Uncertain Switched Fuzzy Systems: A Robust Output Feedback Control Design

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Abstract The problem of robust output feedback control for a class of uncertain switched fuzzy time-delay systems via common Lyapunov function and multiple Lyapunov function methods is solved. Based on employing Parallel Distributed Compensation strategy, the fuzzy output feedback controllers are designed such that the corresponding closed-loop system possesses stability and robustness for all admissible uncertainties. An illustrative example and the respective simulation results are given to demonstrate the effectiveness and feasible control performance of the proposed design synthesis.

1 Introduction

In real world physical and other systems essential phenomena such as nonlinearity, uncertainty, and time-delay often co-exist simultaneously [1]. It is therefore that the issue on how to control a nonlinear time-delay system with uncertainties is a challenging, and important issue. Moreover, it is equally important in theory, in applications, and even more in practical implementations.

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Recently, switching systems, an important class of hybrid systems, which have wide background and applications, have been also one of the main research focuses in the control society. In turn, considerable number of fruitful results in analysis and design of switching systems have been derived too (see for example [3, 9, 10, 14, 15, 22, 35–38], and references therein).

On the other hand, the research activities on fuzzy systems based control, as an important intelligent control approach, combined with some of the math-analytical control theories has attracted great attention. In particular, the class of Takagi-Sugeno (T-S) fuzzy models has been found to be most effective for system modelling in various fuzzy systems based methods. Based on the T-S fuzzy model representations and the feedback control strategy, stability and robust analysis and design as well as handling parameter uncertainties for fuzzy systems have acquired considerable number of fruitful results (see for example [8, 13, 16, 17, 24–28], and references therein).

As a result of the positive notions in using switching systems and fuzzy systems strategies, alone, since the first paper [21], where these two types of strategies are combined, investigations of the synergy of fuzzy and switched systems in the sense of their control synthesis, appeared a logical and natural development. Conceptually, a switched fuzzy system is a type of switching systems in which all of the respective subsystems are fuzzy systems. Many nonlinear systems with switching features can be modelled as switched fuzzy systems or nonlinear systems with fuzzy switching control. However, the results for switched fuzzy systems and switching fuzzy control in the literature seem to be rather limited [23, 32], largely because the first relevant stability results for general nonlinear switched systems have been putted forward fairly recently in [38].

Kazuo et al. [7] and Hiroshi et al. [5], to the best of our knowledge, were the first reports on switching fuzzy control for nonlinear systems. Subsequently, based on the idea of switching Lyapunov function, Hiroshi et al. have finalized their research endeavours in [6]. Yang et al. [31] have contributed the stability solution of a class of uncertain systems based on fuzzy control switching. More recently, in [11], and [12] a new solution type to the robust output feedback control for a class of uncertain switched fuzzy systems was presented. At the same time, authors in [32] has thoroughly elaborated on representation modelling, stability analysis and control design for switched fuzzy systems for both continuous-time and discrete-time cases. Yang et al. [33] contributed a solution to the H_∞ state feedback control for switched fuzzy systems. Authors in [18, 19] have explored and highlighted the influence of the state space partitioning when designing switched fuzzy controllers.

This paper is largely based on the work given in [20]. Inspired by works in [12] and [35], the problem of robust output feedback control for a class of uncertain switched fuzzy time-delay systems whose states are not measurable, hence not available, is further explored and solution given. Sufficient conditions and switching law are derived and formulated in the form of linear matrix inequalities (LMI) based on T-S fuzzy model. These are derived via both common Lyapunov function and multiple Lyapunov function approaches. The fuzzy output feedback controllers are designed by employing the Parallel Distributed Compensation-PDC—strategy. An illustrative example and the respective simulation results are given to demonstrate the effectiveness of the proposed control method and the closed-loop performance it can guarantee.

The next Sect. 2 is dedicated to the presentation of the output fuzzy control design. In Sect. 3, the stability analysis and switching law design are developed. In Sect. 4 the derived results are applied to an illustrative example. Concluding remarks and references follow thereafter.

2 Representation Modelling Preliminaries and Output Feedback Controller Design

2.1 Plant Fuzzy Rule Model and Switching Sequence Classes

In this study, the following class of switched fuzzy time-delay systems with uncertainty is considered:

$$\begin{aligned} &R^i_\sigma : IF\; z_1(\mathrm{t}) \text{ is } M^i_{\sigma 1} \ldots \text{and } z_n(\mathrm{t}) \text{ is } M^i_{\sigma n}, \text{ THEN} \\ &\dot{x}(t) = (A_{\sigma i} + \Delta A_{\sigma i})x(t) + (A_{h\sigma i} + \Delta A_{h\sigma i})x(t-h) \\ &\qquad\quad + (B_{\sigma i} + \Delta B_{\sigma i})\, u_\sigma(t) \\ &y(t) = C_{\sigma i}x(t) \\ &x(t) = \Psi(t), t \in [-h, 0], \quad i = 1, 2, \ldots N_\sigma. \end{aligned} \tag{1}$$

The symbols used in (1) denote: $M^i_{\sigma j}$ represent fuzzy subsets defined by appropriate membership functions; $z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T$ is a vector of the premise variables representing only the measurable system variables and not the entire state vector of the plant process; the sequence $\sigma \in M = \{1, 2, \ldots l\}$ is a piecewise constant function representing the switching signal; $x(t) \in R^n$ is the plant state vector; $u_\sigma(t) \in R^m$ is the input control vector; $y(t) \in R^\vartheta$ is the plant output vector. The plant systems matrices $A_{\sigma i} \in R^{n\times n}$, $A_{h\,\sigma i} \in R^{n\times n}$, $B_{\sigma\, i} \in R^{n\times m}$, $C_{\sigma\, i} \in R^{\vartheta \times n}$; $\Delta A_{\sigma\, i}$, $\Delta A_{h\,\sigma\, i}$, $\Delta B_{\sigma\, i}$ are time-varying matrices of appropriate dimensions that model system uncertainties. Quantity h denotes the constant delay factor present in the plant, and $\Psi(t)$ is initial value of the state vector $x(t)$.

In the real-world plants often the system states are not all measurable. Hence, we consider introducing the switching law of the form $\sigma = \sigma(\hat{x}(t))$, where $\hat{x}(t)$ are observer generated estimates of system states, to generated the switching signal. That is, it is a sequence in time the piecewise constants of which comply with the estimated states. This way employing an output feedback control is enabled. Further, it is assumed a given partition of state space R^n that is denoted as

$\{\tilde{\Omega}_1, \tilde{\Omega}_2, \ldots \tilde{\Omega}_l\}$, that is $\bigcup_{i=1}^{l} \tilde{\Omega}_i = R^n \backslash \{0\}$ and $\tilde{\Omega}_i \bigcap \tilde{\Omega}_j = \Phi, i \neq j$. The switching signal is represented as $\sigma = \sigma(\hat{x}(t)) = r$ if $\hat{x}(t) \in \tilde{\Omega}_r$. The switching signal is subject to the rule:

$$v_r(\hat{x}(t)) = \begin{cases} 1 & \hat{x}(t) \in \tilde{\Omega}_r \\ 0 & \hat{x}(t) \notin \tilde{\Omega}_r \end{cases}, \quad r \in M.$$

That is, $v_r(\hat{x}(t)) = 1$ if and only if $\sigma = \sigma(\hat{x}(t)) = r$. The partition $\{\tilde{\Omega}_1, \tilde{\Omega}_2, \ldots \tilde{\Omega}_l\}$ and the switching law σ will be designed later.

2.2 Employed Output Feedback Controller

For a given $v_r(\hat{x}(t))$ and based on fuzzy-rule inference [34], the considered system (1) can be represented by means of:

$$\begin{aligned} \dot{x}(t) &= \sum_{r=1}^{l} \sum_{i=1}^{N_r} v_r(\hat{x}(t)) \mu_{ri}(z(t)) [(A_{ri} + \Delta A_{ri}) x(t) \\ &\quad + (A_{hri} + \Delta A_{hri}) x(t-h) + (B_{ri} + \Delta B_{ri}) u_r(t)] \\ y(t) &= \sum_{r=1}^{l} \sum_{i=1}^{N_r} v_r(\hat{x}(t)) \mu_{ri}(z(t)) C_{ri} x(t), \end{aligned} \tag{2}$$

where

$$\mu_{ri}(z(t)) = \frac{\prod_{j=1}^{p} M_{rj}^i(z_j(t))}{\sum_{i=1}^{N_r} \prod_{j=1}^{p} M_{rj}^i(z_j(t))},$$

$$0 \leq \mu_{ri}(z(t)) \leq 1, \sum_{l=1}^{N_r} \mu_{ri}(z(t)) = 1,$$

and $M_{rj}^i(z_j(t))$ represents the membership function of $z_j(t)$ belonging to fuzzy subset M_{rj}^i. The following assumption is needed in the sequel for deriving the new result.

Assumption 1 The parameter uncertainty matrices are norm bounded, that is

$$[\Delta A_{ri} \quad \Delta A_{hri} \quad \Delta B_{ri}] = D_{ri} F_{ri}(t) [E_{1ri} \quad E_{hri} \quad E_{2ri}],$$

where D_{ri}, E_{1ri}, E_{hri} and E_{2ri} are constant matrices of appropriate dimensions, $F_{ri}(t)$ are unknown time-varying matrices, satisfying $F_{ri}^T(t) F_{ri}(t) \leq I, \; i = 1, 2, \ldots N_r$.

According to Parallel Distributed Compensation-PDC—design strategy [24, 27, 28], the fuzzy output feedback controllers and observers are designed via the following system architecture:

$$\begin{aligned} u_r(t) &= -\sum_{r=1}^{l}\sum_{i=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))K_{ri}\hat{x}(t) \\ \dot{\hat{x}}(t) &= \sum_{r=1}^{l}\sum_{i=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))\left\{\begin{array}{l} A_{ri}\hat{x}(t) + A_{hri}\hat{x}(t-h) \\ + B_{ri}u_r(t) + L_{ri}[y(t) - C_{ri}\hat{x}(t)] \end{array}\right\} \end{aligned} \tag{3}$$

In here, $\hat{x}(t) \in R^n$ is state vector of the fuzzy observer, and L_{ri} represents observer gain matrix for the ith fuzzy rule of the rth switched plant subsystem. It is known [24] employing such controller designs in the system synthesis can guarantee the global asymptotic stability of the closed-loop system.

3 Stability Analysis and Main Results

In this section, sufficient conditions for global asymptotic stability of the uncertain switched fuzzy time-delay system (1) are given. For the observer equation, a common Lyapunov function is employed such that the observer error $e(t) = x(t) - \hat{x}(t)$ tends to zero under arbitrary switching law. By means of multiple Lyapunov function method, a switching rule is designed based on observed state $\hat{x}(t)$ such that the output-feedback control system is asymptotically stable.

Thus, in turn, the following system representation in closed loop is obtained:

$$\begin{aligned} \dot{x}(t) = \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} & v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t))\{[A_{ri} + \Delta A_{ri} - (B_{ri} \\ & + \Delta B_{ri})K_{rj}]x(t) + (A_{hri} + \Delta A_{hri})x(t-h) \\ & + (B_{ri} + \Delta B_{ri})K_{rj}e(t)]\} \end{aligned} \tag{4}$$

$$\begin{aligned} \dot{\hat{x}}(t) = \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} & v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t))[(A_{ri} - B_{ri}K_{rj})\hat{x}(t) \\ & + A_{hri}\hat{x}(t-h) + L_{ri}C_{rj}e(t)] \end{aligned} \tag{5}$$

$$\begin{aligned} \dot{e}(t) = \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} & v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t))[(A_{ri} + \Delta A_{ri} \\ & - L_{ri}C_{rj})e(t) + (A_{hri} + \Delta A_{hri})e(t-h) \\ & + (\Delta A_{ri} - \Delta B_{ri}K_{rj})\hat{x}(t) + \Delta A_{hri}\hat{x}(t-h)] \end{aligned} \tag{6}$$

In what follows the next assumption and lemma are needed, which are given before proceeding to the proof of asymptotic stability for the proposed designs.

Lemma 1 [29]. Given constant matrices D and E, and a symmetric constant matrix Y of appropriate dimension, the following inequality holds:

$$Y + DFE + E^T F^T D^T < 0,$$

where F satisfies $F^T F \leq R$, *if and only if for some* $\varepsilon > 0$,

$$Y + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0.$$

Lemma 2 [30]. *Given constant matrices X and Y, for exist appropriate dimension positive definite matrix S and arbitrary* $\varepsilon > 0$, *the following inequality*

$$X^T Y + Y^T X \leq \varepsilon X^T S X + \varepsilon^{-1} Y^T S^{-1} Y,$$

holds.

A proof of the closed-loop of system (1) which guarantees global asymptotic stabilization is given as presented below.

Theorem 1 *If there exist the positive definite matrices* P_r, S_r, P_e, Q, *some matrices* K_{ri} *and* L_{ri}, *switching law* $\sigma = \sigma(\hat{x}(t)) \in M = \{1, 2, \ldots l\}$, *some constants* $\beta_{r\lambda}(\, r = 1, 2, \ldots, l, \lambda = 1, 2, \ldots, N_r)$ *that are either all positive or all negative, and a group of positive constants* $\alpha_{rj}, \xi_{ri}, \varepsilon_{rij}(\, i, j = 1, 2, \ldots N_r)$, *such that the following LMI are satisfied*

$$\begin{bmatrix} \Pi_{rij} + \sum\limits_{\substack{\lambda = 1 \\ \lambda \neq r}}^{l} \beta_{r\lambda}(P_\lambda - P_r) & P_r A_{hri} \\ A_{hri}^T P_r & -S_r + E_{hri}^T E_{hri} \end{bmatrix} < 0, \tag{7}$$

$$\begin{bmatrix} \mathrm{T}_{rij} & P_e A_{hri} \\ A_{hri}^T P_e & -Q + E_{hri}^T E_{hri} \end{bmatrix} < 0, \tag{8}$$

where

$$\begin{aligned} \Pi_{rij} = {} & A_{ri}^T P_r + P_r A_{ri} - \alpha_{rj} P_r B_{rj} B_{ri}^T P_r - \alpha_{rj} P_r B_{ri} B_{rj}^T P_r + S_r \\ & + \varepsilon_{rij}^{-1} \xi_{ri}^2 P_r P_e^{-1} C_{ri}^T C_{rj} C_{rj}^T C_{ri} P_e^{-1} P_r + E_{1ri}^T E_{1ri} - \alpha_{rj} P_r B_{rj} E_{2ri}^T E_{1ri} \\ & - \alpha_{rj} E_{1ri}^T E_{2ri} B_{rj}^T P_r + \alpha_{rj}^2 P_r B_{rj} E_{2ri}^T E_{2ri} B_{rj}^T P_r \\ \mathrm{T}_{rij} = {} & A_{ri}^T P_e + P_e A_{ri} - \xi_{ri} C_{rj}^T C_{ri} - \xi_{ri} C_{ri}^T C_{rj} + Q \\ & + E_{1ri}^T E_{1ri} + 4 P_e D_{ri} D_{ri}^T P_e + \varepsilon, \end{aligned}$$

then under the output feedback controller (3), with

$$\left.\begin{array}{l} K_{ri} = \alpha_{ri} B_{ri}^T P_r \\ L_{ri} = \xi_{ri} P_e^{-1} C_{ri}^T \end{array}\right\}, r = 1, 2, \ldots l, i = 1, 2, \ldots N_r, \tag{9}$$

and switching law $\sigma = \sigma(\hat{x}(t)) \in M = \{1, 2, \ldots l\}$, the closed-loop system of system (1) is asymptotically stable for all admissible parameter uncertainty.

Proof It is known from (8) that

$$\begin{bmatrix} e(t) \\ e(t-h) \end{bmatrix}^T \begin{bmatrix} \mathrm{T}_{rij} & P_e A_{hri} \\ A_{hri}^T P_e & -Q + E_{hri}^T E_{hri} \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-h) \end{bmatrix} < 0 \tag{10}$$

holds for any $e(t) \neq 0$. Then, under arbitrary switching law, the observer error satisfies $\lim_{t \to \infty} e(t) = 0$.

Without loss of generality, we assume that $\beta_{r\lambda} \geq 0$. It is apparent that for any $\hat{x}(t) \neq 0$ there exists at least one $r \in M$, such that $\hat{x}^T(t)(P_\lambda - P_r)\hat{x}(t) \geq 0$, $\forall \lambda \in M$. Applying inequality (7) yields

$$\begin{bmatrix} \hat{x}(t) \\ \hat{x}(t-h) \end{bmatrix}^T \begin{bmatrix} \Pi_{rij} & P_r A_{hri} \\ A_{hri}^T P_r & -S_r + E_{hri}^T E_{hri} \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{x}(t-h) \end{bmatrix} < 0. \tag{11}$$

For an arbitrary $r \in M$, let

$$\Omega_r = \{\hat{x}(t) \in R^n \,|\, \hat{x}^T(t)(P_\lambda - P_r)\hat{x}(t) \geq 0, \quad \forall \hat{x}(t) \neq 0\},$$

and thus $\bigcup_r \Omega_r = R^n \backslash \{0\}$. Thereafter, we construct the sets $\tilde{\Omega}_1 = \Omega_1, \ldots, \tilde{\Omega}_r = \Omega_r - \bigcup_{i=1}^{r-1} \tilde{\Omega}_i$. Obviously, we have

$$\bigcup_{i=1}^{l} \tilde{\Omega}_i = R^n \backslash \{0\}, \text{ and } \quad \tilde{\Omega}_i \bigcap \tilde{\Omega}_j = \Phi, i \neq j.$$

Next, we design a switching law as follows:

$$\sigma(\hat{x}(t)) = r \quad \text{when} \quad \hat{x}(t) \in \tilde{\Omega}_r, r \in M. \tag{12}$$

Consequently, we consider the following Lyapunov functional

$$\begin{aligned} V_r(t) &= \hat{x}^T(t) P_r \hat{x}(t) + \int_{t-h}^{t} \hat{x}^T(\tau) S_r \hat{x}(\tau) d\tau \\ &\quad + e^T(t) P_e e(t) + \int_{t-h}^{t} e^T(\tau) Q e(\tau) d\tau \end{aligned} \tag{13}$$

where P_r, S_r, P_e and Q are positive definite matrices, and let

$$V_{1r}(\hat{x}(t)) = \hat{x}^T(t)P_r\hat{x}(t) + \int\limits_{t-h}^{t} \hat{x}^T(\tau)S_r\hat{x}(\tau)d\tau,$$

$$V_2(e(t)) = e^T(t)P_e e(t) + \int\limits_{t-h}^{t} e^T(\tau)Qe(\tau)d\tau.$$

Then it follows:

$$\dot{V}_r(t) = \dot{V}_{1r}(\hat{x}(t)) + \dot{V}_2(e(t)). \tag{14}$$

Furthermore notice the following.

(1) The time derivative of $V_{1r}(\hat{x}(t))$ satisfies

$$\begin{aligned}\dot{V}_{1r}(\hat{x}(t)) &= \dot{\hat{x}}^T(t)P_r\hat{x}(t) + \hat{x}^T(t)P_r\dot{\hat{x}}(t)\\ &\quad + \hat{x}^T(t)S_r\hat{x}(t) - \hat{x}^T(t-h)S_r\hat{x}(t-h)\\ &= \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t))\\ &\quad \times \{\hat{x}^T(t)[(A_{ri} - B_{ri}K_{rj})^T P_r + P_r(A_{ri} - B_{ri}K_{rj})]\hat{x}(t)\\ &\quad + \hat{x}^T(t-h)A_{hri}^T P_r\hat{x}(t) + \hat{x}^T(t)P_r A_{hri}\hat{x}(t-h)\\ &\quad + e^T(t)(L_{ri}C_{rj})^T P_r\hat{x}(t) + \hat{x}^T(t)P_r L_{ri}C_{rj}e(t)\\ &\quad + \hat{x}^T(t)S_r\hat{x}(t) - \hat{x}^T(t-h)S_r\hat{x}(t-h)\}.\end{aligned} \tag{15}$$

According to (9) and Lemma 2, it follows

$$\begin{aligned}\dot{V}_{1r}(\hat{x}(t)) \le & \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t))\\ & \times \{\hat{x}^T(t)[A_{ri}^T P_r + P_r A_{ri} - \alpha_{rj}P_r B_{rj}B_{ri}^T P_r - \alpha_{rj}P_r B_{ri}B_{rj}^T P_r\\ & + S_r + \varepsilon_{rij}^{-1}\xi_{ri}^2 P_r P_e^{-1} C_{ri}^T C_{rj} C_{rj}^T C_{ri} P_e^{-1} P_r]\hat{x}(t)\\ & + \hat{x}^T(t-h)A_{hri}^T P_r\hat{x}(t) + \hat{x}^T(t)P_r A_{hri}\hat{x}(t-h)\\ & + \varepsilon_{rij}e^T(t)e(t) - \hat{x}^T(t-h)S_r\hat{x}(t-h)\}.\end{aligned} \tag{16}$$

(2) The time derivative of $V_2(e(t))$ can be found as follows

$$\begin{aligned}
\dot{V}_2(e(t)) &= \dot{e}^T(t)P_e e(t) + e^T(t)P_e\dot{e}(t) \\
&\quad + e^T(t)Qe(t) - e^T(t-h)Qe(t-h) \\
&= \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t)) \\
&\quad \times \{e^T(t)[(A_{ri}+\Delta A_{ri}-L_{ri}C_{rj})^T P_e \\
&\quad + P_e(A_{ri}+\Delta A_{ri}-L_{ri}C_{rj})]e(t) \\
&\quad + e^T(t-h)(A_{hri}+\Delta A_{hri})^T P_e e(t) \\
&\quad + e^T(t)P_e(A_{hri}+\Delta A_{hri})e(t-h) \\
&\quad + \hat{x}^T(t)(\Delta A_{ri}-\Delta B_{ri}K_{rj})^T P_e e(t) \\
&\quad + e^T(t)P_e(\Delta A_{ri}-\Delta B_{ri}K_{rj})\hat{x}(t) \\
&\quad + \hat{x}^T(t-h)\Delta A_{hri}^T P_e e(t) + e^T(t)P_e\Delta A_{hri}\hat{x}(t-h) \\
&\quad + e^T(t)Qe(t) - e^T(t-h)Qe(t-h)\}.
\end{aligned} \tag{17}$$

According to Lemma 1 and 2, and with regards to (9) and Assumption 1, one can derive

$$\begin{aligned}
\dot{V}_2(e(t)) &\le \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t)) \\
&\quad \times \{e^T(t)[A_{ri}^T P_e + P_e A_{ri} - \xi_{ri}C_{rj}^T C_{ri} - \xi_{ri}C_{ri}^T C_{rj} + Q \\
&\quad + E_{1ri}^T F_{ri}^T(t)D_{ri}^T P_e + P_e D_{ri}F_{ri}(t)E_{1ri}]e(t) \\
&\quad + e^T(t-h)A_{hri}^T P_e e(t) + e^T(t)P_e A_{hri}e(t-h) \\
&\quad + e^T(t-h)E_{hri}^T F_{ri}^T(t)D_{ri}^T P_e e(t) \\
&\quad + e^T(t)P_e D_{ri}F_{ri}(t)E_{hri}e(t-h) \\
&\quad + \hat{x}^T(t)(E_{1ri}^T - \alpha_{rj}P_r B_{rj}E_{2ri}^T)F_{ri}^T(t)D_{ri}^T P_e e(t) \\
&\quad + e^T(t)P_e D_{ri}F_{ri}(t)(E_{1ri} - \alpha_{rj}E_{2ri}B_{rj}^T P_r)\hat{x}(t) \\
&\quad + \hat{x}^T(t-h)E_{hri}^T F_{ri}^T(t)D_{ri}^T P_e e(t) \\
&\quad + e^T(t)P_e D_{ri}F_{ri}(t)E_{hri}\hat{x}(t-h) - e^T(t-h)Qe(t-h)\} \\
&\le \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t)) \\
&\quad \times \{e^T(t)[A_{ri}^T P_e + P_e A_{ri} - \xi_{ri}C_{rj}^T C_{ri} - \xi_{ri}C_{ri}^T C_{rj} + Q \\
&\quad + E_{1ri}^T E_{1ri} + 4P_e D_{ri}D_{ri}^T P_e]e(t) + e^T(t-h)A_{hri}^T P_e e(t) \\
&\quad + e^T(t)P_e A_{hri}e(t-h) - e^T(t-h)(Q - E_{hri}^T E_{hri})e(t-h) \\
&\quad + \hat{x}^T(t)(E_{1ri}^T - \alpha_{rj}P_r B_{rj}E_{2ri}^T)(E_{1ri} - \alpha_{rj}E_{2ri}B_{rj}^T P_r)\hat{x}(t) \\
&\quad + \hat{x}^T(t-h)E_{hri}^T E_{hri}\hat{x}(t-h)\}
\end{aligned} \tag{18}$$

Substituting (16) and (18) into (14) yields

$$
\begin{aligned}
\dot{V}_r(t) \leq & \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t)) \\
& \times \{\hat{x}^T(t)[A_{ri}^T P_r + P_r A_{ri} - \alpha_{rj} P_r B_{rj} B_{ri}^T P_r - \alpha_{rj} P_r B_{ri} B_{rj}^T P_r \\
& + S_r + \varepsilon_{rij}^{-1}\xi_{ri}^2 P_r P_e^{-1} C_{ri}^T C_{rj} C_{rj}^T C_{ri} P_e^{-1} P_r + E_{1ri}^T E_{1ri} \\
& - \alpha_{rj} P_r B_{rj} E_{2ri}^T E_{1ri} - \alpha_{rj} E_{1ri}^T E_{2ri} B_{rj}^T P_r \\
& + \alpha_{rj}^2 P_r B_{rj} E_{2ri}^T E_{2ri} B_{rj}^T P_r]\hat{x}(t) + \hat{x}^T(t-h) A_{hri}^T P_r \hat{x}(t) \\
& + \hat{x}^T(t) P_r A_{hri}\hat{x}(t-h) - \hat{x}^T(t-h)(S_r - E_{hri}^T E_{hri})\hat{x}(t-h)\} \\
& + \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t)) \\
& \times \{e^T(t)[A_{ri}^T P_e + P_e A_{ri} - \xi_{ri} C_{rj}^T C_{ri} - \xi_{ri} C_{ri}^T C_{rj} + Q \\
& + E_{1ri}^T E_{1ri} + 4 P_e D_{ri} D_{ri}^T P_e + \varepsilon_{rij} I] e(t) + e^T(t-h) A_{hri}^T P_e e(t) \\
& + e^T(t) P_e A_{hri} e(t-h) - e^T(t-h)(Q - E_{hri}^T E_{hri}) e(t-h)\} \\
= & \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t)) \\
& \times \begin{bmatrix} \hat{x}(t) \\ \hat{x}(t-h) \end{bmatrix}^T \begin{bmatrix} \Pi_{rij} & P_r A_{hri} \\ A_{hri}^T P_r & -S_r + E_{hri}^T E_{hri} \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{x}(t-h) \end{bmatrix} \\
& + \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t)) \\
& \times \begin{bmatrix} e(t) \\ e(t-h) \end{bmatrix}^T \begin{bmatrix} \mathrm{T}_{rij} & P_e A_{hri} \\ A_{hri}^T P_e & -Q + E_{hri}^T E_{hri} \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-h) \end{bmatrix}
\end{aligned} \tag{19}
$$

From (10) and (11), we can infer that under the switching law (12), $\dot{V}(t) < 0$ holds for arbitrary $\hat{x}(t) \neq 0$ and $e(t) \neq 0$, i.e. $x(t) \neq 0$. Therefore, the closed-loop system of system (1) is asymptotically stable, and the observer error $e(t)$ asymptotically converges to zero. This concludes the proof. □

The stability conditions of Theorem 1 can be transformed into linear matrix inequalities (LMIs). In fact, in view of (8), and using Schur's complement [2], we obtain the following LMIs

$$
\begin{bmatrix} \Theta_{rij} & P_e D_{ri} & P_e A_{hri} \\ D_{ri}^T P_e & -0.25 I & 0 \\ A_{hri}^T P_e & 0 & -Q + E_{hri}^T E_{hri} \end{bmatrix} < 0, \tag{20}
$$

where

$$\Theta_{rij} = A_{ri}^T P_e + P_e A_{ri} - \xi_{ri} C_{rj}^T C_{ri} - \xi_{ri} C_{ri}^T C_{rj} + Q + E_{1ri}^T E_{1ri} + \varepsilon_{rij} I$$

Upon substitution of the solutions P_e of the LMI (20) into inequality (7) and multiplying both sides of inequality (7) by the matrix $diag[X_r \quad X_r]$ with $X_r = P_r^{-1}$, $W_r = X_r S_r X_r$, we obtain the following inequality result

$$\begin{bmatrix} \Psi_{rij} & A_{hri} X_r \\ X_r A_{hri}^T & -W_r + X_r E_{hri}^T E_{hri} X_r \end{bmatrix} < 0, \tag{21}$$

where

$$\begin{aligned} \Psi_{rij} = {} & X_r A_{ri}^T + A_{ri} X_r - \alpha_{rj} B_{rj} B_{ri}^T - \alpha_{rj} B_{ri} B_{rj}^T + W_r \\ & + \varepsilon_{rij}^{-1} \xi_{ri}^2 P_e^{-1} C_{ri}^T C_{rj} C_{rj}^T C_{ri} P_e^{-1} + X_r E_{1ri}^T E_{1ri} X_r \\ & - \alpha_{rj} B_{rj} E_{2ri}^T E_{1ri} X_r - \alpha_{rj} X_r E_{1ri}^T E_{2ri} B_{rj}^T + \alpha_{rj}^2 B_{rj} E_{2ri}^T E_{2ri} B_{rj}^T \\ & + \sum_{\substack{\lambda=1 \\ \lambda \neq r}}^{l} \beta_{r\lambda} (X_r P_\lambda X_r - X_r) \end{aligned}$$

Now with regard to (21), we have

$$\begin{aligned} & \begin{bmatrix} \Psi_{rij} & A_{hri} X_r \\ X_r A_{hri}^T & -W_r + X_r E_{hri}^T E_{hri} X_r \end{bmatrix} \\ & = \begin{bmatrix} \mathrm{N}_{rij} & A_{hri} X_r \\ X_r A_{hri}^T & -W_r \end{bmatrix} + \begin{bmatrix} X_r E_{1ri}^T E_{1ri} X_r & 0 \\ 0 & X_r E_{hri}^T E_{hri} X_r \end{bmatrix} \\ & = \begin{bmatrix} \mathrm{N}_{rij} & A_{hri} X_r \\ X_r A_{hri}^T & -W_r \end{bmatrix} + \begin{bmatrix} X_r E_{1ri}^T & 0 \\ 0 & X_r E_{hri}^T \end{bmatrix} \begin{bmatrix} E_{1ri} X_r & 0 \\ 0 & E_{hri} X_r \end{bmatrix} \\ & < 0, \end{aligned}$$

where

$$\begin{aligned} \mathrm{N}_{rij} = {} & X_r A_{ri}^T + A_{ri} X_r - \alpha_{rj} B_{rj} B_{ri}^T - \alpha_{rj} B_{ri} B_{rj}^T + W_r \\ & + \varepsilon_{rij}^{-1} \xi_{ri}^2 P_e^{-1} C_{ri}^T C_{rj} C_{rj}^T C_{ri} P_e^{-1} - \alpha_{rj} B_{rj} E_{2ri}^T E_{1ri} X_r \\ & - \alpha_{rj} X_r E_{1ri}^T E_{2ri} B_{rj}^T + \alpha_{rj}^2 B_{rj} E_{2ri}^T E_{2ri} B_{rj}^T \\ & + \sum_{\substack{\lambda=1 \\ \lambda \neq r}}^{l} \beta_{r\lambda} (X_r P_\lambda X_r - X_r). \end{aligned}$$

Next, via applying Schur's complement again, the LMI is obtained as follows

$$\begin{bmatrix} \Xi_{rij} & X_r & \cdots & X_r & A_{hri}X_r & X_rE_{1ri}^T & 0 \\ X_r & -\beta_{r1}^{-1}X_1 & \cdots & 0 & 0 & 0 & X_rE_{hri}^T \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ X_r & 0 & \cdots & -\beta_{rl}^{-1}X_l & 0 & 0 & 0 \\ X_rA_{hri}^T & 0 & \cdots & 0 & -W_r & 0 & 0 \\ E_{1ri}X_r & 0 & \cdots & 0 & 0 & -I & 0 \\ 0 & E_{hri}X_r & \cdots & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{22}$$

where

$$\begin{aligned}\Xi_{rij} &= X_rA_{ri}^T + A_{ri}X_r - \alpha_{rj}B_{rj}B_{ri}^T - \alpha_{rj}B_{ri}B_{rj}^T + W_r \\ &\quad + \varepsilon_{rij}^{-1}\xi_{ri}^2P_e^{-1}C_{ri}^TC_{rj}C_{rj}^TC_{ri}P_e^{-1} - \alpha_{rj}B_{rj}E_{2ri}^TE_{1ri}X_r \\ &\quad -\alpha_{rj}X_rE_{1ri}^TE_{2ri}B_{rj}^T + \alpha_{rj}^2B_{rj}E_{2ri}^TE_{2ri}B_{rj}^T - \sum_{\substack{\lambda=1 \\ \lambda\neq r}}^{l} \beta_{r\lambda}X_r.\end{aligned}$$

and $X_\lambda = P_\lambda^{-1}, \lambda = 1, 2, \ldots, l,\ \lambda \neq r$. Thus, the stability conditions of the uncertain switched fuzzy time-delay system are transformed into the LMI (20), and (22). These LMI are tractable by means of the LMI Toolbox of the MATLAB [4].

4 Illustrative Example and Simulation Results

The given uncertain switched fuzzy time-delay system is:

$$\begin{aligned} &R_1^1 : \text{if } x_1(\text{t}) \text{ is } M_{11}^1, \text{ then} \\ &\dot{x}(t) = (A_{11} + \Delta A_{11})x(t) + (A_{h11} + \Delta A_{h11})x(t-h) \\ &\qquad + (B_{11} + \Delta B_{11})u(t) \\ &y(t) = C_{11}x(t) \end{aligned} \tag{23a}$$

$$\begin{aligned} &R_1^2 : \text{if } x_1(\text{t}) \text{ is } M_{11}^2, \text{ then} \\ &\dot{x}(t) = (A_{12} + \Delta A_{12})x(t) + (A_{h12} + \Delta A_{h12})x(t-h) \\ &\qquad + (B_{12} + \Delta B_{12})u(t) \\ &y(t) = C_{12}x(t) \end{aligned} \tag{23b}$$

$$\begin{aligned} &R_2^1: \text{if } x_1(\text{t}) \text{ is } M_{21}^1, \text{ then} \\ &\dot{x}(t) = (A_{21} + \Delta A_{21})x(t) + (A_{h21} + \Delta A_{h21})x(t-h) \\ &\qquad + (B_{21} + \Delta B_{21})u(t) \\ &y(t) = C_{21}x(t) \end{aligned} \tag{23c}$$

$$\begin{aligned} &R_2^2: \text{if } x_1(\text{t}) \text{ is } M_{21}^2, \text{ then} \\ &\dot{x}(t) = (A_{22} + \Delta A_{22})x(t) + (A_{h22} + \Delta A_{h22})x(t-h) \\ &\qquad + (B_{22} + \Delta B_{22})u(t) \\ &y(t) = C_{22}x(t) \end{aligned} \tag{23d}$$

along with the system model matrices

$$A_{11} = \begin{bmatrix} 6.2 & -2.1 \\ 2.3 & -2.8 \end{bmatrix}; A_{12} = \begin{bmatrix} 6.5 & -2.8 \\ 2.7 & -3.0 \end{bmatrix}; \tag{24a}$$

$$A_{21} = \begin{bmatrix} -2.9 & 7.8 \\ -5.1 & 0.5 \end{bmatrix}; A_{22} = \begin{bmatrix} 2.2 & -7.6 \\ 3.5 & -4.6 \end{bmatrix}; \tag{24b}$$

$$A_{h11} = \begin{bmatrix} 1.1 & 0.8 \\ 0.1 & 0.5 \end{bmatrix}; A_{h12} = \begin{bmatrix} 0.6 & 0.5 \\ 1.1 & 0.2 \end{bmatrix}; \tag{25a}$$

$$A_{h21} = \begin{bmatrix} 0.1 & 1.6 \\ 1.3 & 0.6 \end{bmatrix}; A_{h22} = \begin{bmatrix} 1.1 & 1.4 \\ 0.4 & 0.2 \end{bmatrix}; \tag{25b}$$

$$B_{11} = \begin{bmatrix} 4 & 0.2 \\ 0.5 & 2 \end{bmatrix}; B_{12} = \begin{bmatrix} 4 & 0.1 \\ 0.2 & 2 \end{bmatrix}; \tag{26a}$$

$$B_{21} = \begin{bmatrix} 4 & 0.2 \\ 0.1 & 2 \end{bmatrix}; B_{22} = \begin{bmatrix} 4 & 0.1 \\ 0.4 & 2 \end{bmatrix}; \tag{26b}$$

$$C_{11} = [2 \quad 0]; C_{12} = [2 \quad 0]; C_{21} = [2 \quad 0]; C_{22} = [2 \quad 0]; \tag{27}$$

$$D_{11} = D_{12} = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}; D_{21} = D_{22} = \begin{bmatrix} 0 & 0.2 \\ 0.1 & 0 \end{bmatrix}; \tag{28}$$

$$E_{111} = E_{112} = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}; E_{121} = E_{122} = \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0 \end{bmatrix}; \tag{29a}$$

$$E_{211} = E_{212} = \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0 \end{bmatrix}; E_{221} = E_{222} = \begin{bmatrix} 0 & 0.2 \\ 0.1 & 0 \end{bmatrix}; \tag{29b}$$

$$E_{h11} = E_{h12} = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}; E_{h21} = E_{h22} = \begin{bmatrix} 0 & 0.6 \\ 0.1 & 0 \end{bmatrix}; \tag{30}$$

$$F_{11}(t) = F_{12}(t) = F_{21}(t) = F_{22}(t) = \begin{bmatrix} \sin t & 0 \\ 0 & \cos t \end{bmatrix}. \tag{31}$$

The membership functions of fuzzy subsets are chosen as follows:

$$\mu_{11}(x_1(\text{t})) = \mu_{21}(x_1(\text{t})) = 1 - 1/(1 + e^{-4x_1(\text{t})});$$

$$\mu_{12}(x_1(\text{t})) = \mu_{22}(x_1(\text{t})) = 1/(1 + e^{-4x_1(\text{t})}).$$

Next, let it be chosen $\xi_{11} = \xi_{12} = 1$, $\xi_{21} = \xi_{22} = 0.8$. Solving LMI (20) yields the positive definite matrix

$$P_e = \begin{bmatrix} 0.4397 & -0.3416 \\ -0.3416 & 0.9283 \end{bmatrix}.$$

The fuzzy state observer is designed as

$$\begin{aligned} \dot{\hat{x}}(t) = \sum_{r=1}^{2} \sum_{i=1}^{2} v_r(\hat{x}(t)) \mu_{ri}(x_1(t)) \{ A_{ri}\hat{x}(t) + A_{hri}\hat{x}(t-h) \\ + B_{ri}u_r(t) + L_{ri}[y(t) - C_{ri}\hat{x}(t)] \}. \end{aligned}$$

With regard to (9), the obtained observer gain matrices are:

$$L_{11} = \begin{bmatrix} 6.3692 \\ 2.3440 \end{bmatrix}; L_{12} = \begin{bmatrix} 6.3692 \\ 2.3440 \end{bmatrix};$$
$$L_{21} = \begin{bmatrix} 5.0954 \\ 1.8752 \end{bmatrix}; L_{22} = \begin{bmatrix} 5.0954 \\ 1.8752 \end{bmatrix}.$$

By substituting P_e into LMI (7), upon choosing $\alpha_{11} = \alpha_{12} = 6$, $\alpha_{21} = \alpha_{22} = 8$, $\varepsilon_{1ij} = \varepsilon_{2ij} = 2 (i, j = 1, 2)$, $\beta_{12} = \beta_{21} = 10$, the following positive definite matrices are obtained:

$$P_1 = \begin{bmatrix} 0.6191 & 0.1043 \\ 0.1043 & 1.2678 \end{bmatrix}; P_2 = \begin{bmatrix} 0.6103 & 0.0753 \\ 0.0753 & 2.9155 \end{bmatrix}.$$

Following Sect. 3, the partition is adopted as:

$$\Omega_1 = \{ \hat{x}(t) \in R^2 | \hat{x}^T(t)(P_2 - P_1)\hat{x}(t) \geq 0, \hat{x}(t) \neq 0 \},$$
$$\Omega_2 = \{ \hat{x}(t) \in R^2 | \hat{x}^T(t)(P_1 - P_2)\hat{x}(t) \geq 0, \hat{x}(t) \neq 0 \},$$

hence $\Omega_1 \bigcup \Omega_2 = R^2 \backslash \{0\}$.

Thus, we design a switching law as follows:

$$\sigma(\hat{x}(t)) = \begin{cases} 1, \hat{x}(t) \in \Omega_1; \\ 2, \hat{x}(t) \in \Omega_2 \backslash \Omega_1. \end{cases}$$

Also, the output feedback is designed as follows:

$$u_r(t) = -\sum_{r=1}^{2} \sum_{i=1}^{2} v_r(\hat{x}(t)) \mu_{ri}(x_1(t)) K_{ri} \hat{x}(t).$$

With regard to (9), the following output feedback controller gains are obtained:

$$K_{11} = \begin{bmatrix} 15.1709 & 6.3978 \\ 1.9950 & 15.6993 \end{bmatrix}; K_{12} = \begin{bmatrix} 14.9831 & 4.0616 \\ 1.6236 & 15.6367 \end{bmatrix};$$
$$K_{21} = \begin{bmatrix} 19.5894 & 4.7416 \\ 2.1811 & 46.7692 \end{bmatrix}; K_{22} = \begin{bmatrix} 19.7701 & 11.7389 \\ 1.6928 & 46.7090 \end{bmatrix}.$$

Computer simulation investigation was carried out using the initial condition state vector $x(0) = [5, -1]^T$ and the value $h = 0.6$.

The obtained computer simulation results are depicted on Figs. 1, 2, 3, 4, and 5. Figures 1 and 2 show the evolution of system state and the observer state trajectories with regard to time, respectively. Figure 3 depicts the time evolution of the control signals, whereas Fig. 4 gives an insight on how and when the switching signals changes its values between 1 and 2, selecting respectively between regions Ω_1 and Ω_2. According the result given on Fig. 5, it is evident that the time evolutions of the state observer errors, converge asymptotically to zero.

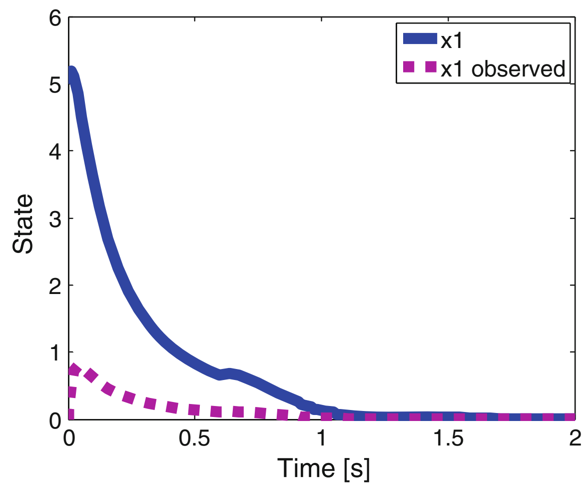

Fig. 1 Time evolutions of state $x_1(t)$ and observed state $\hat{x}_1(t)$

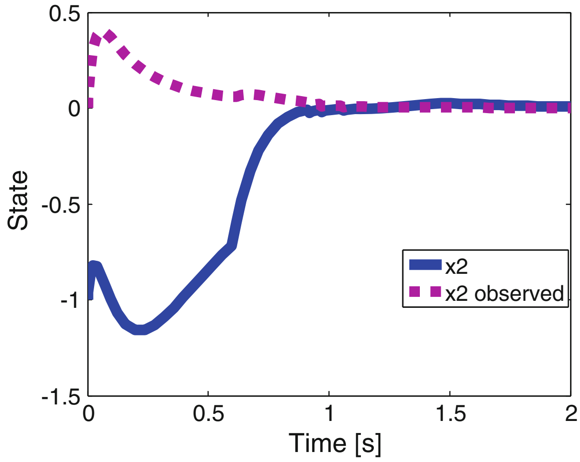

Fig. 2 Time evolutions of state $x_2(t)$ and observed state $\hat{x}_2(t)$

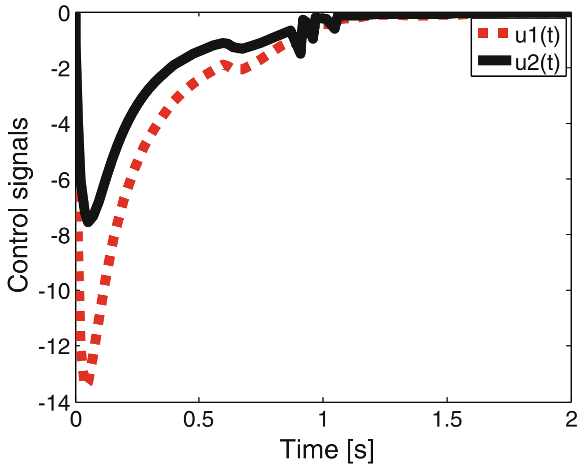

Fig. 3 Time evolutions of the control inputs $u_1(t)$, and $u_2(t)$

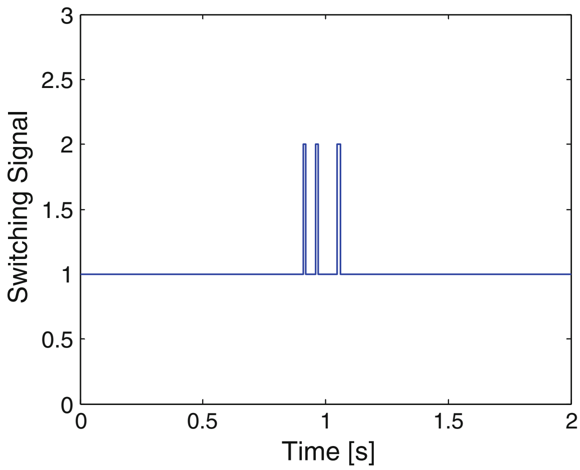

Fig. 4 Time evolution of the switching signal, changing its value between 1 and 2, as it selects between region Ω_1 and Ω_2, respectfully

These simulation results demonstrate that the uncertain switching fuzzy time-delay system (23a–d) is asymptotically stabilized under the proposed design of output feedback controller and the appropriate switching law. In this way we showed the effectiveness of the proposed concept.

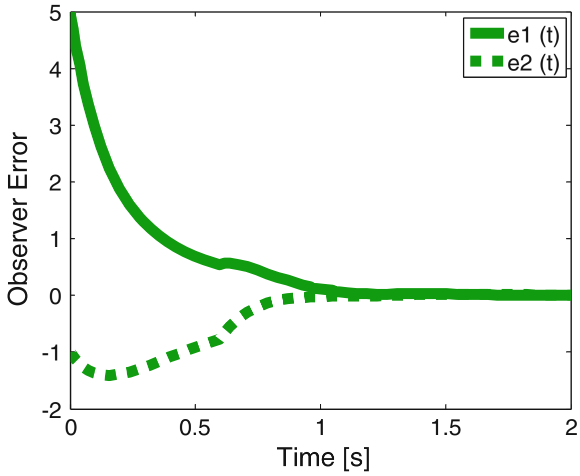

Fig. 5 Time evolutions of the observer errors $e_1(t)$, $e_2(t)$

5 Concluding Remarks

On the grounds of fuzzy T-S model the problem of robust output feedback control for a class of uncertain switched fuzzy time-delay systems whose states are not measurable, hence not available, has been further explored and solved. Sufficient stability conditions and switching law are derived and reformulated as linear matrix inequalities. These are derived by using common Lyapunov function and multiple Lyapunov function approaches. The fuzzy output feedback controllers are designed by employing the strategy of Parallel Distributed Compensation. An illustrative example along with the respective simulation results demonstrates the effectiveness of the proposed control synthesis and the system performance in closed-loop achieved.

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