MATHEMATICAL MODEL FOR THE ANALYSIS OF UNSTEADY FLOW DURING A PRESSURE PIPELINE BREAK USING THE METHOD OF CHARACTERISTICS

GOCE TASESKI [1], NIKOLA KRSTOVSKI [2]

[1] *Faculty of Civil Engineering Skopje, N. Macedonia,* taseski@gf.ukim.edu.mk

[2] *Faculty of Civil Engineering Skopje, N. Macedonia,* nikolatudence@gmail.com

1 Abstract

The goal of developing a mathematical model to analyze the occurrence of non-stationary flow is to conduct an analysis of an existing pressure pipeline - specifically, a siphon from an irrigation system in Macedonia. In this system, when a defect occurs in the pipeline, particularly in the section experiencing the highest pressure, the phenomenon of non-stationary flow arises - creating a significant vacuum within the pipeline, leading to a massive breakdown. Specifically, this concerns a steel pipeline with a diameter of D=1640 mm, operating at a maximum working pressure of 8.2 bar in the lowest part of the siphon. When non-stationarity occurs, a substantial negative pressure (vacuum) emerges, resulting in complete destruction of the pipeline over a length of 500 m.

Keywords: method of characteristics, unsteady flow, water hammer, negative pressure

2 Introduction

The appearance of unsteady flow in a pressurized pipeline is a constant occurrence and, depending on the magnitude of pressure and flow changes in the pipeline, it can cause significant damage to the pipeline. Therefore, even in pipelines where large oscillations in flow and pressure are least expected to occur within a short time interval, it is necessary to conduct hydraulic analyses for unsteady flow. These analyses are much more complex than those for steady flow and require more time, which is why they are not always performed.

However, with today's capabilities of computer technology, it is possible to perform hydraulic analyses for unsteady flow by appropriately applying the basic hydraulic equations and methods for solving them within previously defined boundary conditions that vary depending on each specific case.
In such analyses, the first step is to detect the causes of the occurrence of unsteady flow, which can be various and include:

- secondary opening or closing of a valve at the beginning, middle or end of the pipeline
- improper commissioning of the pipeline
- defect occurrence of a pipeline, etc.

According to the aforementioned, in this paper, by detecting the causes of unsteadiness and the geometric characteristics of the system – the pipeline, the procedure for creating a hydraulic model for unsteady flow will be presented. This model will be applied to a real pipeline to detect the causes of failures within it.

Additionally, with the mathematical model thus created, it will be possible to determine whether the occurrence of unsteady flow will cause damage to part or the entire pipeline. This primarily depends on the material and diameter of the pipeline, as well as the initiator of the unsteadiness, which are the basic input parameters in the model.

3 Basic equations for unsteady flow

According to Wylie [3] (1993), the water hammer is defined as the hydraulic variable occurrence of flow, which causes an increase of overpressure in a pipeline system. The unsteady flow can be generated by certain operational measures such as: opening or closing of the valve, turning the pumps on or off, abrupt cracking of the pipe etc.

Starting points in the mathematical description of the unsteady flow [4] are the basic laws in the mechanics of fluids:

- Dynamic equation – equation of motion and
- Continuity equation

The final form of the dynamic equation for unsteady flow in closed systems under pressure:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + \frac{\lambda}{2D}V|V| = 0 \tag{1}$$

The convective acceleration $V\partial V/\partial x$ or acceleration along the pipe is significantly lower compared to the local acceleration $\partial V/\partial t$ or acceleration over time, so mostly that convective acceleration is overlooked, and the dynamic equation is written:

$$\frac{\partial V}{\partial t} + g\frac{\partial P}{\partial x} + \frac{\lambda}{2D}V|V| = 0 \tag{2}$$

Assuming that the density of the fluid changes very little in terms of piezometric height (ρ = const), the equation of continuity gets the following form:

$$V\frac{\partial P}{\partial x} + \frac{\partial P}{\partial t} - V sin\alpha + \frac{a^2}{g}\frac{\partial V}{\partial x} = 0 \tag{3}$$

Where a is the speed of propagation of the pressure wave and it is determined by the ratio of compression of the fluid and the module of elasticity of the tube:

$$a = \sqrt{\frac{K}{\rho\left(1 + \frac{K}{E}\frac{D}{e}c_1\right)}} \tag{4}$$

The coefficient c_1 depends of the pipe anchorage and is equal to:

- $c_1=1-\mu/2$ – pipe anchorage only at the upstream
- $c_1=1-\mu^2$ – pipe anchorage throughout against axial movement
- $c_1=1$ – pipe anchorage with expansion joints throughout

In Eq. (1) to (4), P denotes the static pressure at the centerline of the pipeline at location x and time t, V is the average velocity of flow, D is the pipe diameter, λ is the friction factor in the Darcy-Weisbach formula, x is the distance along the centerline of the pipe, α is the angle between the horizontal and the centreline of the pipe, taken as positive for the pipe sloping downwards in the direction of positive x, g is the gravitational constant; and a is the celerity of the pressure surge, i.e. the velocity with which the surge is propagated relative to the liquid. The positive direction for V coincides with that for x.

3.1 Method of characteristics for solving basic equations of unsteady flow

With the method of characteristics [5] the basic partial differential equations which are not integrable in closed form, are transformed into ordinary differential equations which have a solution in a closed form.

The basic equations, the equation of continuity and the dynamic equation can be designated with L_1 и L_2:

$$L_1 = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial P}{\partial x} + \frac{\lambda}{2D}V|V| = 0 \tag{5}$$

$$L_2 = \frac{\partial P}{\partial t} + V\frac{\partial P}{\partial x} - Vsin\alpha + V\frac{a^2}{g}\frac{\partial V}{\partial x} = 0 \tag{6}$$

These linear equations can be combined as follows:

$$L = L_1 + \chi L_2 = \chi\left[\left(V + \frac{g}{\chi}\right)\frac{\partial \Pi}{\partial x} + \frac{\partial \Pi}{\partial t}\right] + \left[\left(V + \chi\frac{a^2}{g}\right)\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t}\right] + \frac{\lambda}{2D}V|V| - \chi V\,sin\,\alpha = 0 \tag{7}$$

The two dependent variables, the speed V and the pressure Π are in a function from the position and time, V=V(x,t) и Π=Π(x,t). The material statements of these dependent variables are total accelerations which are determined by the convective and local acceleration:

$$\frac{dP}{dt} = \frac{\partial P}{\partial x}\frac{dx}{dt} + \frac{\partial P}{\partial t} \tag{8}$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial t} \tag{9}$$

Comparing the expression of the convective acceleration of equation (7) to those of equations (8) and (9), follows:

$$\frac{dx}{dt} = V + \frac{g}{\chi} = V + \frac{\chi a^2}{g} \tag{10}$$

Then equation (7) is written:

$$\chi\frac{dP}{dt} + \frac{dV}{dt} + \frac{\lambda}{2D}V|V| - \chi g\,sin\,\alpha = 0 \tag{11}$$

The solution of equation (11) is:

$$\chi = \pm\frac{g}{a} \tag{12}$$

$$\frac{dx}{dt} = V \pm a \tag{13}$$

From the previous equation it can be concluded that it’s about two families of curves that are practically straight lines, where the speed of propagation is constant and many times faster than the basic flow, so the system of two partial differential equations are transformed into system of ordinary four differential equations which are marked with a C+ and C– and determine straight lines:

$$\left.\begin{aligned}\frac{dP}{dt} + \frac{a}{g}\frac{dV}{dt} + \frac{\lambda}{2D}V|V| = 0\\ \frac{dx}{dt} = +a\end{aligned}\right\} C^{+} \tag{14}$$

$$\left.\begin{aligned}\frac{dP}{dt} - \frac{a}{g}\frac{dV}{dt} + \frac{\lambda}{2D}V|V| = 0\\ \frac{dx}{dt} = -a\end{aligned}\right\} C^{-} \tag{15}$$

According the given numerical network, equations (14) and (15) can be written as follows:

$$\frac{d}{dt}\left(P \pm \frac{a}{g}V\right) + \lambda \frac{a}{D}\frac{V|V|}{2g} \mp V \sin\alpha = 0 \tag{16}$$

The previous equation can be integrated along the positive and negative characteristics, i.e. along the length of the lines AP and BP, as follows:

$$\int_{tA}^{tP} \frac{d}{dt}\left(P + \frac{a}{g}V\right) dt + \int_{tA}^{tP} \left(\lambda \frac{a}{D}\frac{V|V|}{2g} - V \sin\alpha\right) dt = 0 \tag{17}$$

$$\int_{tB}^{tP} \frac{d}{dt}\left(P - \frac{a}{g}V\right) dt + \int_{tB}^{tP} \left(\lambda \frac{a}{D}\frac{V|V|}{2g} + V \sin\alpha\right) dt = 0 \tag{18}$$

After integration, equations of positive and negative characteristic are written:

$$\frac{P_P - P_A}{\Delta t} + \frac{a}{g}\frac{V_P - V_A}{\Delta t} + \frac{\lambda a}{2gD} V_A|V_A| = 0 \tag{19}$$

$$\frac{P_P - P_B}{\Delta t} + \frac{a}{g}\frac{V_P - V_B}{\Delta t} + \frac{\lambda a}{2gD} V_B|V_B| = 0 \tag{20}$$

If it is known that in the hydraulic analysis it is important to determine the flow change and height position of the hydrodynamic line in any section along the pipe, and in a certain time interval, additional approximation is introduced that the cross section of the pipe along its entire length is constant, and if it is known that median speed can be determined by the equation *V=Q/A*, the previously stated equations, knowing the numerical network, for the pressure, can be written in the following form:

$$P_i^{n+1} = P_{i-1}^n - B(Q_i^{n+1} + Q_{i-1}^n) - MQ_{i-1}^n|Q_{i-1}^n| = 0 \tag{21}$$

$$P_i^{n+1} = P_{i-1}^n + B(Q_i^{n+1} - Q_{i-1}^n) + MQ_{i+1}^n|Q_{i+1}^n| = 0 \tag{22}$$

If:

$$B = \frac{a}{gA} \qquad \text{and} \qquad M = \frac{\lambda \Delta x}{2gDA^2} = 0 \tag{23}$$

Using the previous equations, for the pressure, i.e. for the height position of the hydrodynamic line, it can be written:

$$P_i^{n+1} = CP - BQ_i^{n+1} \tag{24}$$

$$P_i^{n+1} = CM + BQ_i^{n+1} \tag{25}$$

In the previous equations the article which include the slope of the pipes (sinα) is very small and often overlooked, so the equations (24) and (25) are written:

$$CP = P_{i-1}^n - BQ_{i-1}^n - MQ_{i-1}^n|Q_{i-1}^n| = 0 \tag{26}$$

$$CM = P_{i+1}^n - BQ_{i+1}^n + MQ_{i+1}^n|Q_{i+1}^n| = 0 \tag{27}$$

Knowing the piezometric height (Pi) in the time period (n+1), the flow (Qi) is determined by equations (26) and (27).

$$P_i^{n+1} = \frac{CP + CM}{2} \tag{28}$$

3.2 Borderline Conditions

The conditions of the flow that govern within the boundary of the system under pressure – the water supply system is defined as boundary conditions. Their definition is of crucial importance for getting the solution at the points in the system. Follow-on are the most common cases of boundary conditions encountered in the water supply systems [1, 2].

Serial connection of two pipes in a junction

$$Pressure: CP = \Pi_{1,N}^{n+1} = \Pi_{2,1}^{n+1} = \Pi^{n+1} \tag{29}$$

$$\text{Flow: } Q_{1,N}^{n+1} = Q_{2,1}^{n+1} = Q^{n+1} = \frac{CP_1 - CM_2}{B_1 + B_2} \tag{30}$$

Reservoir (open channel) at the end of pipeline

$$Pressure: \Pi_1^{n+1} = \Pi_R \tag{31}$$

$$\text{Flow: } \quad Q_1^{n+1} = \frac{(\Pi^{n+1} - CM)}{B} \tag{32}$$

Valve at the middle of the pipeline (pipe break point)

$$\begin{gathered}\text{Pressure: } \Pi_{1,N}^{n+1} = CP_1 - B_1 Q_{1,N}^{n+1}, \quad \Pi_{2,1}^{n+1} = CM_2 - B_2 Q_{2,1}^{n+1}, \\ CP_1 - B_1 Q^{n+1} - CM_2 - B_2 Q^{n+1} - C_1 Q^{n+1}|Q^{n+1}| = 0\end{gathered} \tag{33}$$

$$Flow: Q^{n+1} = \frac{-(B_1 + B_2) + \sqrt{(B_1 + B_2)^2 + 4C_1(CP_1 - CM_2)}}{2C_1} \tag{34}$$

4 Development and application of Mathematical Model

Unsteady flow in pipelines that transport pressurized water is a constant occurrence, regardless of the geometric characteristics of the pipeline, the amount of water being transported, the change in hydrostatic and hydrodynamic pressure along the length of the pipeline during steady-state conditions, as well as the boundary conditions at the entry and exit of the siphon. However, the magnitude of the changes in pressure and water flow in the siphon primarily depends on the cause of the unsteady flow, i.e., the initiator of the unsteadiness, which can be varied, including:

- Sudden release of water during siphon filling
- Opening/closing of a valve along the length or at the end of the siphon
- Defect in the pipeline – bursting of the pipe and sudden draining of the siphon

The application of the hydraulic model for unsteady flow aims to analyze the oscillations in flow and pressure in a pressurized pipeline - siphon in the event of a defect along the length of the pipeline - Case study of the siphon from the irrigation system "Makarija". Specifically, this involves analyzing a pipeline constructed in 1965 with a length of 2150 m and a maximum flow rate of 3.8 m^3/s. The pipeline is constructed from two materials:

- In the section where pressures are lower than 3.5 bars, reinforced concrete pipes lined on the inside with a steel sheet, 4 mm tgick and with a diameter of 1640 mm, have been applied.
- For the sections where the pressure in the pipeline exceeds 3.5 bars, steel pipes with a diameter of 1640 mm are used. The thickness of the pipe varies depending on the pressure: 6 mm, 8 mm and 9 mm.

On May 27, 2023, a major incident occurred in the lowest part of the pipeline - the siphon (see Fig.1), resulting in damage to a total length of 400-450 meters of the pipeline (see Fig.2). Although it was initially considered that such a failure was likely due to multiple factors such as aging of the pipe, local deformations, or torsion in the cross-section of the pipeline, the question arises as to why the damage extended over such a length if the defect occurred only in a small part of the pipe.

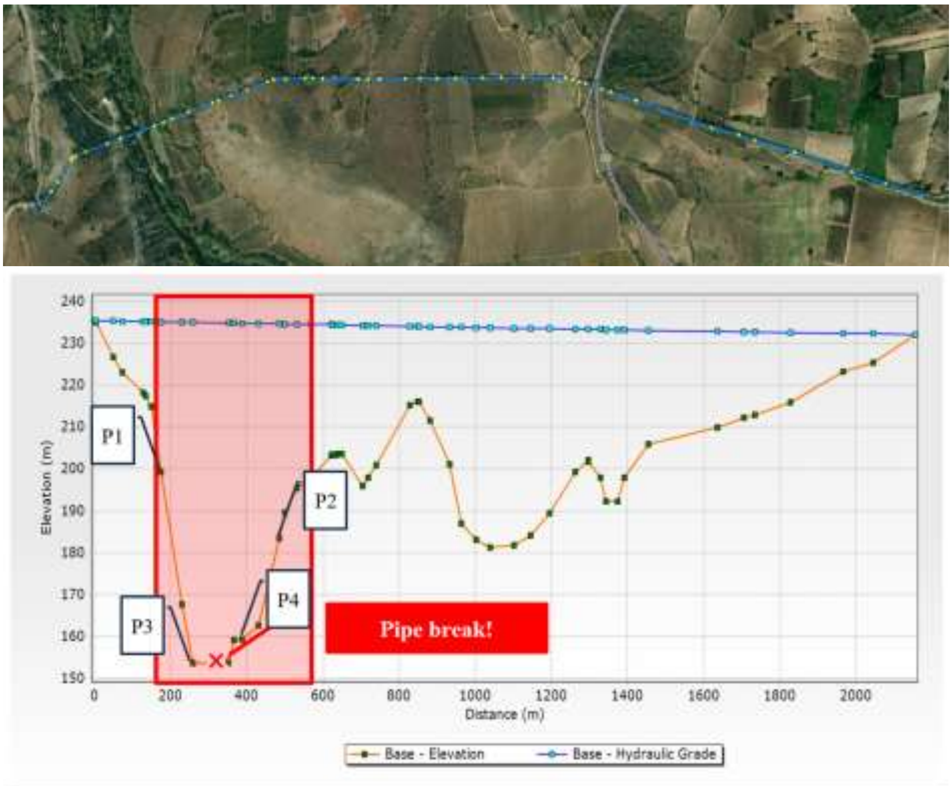

Figure 1. Layout and profile of the pipeline location of the defect, and the par that is damaged

It is important to note that no issues were observed in the operation of the siphon before the incident. Practically, for the siphon operator, such a major failure is a surprise and challenge to define the exact causes of the failure, and it is crucial to prevent its recurrence in the future.

Figure 2. Condition of the pipeline after the accident

According to basic hydrotechnical observations of the pipeline's condition after the accident, it is clear that the cause of such a failure is the occurrence of a vacuum in the pipeline, or the presence of unsteady flow. Therefore, the primary goal of this study is to develop a hydraulic model to analyze unsteady flow in a pressurized pipeline, focusing on examining changes in flow and pressure when unsteadiness occurs due to a pipeline defect. In the hydraulic modeling process, the pipeline defect is simulated as a boundary condition, acting like a valve that opens within fractions of a second. Its dimensions correspond to the size of the opening where the defect occurred.

For the analysis of unsteady flow, the characteristics of both the pipeline and the water as a fluid are also significant, as provided in the following Table 1.

Table 1. Material characteristics of pipelines and water as fluid

Characteristics	Value
STEEL PIPES	
Young modulus of elasticity	207 GPa
Poisson factor	0.30
FLUID – WATER	
Temperature	20°C
Density	998 kg/m^3
Modulus of elasticity	2.19 GPa
Kinematic viscosity	1.01×10^{-6} m^2/s

5 Results and Discussion

In hydraulic analysis, the first step involves calibration and verification of the model under steady-state operation of the system. Through the model, it was practically confirmed that the maximum capacity of the pipeline is 3.8 m^3/s. This calibrated model serves as the basis for hydraulic analysis under unsteady flow conditions.

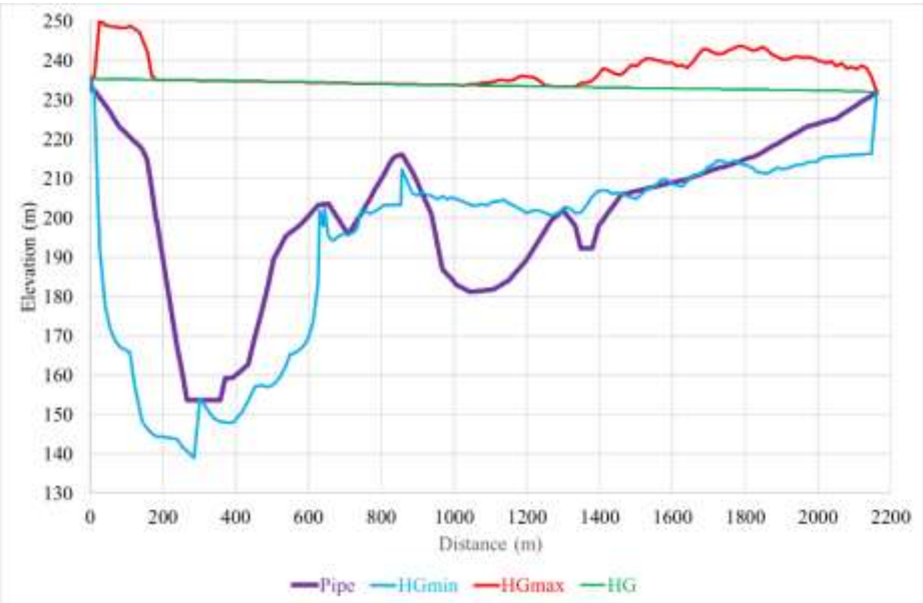

Figure 3. Maximum and minimum possible pressures in each part of the pipeline

Based on the results for the maximum and minimum possible pressures for each part of the pipeline shown in the graph (Figure 3), it can be concluded that the occurrence of the highest negative pressure – vacuum – appears from the very beginning of the pipeline and continues up to after the 600th meter.

The graph for the maximum and minimum possible pressures also shows the occurrence of negative pressure ranging from -1 to -1.5 bar in the last 200-300 meters of the final section of the pipeline. Although it was initially assumed that there was no damage in this section, after obtaining the results from the model and conducting a follow-up field inspection, minor insignificant damages were found in this part of the pipeline. This is primarily due to the fact that the minimum pressures are not sufficiently high to cause a complete collapse of the pipeline in that section.

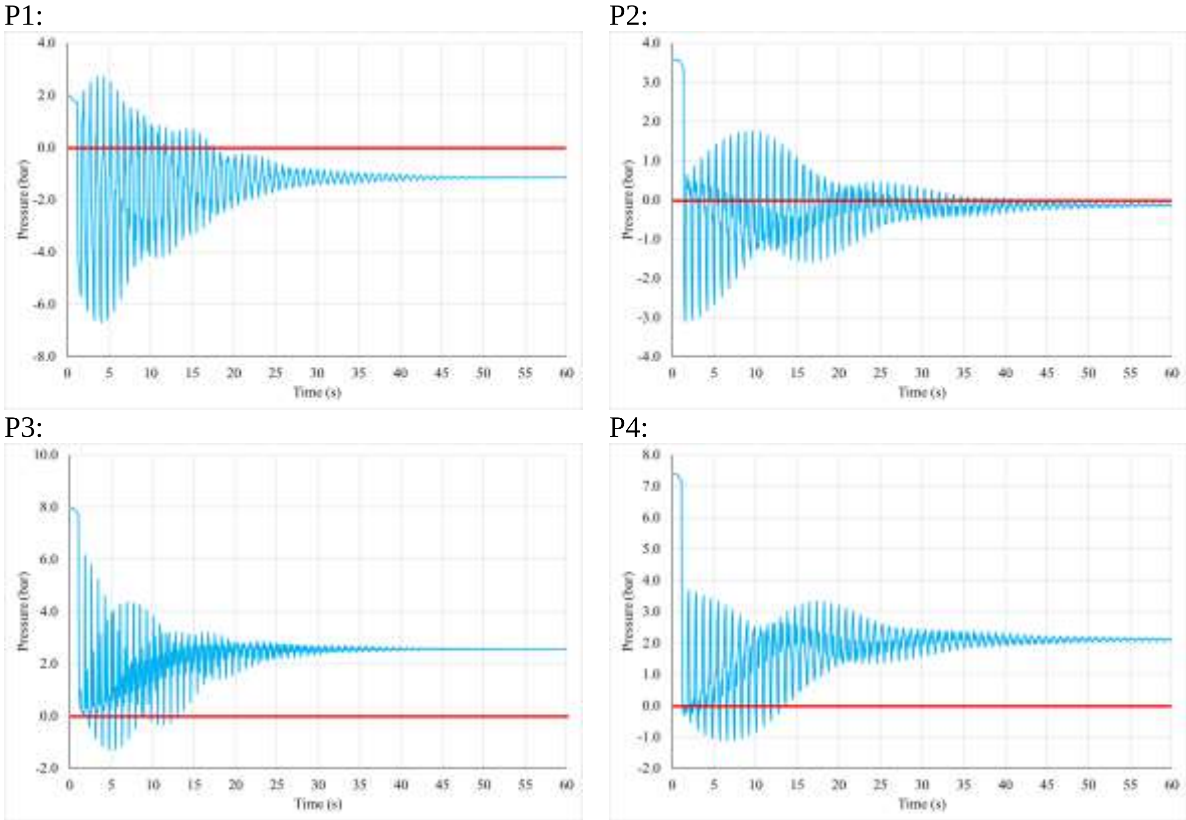

Figure 4. Results from the hydraulic model at characteristic points along the pipeline

6 Conclusions

The occurrence of unsteady flow in a pressurized pipeline, is a common phenomenon. However, the occurrence of such rapid accidental draining of the pipeline and the resulting unsteadiness with significant negative pressures is an accidental event. Such occurrences are not typically anticipated during the design phase of these systems.

The main cause of such a major accident is the occurrence of the initial defect - the bursting of the pipeline, specifically in the section where maximum pressures are present. This created conditions for the onset of significant negative pressures in a very short period - fractions of a second, where there was no opportunity to eliminate them, leading to a catastrophic failure of substantial proportions.

The occurrence of such a failure – a rupture of a large section of the pipeline – is highly unlikely and practically impossible to predict during the design phase, as well as later during the operational phase, with technical measures that would protect the pipeline while still being economically justified.

References:

[1] Goce Taseski (2015). „Characteristics of water hammers in water supply networks“. Doctorate dissertation, Skopje, Macedonia,

[2] Cvetanka Poposka (2000). „Hydraulics“. Publisher Civil Engineering Faculty, University Ss. Cyril and Methodius, ISBN 9989-43-100-0.

[3] Loh. Lohrasbi, A. & Attarnejad, R. (2008). Water Hammer Analysis by Characteristic Method.

American Journal of Engineering and Applied Sciences, 1(4), 287- 294.
[4] Josef Zaruba (1993). "Water hammer in pipe-line systems", Publisher: Elsevier Science, ISBN-13: 978-0444987228.
[5] A.R.D. Thorley (2004). Fluid Transients in Pipeline Systems, Second Edition (Pipelines and Pressure Vessels), Publisher: The American Society of Mechanical Engineers; 2nd edition, ISBN-13: 978-0791802106.
[6] Kaveh Hariri Asli (2013). "Water Hammer Research: Advances in Nonlinear Dynamics Modeling", Publisher: Apple Academic Press, ISBN 9781926895314 - CAT# N10708.
[7] Izquierdo, J., Inglesias, L.P. (2004), "Mathematical modelling of hydraulic transients in complex systems". Mathematical and Computer Modelling, 39,529-540.
[8] Thomas Repp, (2011). Fluid Dynamic Water Hammer Simulations with Consideration of Fluid-Structure Interaction, https://www.hzdr.de/FWS/publikat/JB98/jb08.pdf.