studied the dynamic analytical solution of a piezoelectric stack utilized in an actuator and a generator based on the linear piezo-elasticity theory. Complementary, the nonlinear flexural vibrations behavior of piezoceramic actuators has been studied by Parashar et al. [4]. Mukherjee and Chaudhuri [5] demonstrated the effect of large deformations on piezoelectric materials and structures under time-varying loads. describes the design, fabrication, and testing of micro-electrostatic vibration-to-electricity converters.

Improvement in the performance of energy harvesting can be seen in [6] where the authors use a frequency-up conversion effect on stack-based piezoelectric generators. S.Zhao and A.Erturk [7] present a theoretical and experimental investigation of piezoelectric energy harvesting from broadband random vibrations using cantilever bimorphs. Within [8] piezoelectric energy harvesting is achieved from vortex-induced vibrations. Within [9] S.Roundy describes the design, fabrication, and testing of a micro electromagnetic system that converts ambient vibration electricity. The feasibility of low-frequency MEMS energy harvesting has been demonstrated by L.M. Miller in [10] where they experimentally tested a device by subjecting it to low-frequency vibrations of known amplitude an frequency. The addition of a proof mass on a cantilever beam and the effect of its variation on the energy harvesting performance is presented in [11], where the authors concluded that a maximum power output is achieved when a proof mass of 25% of the beam's total mass is added.M Microfabricated piezoelectric vibration energy harvesters and their performance are investigated by DuToit and Wardle in [12]. The feasibility of energy harvesting using impact events in composite beams with piezoelectric transducers are investigated by Margelis in [13]. Energy can be harvested from random vibrations using piezoelectric materials. This type of setup is investigated in [14] where the authors analyze theoretical and experimental aspects of energy harvesting from random vibrations. The summary of the state of the art as well as the results obtained by the various authors is given in table 1.

Table 1 Benchmark models for energy

Reference	Methodology	Results
[6]	Stacked piezoelectric transducer with frequency up generation	7.8 mW/cm^3 with frequency up 2.7mW/cm^3 without frequency up
[7]	Broadband vibrations with cantilever beam	5.7mW peak 1.28mW RMS
[8]	Energy harvesting from vortex induced vibrations	150µW at wind speed of 3.2m/s
[9]	Parallel plate capacitor that moves in response to vibrations	116µW
[10]	Low frequency vibration excitation of parallel plate capacitor with a proof mass attached to one of the plates	0.3mW at 1Hz
[11]	Cantilever beam with proof mass	0.5mW
[12]	Cantilever beam with proof mass	0.6mW
[13]	Impact excitation in composite cantilever beam	0.8mW
[14]	Energy harvesting from random vibrations	2.9mW

Having reviewed the state of the art in the field of energy harvesting with piezoelectric transducers the methodology for this paper was formed. The aim of this paper is to design an energy harvesting system that can be used in industry to power low power sensors. The proposed energy harvesting system can be easily coupled with an existing machine. The use of a cantilever beam has been employed since the vibrational frequency of the first mode of vibration can be controlled simply by varying the length of the beam. It has been decided to use a rotational machine as an excitation source and the cantilever beam has been modeled to reach maximum deformation, first mode of vibration, depending on the frequency of rotation of the excitation machine

In this paper, a mathematical model of an energy harvesting system has been calculated, designed using FEM simulations, and validated by an experimental model. The Euler-Bernoulli method has been used in order to create an analytical model of the cantilever energy harvesting system. Within the experimental model, a piezoelectric transducer is used to transform vibrational to electrical energy. The transducer is placed on a cantilever beam that is coupled together with a rotational machine

II. MATHEMATICAL MODELING

A. Mathematical model of the beam

Mechanical systems with continuously distributed mass theoretically have infinite degrees of freedom. Solving the dynamics of these systems requires certain approximations such as material homogeneity and an ideally constant width of the beam along the entire length. Using the Euler- Bernoulli method, the bending energy of the structure and the kinetic energy of the transversal movement of the beam is considered. The differential equation of motion of a continuous beam according to the Euler-Bernoulli method is shown with the following expression:

$$EI_y \frac{\partial^4 y(x,t)}{\partial x^4} = f(x,t) - \rho A \frac{\partial^2 y(x,t)}{\partial t^2} \qquad (1)$$

where y is the displacement of the beam ρ_b is its density and A_b is its area of the cross-section, whereas $\rho_b A_b \frac{\partial^2 y(x,t)}{\partial t^2}$, is kinetic energy $E_b I_b \frac{\partial^4 y(x,t)}{\partial x^4}$ is potential energy and $f(x,t)$ is the excitation force.

If the excitation force is a harmonic function the differential equation can be rewritten as:

$$\frac{E_b I_b}{\rho_b A_b} \frac{\partial^4 y(x,t)}{\partial x^4} + \frac{\partial^2 y(x,t)}{\partial t^2} = \frac{F_0}{\rho_b A_b} \sin(\omega t)\, \delta\left(x - L_f\right) \qquad (2)$$

The general solution of the equation (2) can be expressed as:

$$y(x,t) = \sum_{i=1}^{\infty} T_i(t) Y_i(x) \qquad (3)$$

where $T_i(t)$ is the ith modal coordinate of the shape and $Y_i(t)$ is the ith mode shape of the beam

B. Mathematical modeling of the piezoelectric transducer

According to the study in [15], the mathematical model for a piezoelectric transducer is written below and shows the relationship between the moment of the beam and the generated voltage as a function of time:

$$V(t) = -\frac{6 g_{31} \frac{E_b t_b}{E_p t_p}\left(1 + \frac{t_b}{t_p}\right)}{b_p t_p \left\{1 + \left(\frac{E_b t_b}{E_p t_p}\right)^2 \cdot \left(\frac{t_b}{t_p}\right)^2 + 2 \frac{E_b t_b}{E_p t_p}\left[2 + 3\frac{t_b}{t_p} + 2\left(\frac{t_b}{t_p}\right)^2\right]\right\}} \cdot M(t) \qquad (4)$$

where $g_{31}\left[V \cdot \frac{m}{N}\right]$ is the voltage constant, $E_b\left[\frac{N}{mm^2}\right]$ is Young's modulus of the material of the beam, and $E_p\left[\frac{N}{m^2}\right]$ is Young's modulus of the piezoelectric material. The geometry of the system is represented by $t_b[m]$, $t_p[m]$ *and* $b_p[m]$, thickness of the beam, thickness and width of the piezoelectric transducer, respectively.

The generated power can be calculated as:

$$P(t) = V(t) \cdot I(t) \tag{5}$$

$$I(t) = C_P \cdot \frac{dV}{dt} \tag{6}$$

$$C_P = \frac{d_{31} \cdot A_p}{g_{31} \cdot t_p} \tag{7}$$

Where I(t) is the current expressed as a function of time, C_p is the capacitance of the piezoelectric material, A_p is the effective area of the piezoelectric material and d_{31} is the piezoelectric strain coefficient.

C. Analytical modeling using Matlab

Analytical modeling has been made in Matlab in order to present a relationship between the characteristics of the piezoelectric transducer and the cantilever and to generate maximal power output from the system. The input parameters used in the calculations are presented in table 2.

Table 2. Parameters of the beam and piezoelectric transducer

Parameter	Notation	Value	Unit
Cantilever beam			
Parameter	**Notation**	**Value**	**Unit**
Length	L_b	0.230	m
Width	b_b	0.04	m
Thickness	t_b	0.0015	m
Density	ρ_b	7850	kg/m^3
Young's Modulus	E_b	$2{\cdot}10^{11}$	Pa
PZT (P-876.A12)			
Parameter	**Notation**	**Value**	**Unit**
Young's Modulus	E_p	23.3	GPa
Length	L_p	0.061	m
Width	b_p	0.35	m
Thickness	t_p	0.0005	m
Resistance	R	1000	kΩ

Using these parameters, an optimal thickness and length ratio between the piezoelectric transducer and cantilever beam have been calculated which is shown in figures 1 and 2

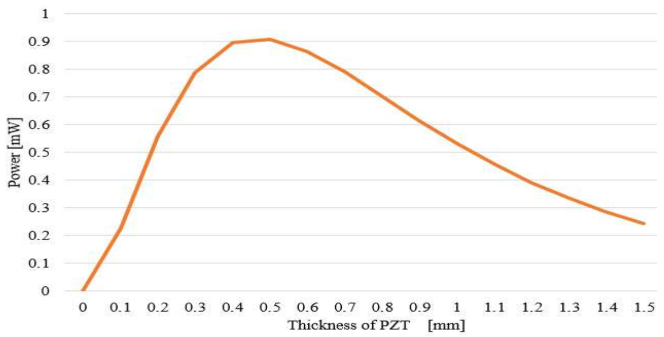

Figure. 1. Output electrical power [mW] as a function of thickness of PZT [mm]

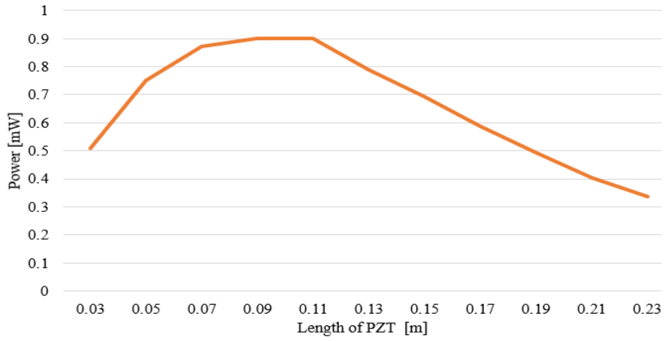

Figure. 2. Output electrical power [mW] as a function of length of PZT [mm]

Both graphs for the thickness and length ratio have similar characteristic as a function and behave correspondingly. By increasing the dimensions of the PZT with respect to the cantilever beam, the generated power output increases until a certain crossing point. Once this crossing point has been reached, further increase in the dimensions of the PZT results in decreased power output. The optimal dimensions for the PZT for a cantilever beam of 230 [mm] have been found to be a length of 100 [mm] and a thickness of 0.5 [mm]. In the experimental model a PZT-A12 has been used where the thickness corresponds to the optimal dimensions but the length is 61 [mm].

D. FEM Simulations

A simulation of the behavior of the system has been developed using a Finite Element Method software. The cantilever beam coupled together with the piezoelectric transducer with the dimensions of the real system are shown on figure 3.

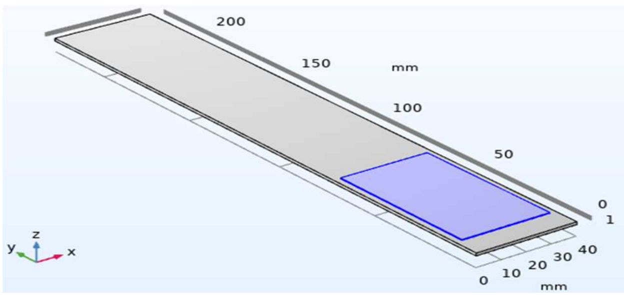

Figure 3. FEM model of the energy harvesting cantilever beam with PZT

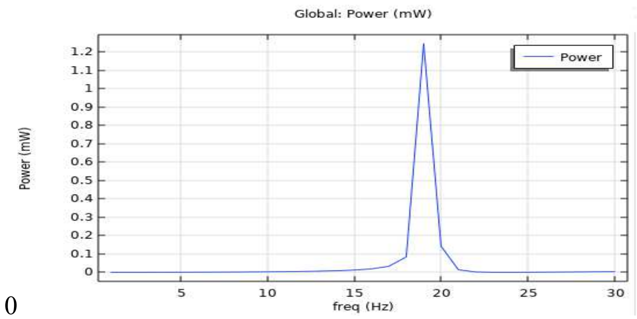

Figure 4. Generated electrical power from FEM model of the energy harvesting cantilever beam

The obtained results from the FEM simulations (Figure 4) show an electrical power output of 1.25 [mW] excited by a periodic force with an amplitude of 3.4 [mm/s] and a frequency of 20 [Hz].

III. Experimental model

A. Impact "bump-test" method

In order to determine the real resonant frequency of the cantilever beam together with the PZT, an experimental method is used which methodology is explained in [16]. The hammer impact testing method or bump test method is used to check whether the previously defined dimensions of the beam are appropriate for the generated vibrations of the given rotational machine. The frequency of the rotational system can be defined by the user using a frequency regulator. However, considering the loses of the real system it is chosen to be around 20 [Hz], which is a standard value for these machines. Using the "bump test" method, the propagation of vibrations throughout the beam is measured and the resonant frequency is shown in the figure 5

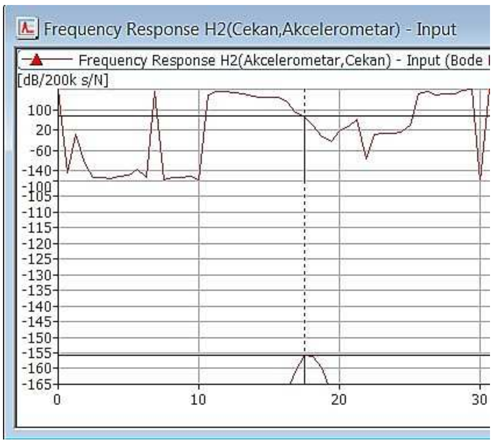

Figure 5. Results from measurement of natural frequency of the beam using bump test method.

As shown in the figure, the natural frequency of the cantilever beam together with the PZT is around 18 [Hz]. This value slightly differs from the result of the simulations, which is due to the adhesive and the non-homogeneity of the real materials themselves.

B. Experimental setup

The experimental model as shown in [16], has been used for further analysis. It consists of a cantilever beam with one piezoelectric transducer, coupled to a rotational shaft which is the source of the vibrations, figure 6. The PZT gives an AC voltage and an energy harvesting module (PI-E-821) needs to be implemented to convert, the voltage to DC in order for it to be stored or used. The piezoelectric transducer is placed as close as possible to the clamped end, as seen in figure 1 and 2, in order to achieve maximum stress. The frequency of rotation of the machine is set to the resonant frequency of the cantilever beam.

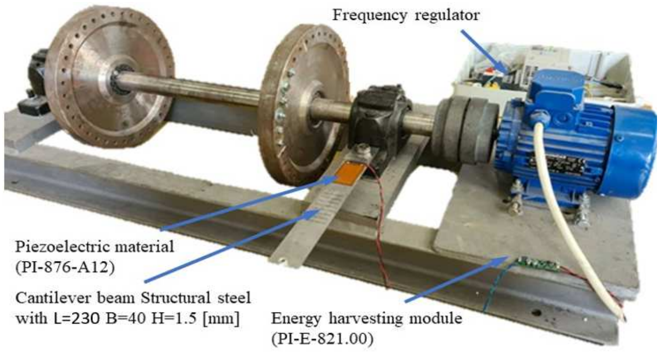

Figure 6. Experimental setup of the energy harvesting cantilever beam [16]

C. Energy harvesting module

The schematics of the energy harvesting module PI-E-821.00 is shown in figure 7. The Graetz bridge (figure 8) converts the AC voltage to DC and the capacitors of the module store the energy.

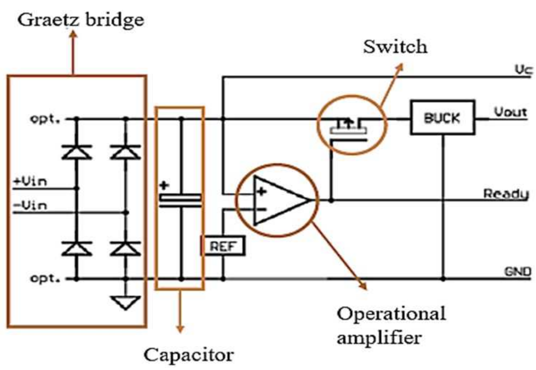

Figure 7. Schematic of PI-E-821.00

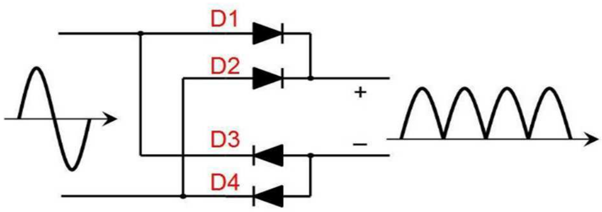

Figure 8. Schematic of Graetz bridge

The module has two capacitors each with a capacitance of C=100 [μF] placed in parallel, resulting with a final capacitance of C=200 [μF]. Another limitation of this module is that the maximum voltage it can store is 15 [V]. The module has an integrated mosfet switch which controls the discharge and charge of the capacitors. The switch turns on and allows discharge of the capacitors only when the module has collected a voltage of at least 12 [V] making this a discharge boundary condition. Once the voltage of the capacitors falls below 6 [V] the switch turns off allowing time for the capacitors to be charged making this the charge boundary condition. Having this configuration of the energy harvesting module the power generated towards the consumer can be represented with the equation:

$$W_{el} = \frac{C}{2} \cdot (V_h^2 - V_l^2) = \frac{V_r^2}{R} \cdot \Delta t \quad (8)$$

where W_{el} [J] is electric energy, C [F] is capacitance, V_h, V_l [V] are the discharge and charge boundary condition, respectively, V_r [V] is output voltage of the module which is 3.3 V, R [Ω] is the resistance of the connected consumer and Δt [s] is the discharge time.

IV. Results and discussion

To measure the possible power output of the system, an oscilloscope was used for signal acquisition, a 3 [kΩ] consumer connected to the 3.3 [V] output of the energy harvesting module. Important parameters that were observed include the time needed to charge from 0 to 15 [V] (figure 9), and the time during which the system can exert power to the consumer (figure 10).

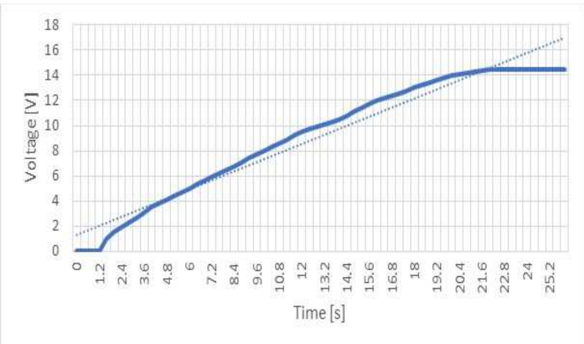

Figure 9. Time needed to charge from 0 to 15 V

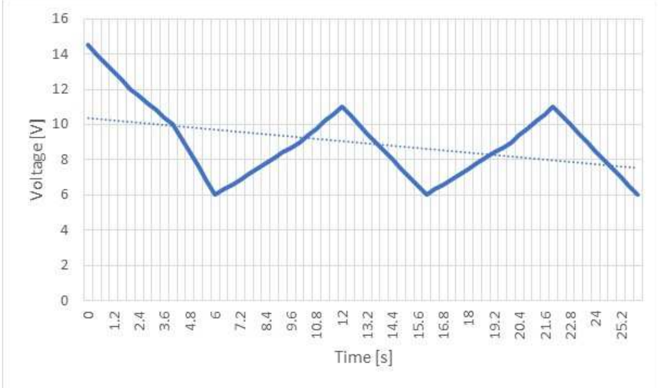

Figure 10. The time during which the system exerts power

Looking at the figure 9, it can be seen that the time needed to charge from 0 to 15 [V] is 23 seconds. While these parameters are informative, the system does not work in this range. As discussed above, the system can give power to the consumer only when it has a charge between 6-12 [V] as shown on the figure 10. It can be seen that the time needed to discharge from 15 [V] to 6 [V] is 6.2 seconds. After this the system needs 8 seconds to recuperate to 12 [V] when the switch turns on allowing for current to flow and another discharge occurs till 6 [V] this time in 3.6 second. According to equation (5), the maximum used energy that can be stored is 22.5 [mJ], however this is not the operating range of the module. Within the operating range of 6 to 12 [V], while the module is simultaneously charged by vibrations and discharged by the consumer, the maximum energy is around 12.5 [mJ]. If all familiar values for the parameters are inputted in equation (5), an equilibrium between the left and the right side is not reached, which is due to the losses that occur within the module.

The Matlab and FEM simulations have given starting information about the system which has been validated by the experiment. Namely the FEM simulations give the frequency at which the system will give a maximal power output and the power which can be achieved while the matlab simulations gave the optimal ratios between the beam and the piezoelectric transducer. This has been validated by the experiment seeing that the optimal values of the power output are indeed at the resonant frequency of the beam shown by figure 4 and figure 5. Furthermore, the power output given by the FEM simulation in figure 4 is closely related to the output of the real system where a continuous power output can be achieved of around 1.3mW

V. Conclusion

Within this paper, an analytical an experimental model of a unimorph piezoelectric cantilever beam energy harvesting system from vibrations has been made. Based on the obtained results, it can be concluded that, in this stage of work, this system can continuously give power to low power consumers with a supply voltage of 3.3V that use less than 2mW power. Devices that consume larger power can work with segmented periods of work and idle time which will depend on the ratio of charge and discharge of the capacitors. Using the equation 8, an example can be given for a device that uses 15 [mW] of power. Within a time, interval of 11 seconds, this device can work for 1 second with an idle time of 10 seconds in which the capacitors will be recharged. Comparing the results with the state of the art it can be seen that larger power is achieved with this setup. Having achieved continuous power of 2mW and a maximum power of 15mW this type of energy harvester can be used to power low power sensors which are placed in difficult to reach locations where ambient vibrations are present from rotational machines.

Further research would investigate the generated voltage and power of a bimorph piezoelectric cantilever beam and the influence various types of PZT have on the obtained outputs. Furthermore, the implementation of such low power devices can be tested in real systems where wireless network sensors will be placed which would send information of the system to the user. This information can later be used for condition monitoring of the systems in which they are used. These low powered devices would be placed in difficult to reach locations where a conventional power system would not be applicable.

References

[1] A. Erturk; D.J. Inman: A distributed parameter electromechanical model for cantilevered piezoelectric energy harvesters, Journal of vibration and acoustics, 130(4) (2008).

[2] U.J. Gogoi and T. Shanmuganantham: Energy harvesting of cantilever with silicon tip mass-based MEMS energy scavengers. 2nd International Conference on Electronics and Communication Systems (ICECS, IEEE), 185-188 (2015).

[3] X. Liu; J. Wang; W. Li; dynamic analytical solution of a piezoelectric stack utilized in an actuator and a generator. Appl. Sci. 2018, 8, 1779

[4] S.K. Parashar; U.V. Wagner; P. Hagedorn; nonlinear shear-induced flexural vibrations of piezoceramic actuators: experiments and modeling. J. Sound vib. 2005, 285, 989–1014

[5] A. Mukherjee; A.S. Chaudhuri; nonlinear dynamic response of piezolaminated smart beams. Comput. Struct. 2005, 83, 1298–1304.

[6] Peng, Y., Xu, Z., Wang, M., Li, Z., Peng, J., Luo, J., Xie, S., Pu, H. and Yang, Z., 2021. Investigation of frequency-up conversion effect on the performance improvement of stack-based piezoelectric generators. Renewable Energy, 172, pp.551-563.

[7] Zhao, S. and Erturk, A., 2012. Electroelastic modeling and experimental validations of piezoelectric energy harvesting from broadband random vibrations of cantilevered bimorphs. Smart Materials and Structures, 22(1), p.015002

[8] Zhang, L.B., Abdelkefi, A., Dai, H.L., Naseer, R. and Wang, L., 2017. Design and experimental analysis of broadband energy harvesting from vortex-induced vibrations. Journal of Sound and Vibration, 408, pp.210-219.

[9] S. ROUNDY; P.K. WRIGHT; K.S. PISTER; Micro-electrostatic vibration-to-electricity converters. In ASME international mechanical engineering congress and exposition (2002, January Vol. 36428, pp. 487-496)

[10] Miller, L.M., Halvorsen, E., Dong, T. and Wright, P.K., 2011. Modeling and experimental verification of low-frequency MEMS energy harvesting from ambient vibrations. Journal of Micromechanics and Microengineering, 21(4), p.045029

[11] Kim, M., Hoegen, M., Dugundji, J. and Wardle, B.L., 2010. Modeling and experimental verification of proof mass effects on vibration energy harvester performance. Smart Materials and Structures, 19(4), p.045023

[12] DuToit, N.E. and Wardle, B.L., 2007. Experimental verification of models for microfabricated piezoelectric vibration energy harvesters. AIAA journal, 45(5), pp.1126-1137

[13] Margelis, N., Plagianakos, T.S., Karydis-Karandreas, P. and Papadopoulos, E.G., 2021. Assessment of impact energy harvesting in composite beams with piezoelectric transducers. Sensors, 21(22), p.7445

[14] Lefeuvre, E., Badel, A., Richard, C. and Guyomar, D., 2007. Energy harvesting using piezoelectric materials: Case of random vibrations. Journal of Electroceramics, 19, pp.349-355.

[15] A. Ignjatovska; D. Pecioski; D. Shishkovski; M. Anachkova; S. Domazetovska; analytical modeling and fem simulations of energy harvesting cantilever beam; BENA international conference GREEDIT, Skopje, 2022, p 129

[16]. M. Anachkova; J. Jovanova; Z. Petreski; impact testing of hydro generators end-winding in different temperature state; journal of vibroengineering, vol. 22, issue 2, 2020, p. 393-402

[17]. D. Pecioski; A. Ignjatovska; D. Shishkovski; S. Domazetovska; M. Anachkova; design of an energy harvesting system using piezoelectric materials; BENA international conference GREEDIT, Skopje, 2022, p 128