

# DYNAMIC STIFFNESS OF HIGH SPEED CUTTING (HSC) LINEAR MOTOR MACHINE TOOL

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**ABSTRACT:** In this paper a model of the feed drive system with disturbance force for High Speed Cutting (HSC) linear motor machine tool is given. The dynamic stiffness for the proposed model is analysed. A simulation of the influence of some parameters on feed drive dynamic stiffness is performed with the simulation program MATLAB & SIMULINK. Correctness of the proposed model is verified with an experimental measurement of the dynamic stiffness of the feed drive on the prototype HSC linear motor machine tool (HSC 11).

**KEYWORDS:** dynamic stiffness, feed drive, HSC linear motor machine tool

## 1 INTRODUCTION

Feed drive systems are widely applied to CNC machine tools, robots, manipulators, assembly machines etc (Brecher & Weck, 2021; Gross et al., 2006; Altintas et al., 2008; Lopez de Lacalle, 2009; Soucek, 2004). The use of linear motors in the feed drive systems is rapidly increasing, especially for the high-speed machine tools. The linear motor direct feed drives have increased sensitivity to the load disturbances. In this case dynamic stiffness of the feed drive is vitally important to attain high accuracy (Lu et al., 2022; Kehne et al., 2020; Ding et al., 2020, Ping et al. 2004; Gao, 1999; Lyu et al. 2020).

Generally feed drive stiffness can be defined as an influence of the disturbance force (torque) on the position (angular position) deviation (Brecher & Weck, 2021; Gross et al., 2006; Altintas et al., 2008; Lopez de Lacalle, 2009; Soucek, 2004; Pandilov et al., 2015; Ebrahimi & Whalley, 2000).

Investigations about feed drive stiffness are very seldom presented in the literature. The results presented in (Gao et al., 2021, Elfizy et al., 2005, Jamaludin et al., 2007; Kakino et al., 1997, 1995; Pislaru et al., 2004, Pritschow et al., 2003; Ebrahimi & Whalley, 2000; Losic, 1985) are of particular interest, but unfortunately the most of them are for the feed drives with classical structure, rotary motor and ball screw-nut.

## 2 A MODEL OF LINEAR MOTOR FEED DRIVE SYSTEM WITH DISTURBANCE FORCE

Figure 1 and Figure 2 show an original model of the linear motor feed drive system with disturbance force.

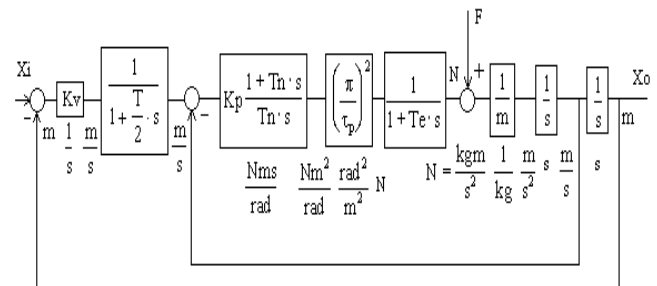


Fig.1 Model of the linear motor feed drive system with disturbance forces

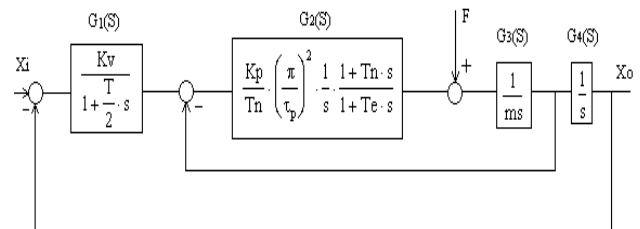


Fig.2 The model from Figure 1 after arranging

All the relevant parameters are given below: Kv-position loop gain [1/s], T-sampling period [s], s-Laplace operator, Kp-proportional gain of the velocity controller [Nms/rad], Tn-time constant of the velocity controller [s],  $\tau_p$ -pole pitch of the linear motor [m], Te-time constant of the current controller [s], F-disturbance force [N], m-mass of the working table and moving part of the linear motor [kg], Xi-input position [m], Xo-output position [m].

The transfer function between output position and disturbance force is given with Equation 1:

$$\frac{Xo(s)}{F(s)} = \frac{G_3(s) \cdot G_4(s)}{1 + G_2(s) \cdot G_3(s) + G_1(s) \cdot G_2(s) \cdot G_3(s) \cdot G_4(s)} \quad (1)$$

With substituting the transfer functions  $G1(s)$ ,  $G2(s)$ ,  $G3(s)$ , and  $G4(s)$ , from Figure 2 in Equation 1 we obtain

$$\frac{Xo(s)}{F(s)} = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \quad (2)$$

Coefficients in the Equation 2 are:

$$\begin{aligned} b_3 &= T_e \cdot \frac{T}{2}, b_2 = T_e + \frac{T}{2}, b_1 = 1, b_0 = 0, a_5 = m \cdot T_e \cdot \frac{T}{2}, \\ a_4 &= m \cdot \left( T_e + \frac{T}{2} \right), a_3 = m + \frac{K_p}{T_n} \cdot \left( \frac{\pi}{\tau_p} \right)^2 \cdot \frac{T}{2}, a_2 = \\ &\frac{K_p}{T_n} \cdot \left( \frac{\pi}{\tau_p} \right)^2 \cdot \left( T_n + \frac{T}{2} \right), a_1 = \frac{K_p}{T_n} \cdot \left( \frac{\pi}{\tau_p} \right)^2 \\ &\text{and } a_0 = \frac{K_p}{T_n} \cdot \left( \frac{\pi}{\tau_p} \right)^2. \end{aligned}$$

### 3 DYNAMIC STIFFNESS OF THE LINEAR MOTOR FEED DRIVE SYSTEM

One of the important requirements with regard to feed drive system concerns its sensitivity to load disturbances. The qualitative measure of this sensitivity is the feed drive stiffness (Brecher & Weck, 2021; Gross et al., 2006; Altintas et al.; Zirn, 2008; Lopez de Lacalle, 2009; Soucek, 2004; Ebrahimi et Whalley, 2000; Gao et al., 2021).

The dynamic feed drive system stiffness can be defined as a measure of influence of disturbance force  $F$  (torque  $T$ ) on the output position  $Xo$  (angular position) deviation in the transient period (Brecher & Weck, 2021; Gross et al., 2006; Altintas et al.; Zirn, 2008; Lu et al., 2022; Kehne et al., 2020; Ding et al., 2020, Pandilov et al., 2015; Ping et al. 2004; Ebrahimi & Whalley, 2000; Losic, 1985):

$$Sd(s) = \frac{F(s)}{Xo(s)} = \frac{T(s)}{\theta o(s)} \quad (3)$$

For the model from Figure 1 and Figure 2 the equation for dynamic stiffness for linear motor feed drive system becomes

$$Sd(s) = \frac{F(s)}{Xo(s)} = \frac{a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{b_3 s^3 + b_2 s^2 + b_1 s + b_0} \quad (4)$$

where the coefficients  $a_5, a_4, a_3, a_2, a_1, a_0, b_3, b_2, b_1$  and  $b_0$  are equal to the coefficients in the Equation 2.

### 4 SIMULATION OF THE LINEAR MOTOR FEED DRIVE DYNAMIC STIFFNESS

The influence of the position loop gain  $K_v$ , sampling period  $T$ , proportional gain of the velocity controller  $K_p$ , time constant of the velocity controller  $T_n$ , time constant of the current controller  $T_e$  and mass of the working table and moving part of the linear motor  $m$  on the dynamic stiffness has been investigated with the simulation program MATLAB & SIMULINK.



**Fig. 3 High Speed Cutting CNC linear motor machine (HSC 11) at the Institute of Production Management, Technology and Machine Tools (PTW) at Technical University Darmstadt, Germany**

As an object for simulation was taken the x-axis of the prototype HSC linear motor machine (HSC 11) (Figure 3) developed on the Institute of Production Management, Technology and Machine Tools (PTW) at Technical University Darmstadt, Germany.

Important parameters for the x-axis are  $K_v=166.7$  [1/s]=10 [(m/min)/mm],  $K_p=9$  [Nms/rad],  $T_n=0.004$  [s],  $T=0.001$  [s],  $T_e=62.5$  [ $\mu$ s],  $m=150$  [kg] and  $=0.036$  [m].

In the simulations one parameter has been changed, and the others were kept constant.

In fact with the simulations a position deviation in time domain  $Xo(t)$  caused by step disturbance force  $F=75.2$  [kg]= 738 [N] is shown.

The simulations are performed on the models presented on Figure 1 and Figure 2.

$$X_o(t) = L^{-1} \left[ \frac{X_o(s)}{F(s)} \cdot \frac{F}{s} \right] \quad [m] \quad (5)$$

To estimate the dynamic stiffness we will use the following equation

$$S_d = \frac{F}{\max X_o(t)} \quad [N/m] \quad (6)$$

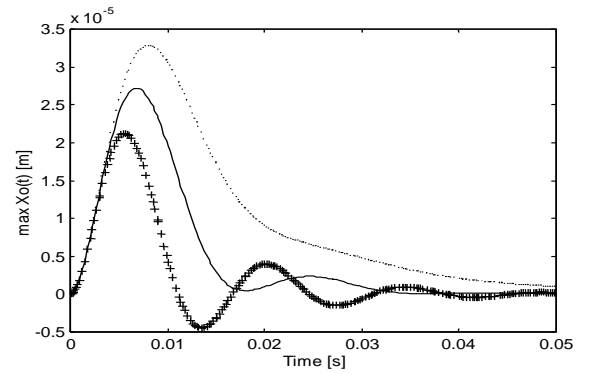
where  $F$  is the disturbance force and  $\max X_o(t)$  is the maximal position deviation caused by the disturbance force.

Simulations have shown that increasing position loop gain  $K_v$  and proportional gain of the velocity controller  $K_p$  increase dynamic stiffness (Figure 4 and Figure 6). Increasing time constant of the velocity controller  $T_n$ , sampling period  $T$ , time constant of the current controller  $T_e$  and mass of the working table and moving part of the linear motor  $m$ , decrease the dynamic stiffness (Figure 4, Figure 5, Figure 7 and Figure 8).

From the simulation results it is obvious that  $K_v$ ,  $K_p$  and  $T_n$  have the most significant influence on the dynamic stiffness (Table 1).

**Table 1 Influence of different parameters on dynamic stiffness  $S_d$**

Changing range	$K_v=2.5-40$ [(m/min)/mm]	$K_p=2.25-36$ [Nms/rad]	$T_n=1-16$ [ms]
Increasing in %	1600	1600	1600
Changing $S_d$ [N/ $\mu$ m]	19.84-47.49	6.03-132.26	61.45-16.29
Changing $S_d$ in %	+139.36	+2093.36	-73.49
Changing range	$T_e=31.25-250$ [ $\mu$ s]	$m=37.5-600$ [kg]	$T=0.25-2$ [ms]
Increasing in %	800	1600	1600
Changing $S_d$ [N/ $\mu$ m]	27.35-26.18	33.04-24.11	28.49-23.88
Changing $S_d$ in %	-4.28	-27.03	-16.18



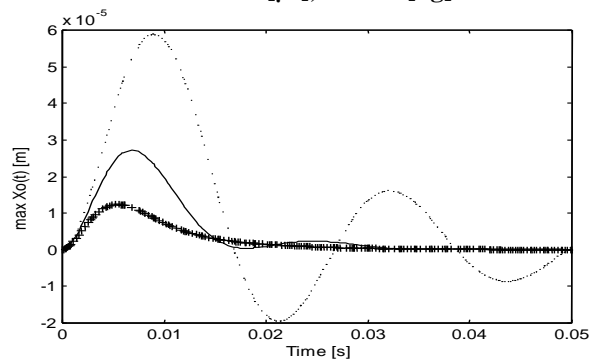
(----)  $K_v=10$ [(m/min)/mm]

(.....)  $K_v=5$ [(m/min)/mm]

(+++ )  $K_v=20$ [(m/min)/mm]

$K_p=9$  [Nms/rad],  $T_n=4$  [ms],  $T=1$  [ms],

$T_e=62.5$  [ $\mu$ s],  $m=150$  [kg]



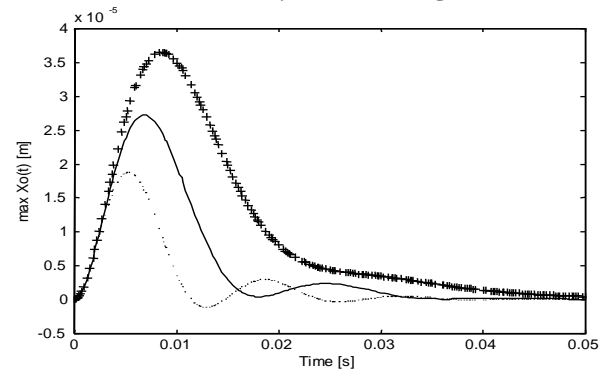
(----)  $K_p=9$ [Nms/rad]

(.....)  $K_p=4.5$ [Nms/rad]

(+++ )  $K_p=18$ [Nms/rad]

$K_v=10$  [(m/min)/mm],  $T_n=4$  [ms],  $T=1$  [ms],

$T_e=62.5$  [ $\mu$ s],  $m=150$  [kg]



(----)  $T_n=4$ [ms]

(.....)  $T_n=2$ [ms]

(+++ )  $T_n=8$ [ms]

$K_v=10$  [(m/min)/mm],  $K_p=9$  [Nms/rad],

$T=1$  [ms],  $T_e=62.5$  [ $\mu$ s],  $m=150$  [kg]

**Fig. 4 Influence of the different values of  $K_v$ ,  $K_p$  and  $T_n$ , on the maximal position deviation  $\max X_o(t)$  caused by disturbance step force  $F=75.2$  kg=738 N**

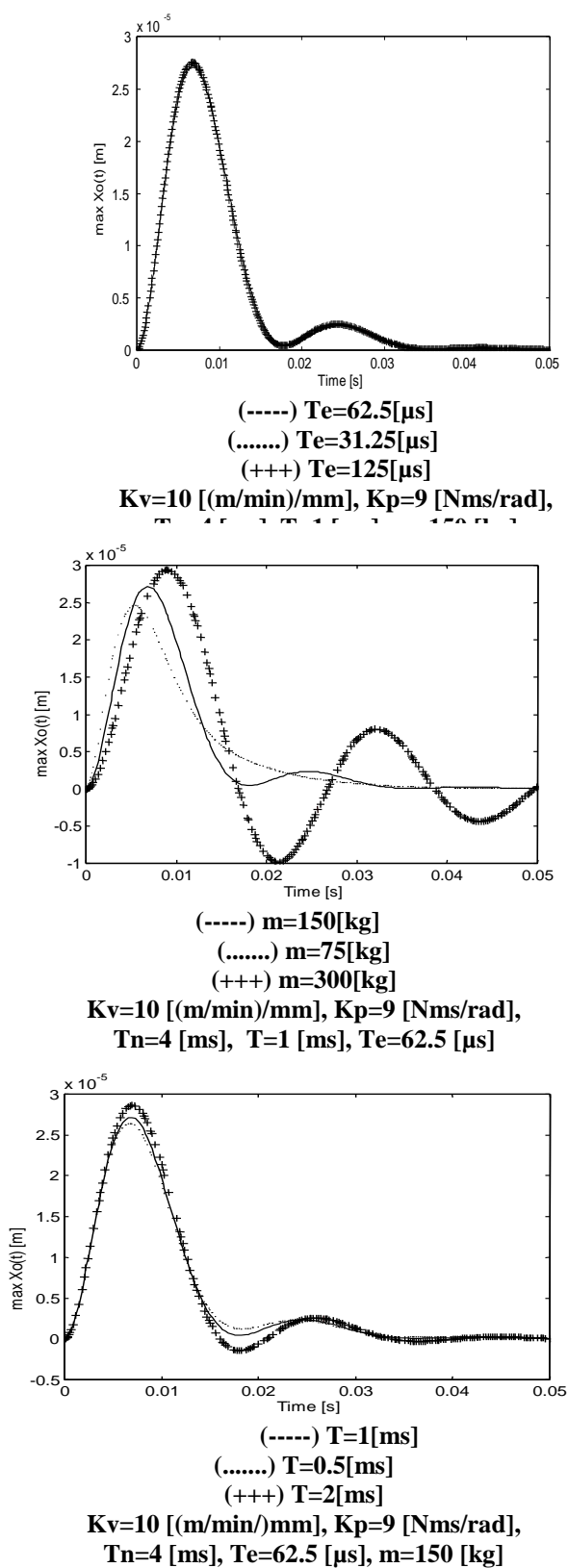


Fig. 5 Influence of the different values of  $T_e$ ,  $m$  and  $T$  on the maximal position deviation  $\max X_o(t)$  caused by disturbance step force  $F = 75.2 \text{ kg} = 738 \text{ N}$

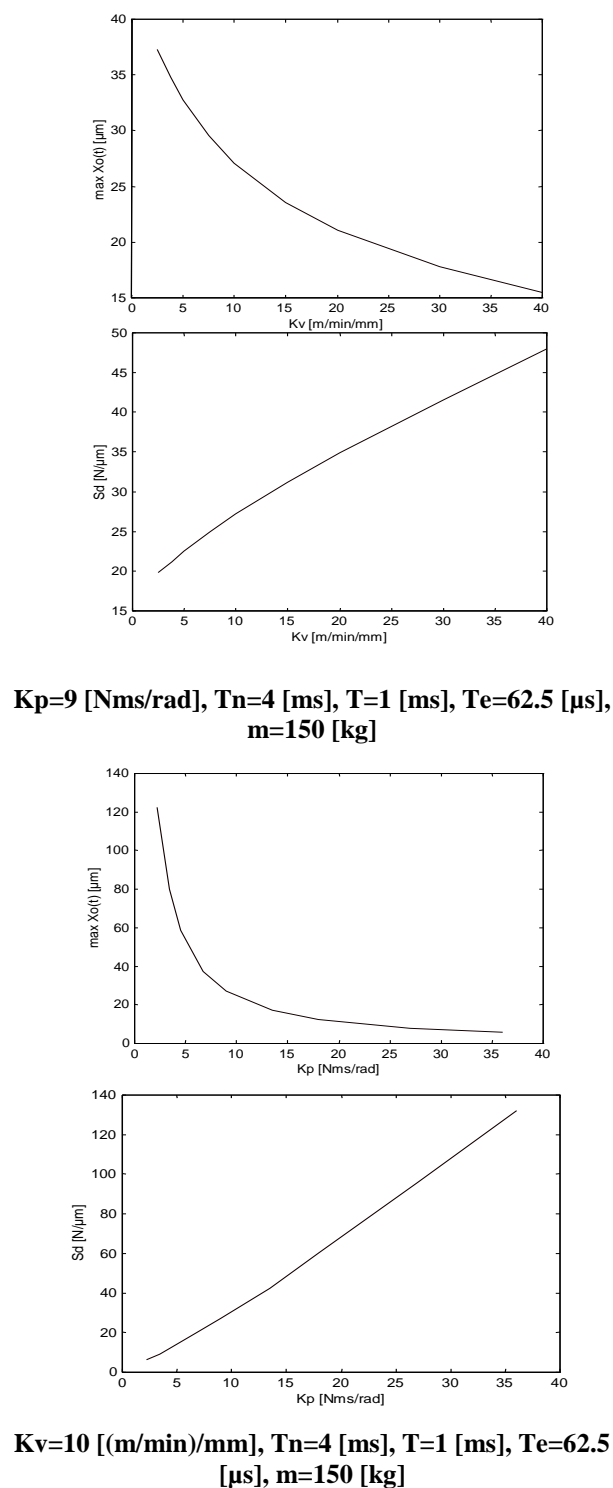
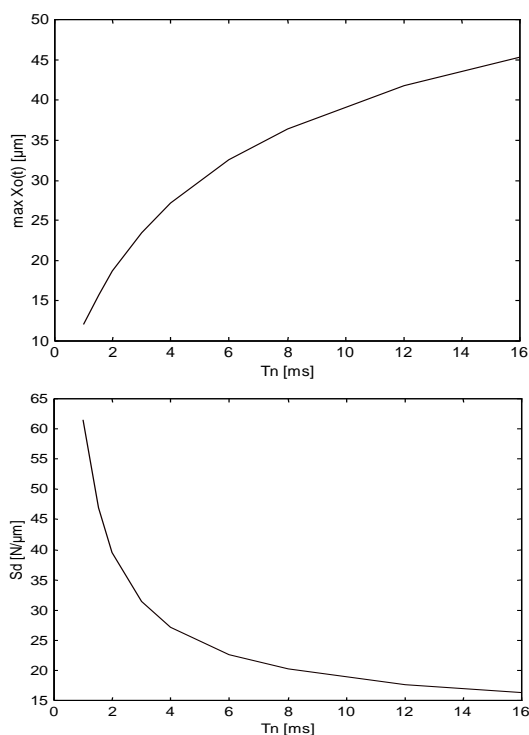
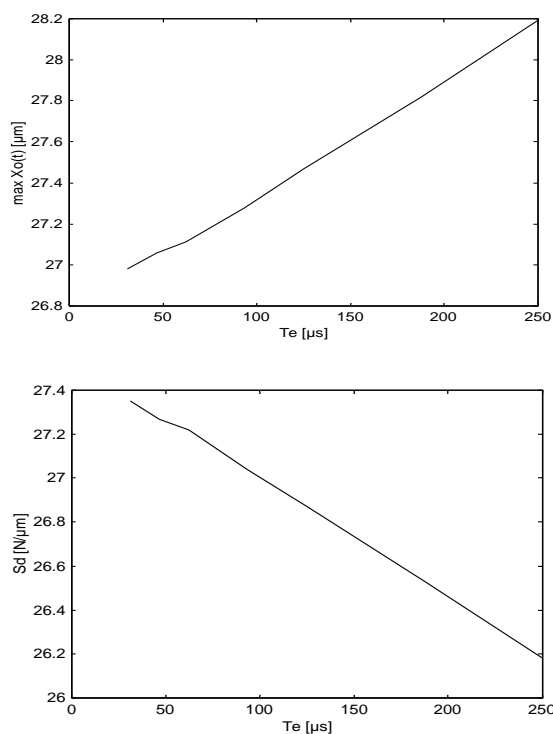


Fig. 6 Influence of the variation of  $K_v$  and  $K_p$  on the maximal position deviation  $\max X_o(t)$  and dynamic stiffness  $S_d$  caused by disturbance step force  $F = 75.2 \text{ kg} = 738 \text{ N}$

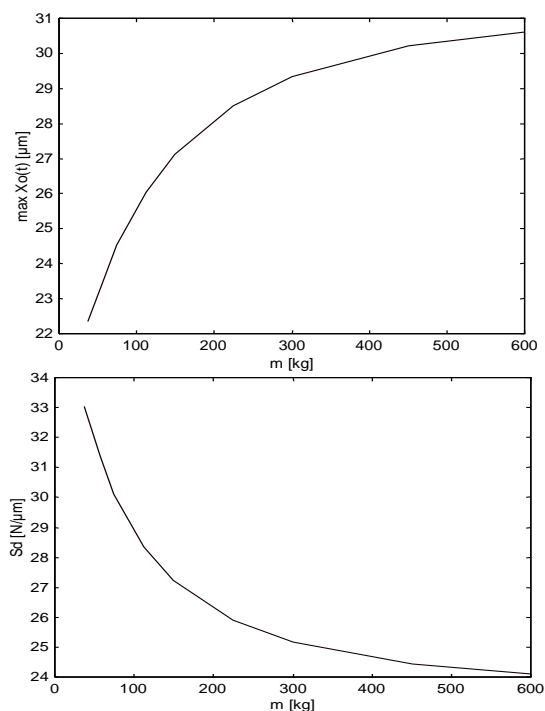


$K_v=10$  [(m/min)/mm],  $K_p=9$  [Nms/rad],  $T=1$  [ms],  
 $T_e=62.5$  [ $\mu\text{s}$ ],  $m=150$  [kg]

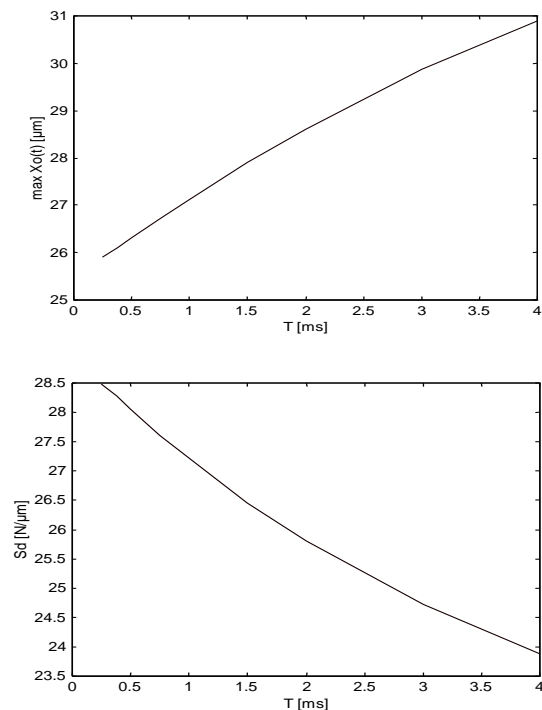


$K_v=10$  [(m/min)/mm],  $K_p=9$  [Nms/rad],  $T_n=4$  [ms],  
 $T=1$  [ms],  $m=150$  [kg]

**Fig. 7** Influence of the variation of  $T_n$  and  $T_e$  on the maximal position deviation  $\max X_o(t)$  and dynamic stiffness  $S_d$  caused by disturbance step force  $F=75.2$  kg=738 N



$K_v=10$  [(m/min)/mm],  $K_p=9$  [Nms/rad],  $T_n=4$  [ms],  
 $T=1$  [ms],  $T_e=62.5$  [ $\mu\text{s}$ ]



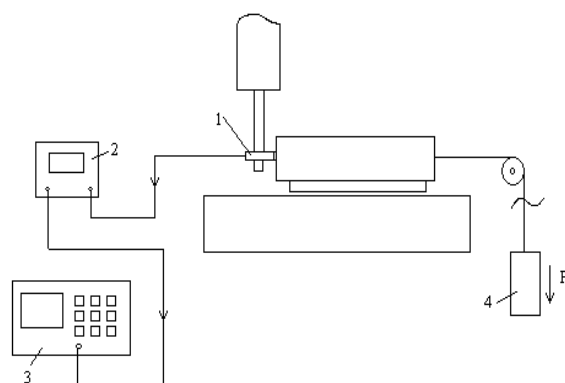
$K_v=10$  [(m/min)/mm],  $K_p=9$  [Nms/rad],  $T_n=4$  [ms],  
 $T_e=62.5$  [ $\mu\text{s}$ ],  $m=150$  [kg]

**Fig. 8** Influence of the variation of  $m$  and  $T$  on the maximal position deviation  $\max X_o(t)$  and dynamic stiffness  $S_d$  caused by disturbance step force  $F=75.2$  kg=738 N

## 5 EXPERIMENTAL DETERMINATION OF THE DYNAMIC STIFFNESS OF THE LINEAR MOTOR DRIVE

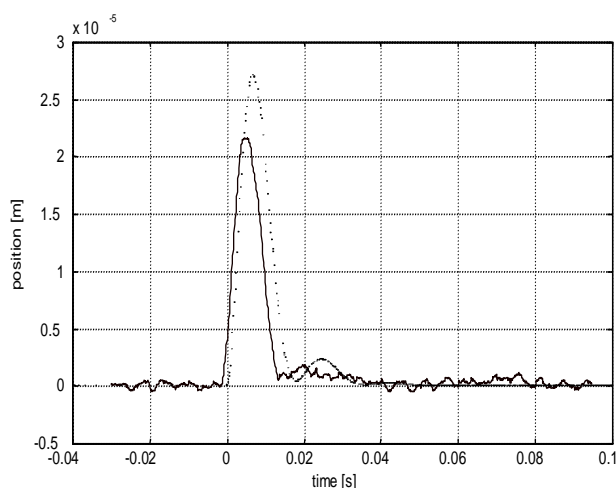
In order to verify the proposed model and simulations of the linear motor feed drive dynamic stiffness, an experiment on the concrete feed drive system (x-axis) of the HSC linear motor machine (HSC 11) was performed. The experimental installation is shown in Figure 9.

The results of the experiment compared with the simulation are given in Figure 10.



**Fig. 9 Experimental installation for determining dynamic stiffness of the linear motor machine feed drive (1. inductive transducer, 2. amplifier EH MW210-13-5,**

**3. dynamic signal analyser HP 35670A, 4. disturbance force (load). )**



**Fig. 10 Comparison of the experimental (-----) and with simulation (.....) obtained maximal position deviation caused by the step disturbance force  $F=75.2$  kg=738 N.**

Table 2 gives a comparison between experimentally and by simulation obtained values of the maximal position deviation  $\max X_o(t)$  and dynamic stiffness  $S_d$ .

**Table 2 Comparison between experimentally and by simulation obtained values of the maximal position deviation  $\max X_o(t)$  and dynamic stiffness  $S_d$**

	maximal position deviation $\max X_o(t)$ [ $\mu\text{m}$ ]	dynamic stiffness $S_d$ [ $\text{N}/\mu\text{m}$ ]
Experiment	21.7 [ $\mu\text{m}$ ]	34 [ $\text{N}/\mu\text{m}$ ]
Simulation	27.11 [ $\mu\text{m}$ ]	27.22 [ $\text{N}/\mu\text{m}$ ]

From Table 2 it is obvious that the difference between experimentally and by simulation obtained dynamic stiffness of the linear motor feed drive system is less than 20% which is acceptable and sufficient for practice.

## 6 CONCLUSION

A model of the linear motor feed drive system with disturbance force was proposed. By simulations it has been shown that bigger values of the position loop gain  $K_v$  and proportional gain of the velocity controller  $K_p$  increase dynamic stiffness. On the other hand increasing time constant of the velocity controller  $T_n$ , sampling period  $T$ , time constant of the current controller  $T_e$  and mass of the working table and moving part of the linear motor  $m$ , decrease dynamic stiffness. Correctness of the proposed model and simulation of the dynamic stiffness of the linear motor feed drives was experimentally verified. The difference between experimentally and by simulation obtained dynamic stiffness is less than 20%, which is completely acceptable.

## 7 ACKNOWLEDGEMENT

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