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DETERMINATION OF CRITICAL PATH IN FUZZY NETWORK DIAGRAMS

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ABSTRACT

The classical critical path method (CPM) is characterized by a deterministic expression of the duration of activities in the network plan. When it comes to planning projects in the field of construction, it is especially difficult, and in some cases even impossible, to accurately predict or determine the duration of certain activities. The occurrence of uncertainty and inaccuracy in the duration of construction activities entails an imprecise determination of the total duration of the project's implementation.

Often, in practice, there is a need for modelling of risks and uncertainties. This need arises from the fact that sometimes, depending on the specificity of the conditions in which certain construction activities are carried out, they cannot be defined with a precisely determined duration. The time of execution of the activities is usually conditioned by the uncertainty of the factors that have an impact on the activity. This is precisely why there is a need to apply fuzzy numbers in the process of planning and project management.

This paper presents one example of application of Fuzzy logic in the process of planning and management of construction projects. The duration of construction activities is being analyzed in conditions of uncertainty and risk. Planning of construction process in cases where the durations of the activities, as part of the project, are given as fuzzy numbers is especially convenient in conditions of uncertainty and risk due to the great subjectivity that occurs during their assessment.

Furthermore, this paper deals with the application of Fuzzy network planning technique. An overview of different procedures and methods for calculation of earliest and latest duration time of project activities will be given, and one specific method will be applied for determination of the critical path in fuzzy network diagram. The results of the conducted analyses show that this method can provide more realistic results thus more reliable planning of the construction time for the whole project.

Keywords: fuzzy logic; fuzzy numbers; fuzzy neural networks; planning; construction; time; critical path; risks; uncertainties.

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1. NETWORK MODELS

In modern construction, the realization of any more serious project cannot be imagined without the application of quality planning and management methods. Network models are an example of the most commonly used methods for planning of the implementation of complex and long-term projects. With the network models, a precise and accurate estimation of the duration of the project is possible, which is actually the basic goal of the project planning process.

The most commonly used network models in construction are: Critical Path Method (CPM) and the PERT method (Project Evaluation and Review Technique). They are used for project time analysis, but can also be successfully used for cost and resource analysis with appropriate conceptual adjustments and modifications. Both methods are characterized by a graphic representation of the project through an appropriately oriented network composed of basic elements such as: activities, events, durations, costs, resources, etc.

The CPM method is used in those cases when the duration of project activities is known, or when it can be precisely determined. It is characterized by a deterministic expression of the duration of the activities in the network plan. After determining the time of individual project activities, the total time required for the realization of the project can be calculated in an easy and simple way.

In those cases, when the duration of the activities is not known or it cannot be explicitly determined, the PERT method is used. The PERT method is used to plan projects that are composed of activities whose duration is random (stochastic). This method is based on probability theory. The main feature of this method is the assessment of three values for the duration of the project activities: optimistic, normal and pessimistic duration, which is the basis for calculating the total time required for the realization of the project. The assessment of the three values of the duration of the activities is performed by experts based on their own knowledge, experience and availability of available information.

Apart from CPM and PERT, other types of network models have been developed, primarily due to the need to solve problems that could not be solved through the previously mentioned techniques of network planning, such as: GERT (Graphical Evaluation and Review Technique) and VERT (Venture Evaluation and Review Technique). Their advantage lies in the possibility of simulating future events based on the current basis, which gives insight into the probability of a certain event occurring, which is particularly significant in the project planning process.

When the duration of project activities is deterministic and known, or can be accurately and precisely determined, CPM and PERT methods are useful graphical tools for planning of the project implementation.

However, when it comes to planning of realization of construction projects, it is particularly difficult, and in some cases even impossible, to accurately predict or determine the durations of certain project activities. The time of execution of the activities is usually conditioned by the uncertainty of the factors that have an impact on the activity. The occurrence of uncertainty and inaccuracy in the duration of construction activities entails an imprecise determination of the total duration of the project's implementation. In order to overcome these challenges, the application of Fuzzy logic in the process of planning the implementation of projects began, thus Fuzzy network models were developed. Their basic characteristic is that the durations of the project activities or the costs of the project are set as fuzzy numbers. Through the use of fuzzy theory instead of probability theory, a new, modified critical path method is developed, called the Fuzzy Critical Path Method (FCPM).

2. FUZZY CRITICAL PATH METHOD

The application of fuzzy logic in the process of planning and management of construction projects enables the analysis of the duration of construction activities in conditions of uncertainty and risk. The main characteristic of this method is the estimation of the duration of the activities and their expression through triangular or trapezoidal fuzzy numbers.

Based on the concepts of fuzzy theory, different methods have been developed for fuzzy network planning, which are based on the application of basic fuzzy algebraic operations to calculate the duration of project activities and to determine the total time for the realization of the project and its critical path.

Fuzzy network planning gives the possibility to achieve more realistic results and obtain more reliable plan-time and cost project indicators.

This paper will explain the procedure for calculation of the earliest and the latest fuzzy time of project activities, through an example of the Tabular method that can be easily applied to determine the critical path in fuzzy network diagrams.

2.1. Tabular method for determination of the critical path in fuzzy network diagrams

2.1.1 Determination of fuzzy critical path

Through the fuzzy network diagram composed of activities set through fuzzy durations, shown in Figure 1, a simple tabular method will be explained through which the critical path can be easily determined. The method was published in 2011 by authors N. Ravi Shankar, K. Srinivasa and Rao S. Siresha.

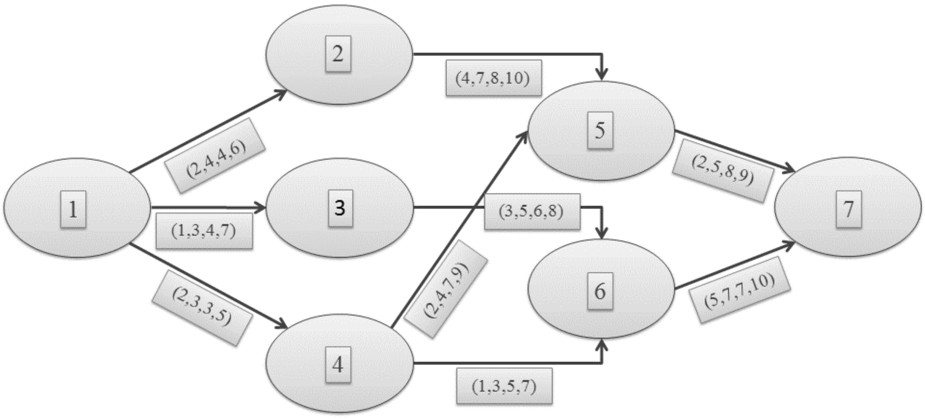

Fig. 1. An example of fuzzy network plan

The first step in this method is the formation of a table consisting of *(n-1)* rows and *(n-1)* columns, where "n" indicates the number of events in the fuzzy network plan. The rows are marked with the index "i", and the columns with the index "j", where: *i=1,2,3....n-1; j=2,3,4.....n*.

The second step is filling the fields in the table, that is, entering the corresponding values of the fuzzy duration $T_{i,j}$ for each activity *i-j*. The values for the fuzzy duration are entered in the first row of the table $T_{1,j}$, ie $T_{1,2}$, $T_{1,3}$ and $T_{1,4}$ (Table 1). Next, the fields from the second row (*i=2*) are filled, that is, the fuzzy durations of all activities that come out of event number 2, $T_{2,5}$, and are calculated by adding the previous fuzzy durations of the second event to the duration of activity 2-5. Same goes for $T_{3,6}$.

$$T_{2,5} = T_{1,2} + t_{2,5} = (2,4,4,6) + (4,7,8,10) = (6,11,12,16)$$

$$T_{3,6} = T_{1,3} + t_{3,6} = (1,3,4,7) + (3,5,6,8) = (4,8,10,15)$$

The calculation of the values from the fourth row of the table (*i=4*) is done by adding the previous fuzzy durations of the fourth event to the durations of activities 4-5 and 4-6, ie $t_{4,j}$.

If an activity has two or more previous activities, then the highest value for the duration of the paths leading to that activity is entered in the table. To compare two or more fuzzy numbers and to determine the greater of them, different methods for comparing fuzzy numbers can be used.

$$T_{4,5} = T_{1,4} + t_{4,5} = (2,3,3,5) + (2,4,7,9) = (4,7,10,14)$$

$$(T_{5,7})_1 = T_{2,5} + t_{5,7} = (6,11,12,16) + (2,5,8,9) = (8,16,20,25)$$

$$(T_{5,7})_2 = T_{4,5} + t_{5,7} = (4,7,10,14) + (2,5,8,9) = (6,12,18,23)$$

$$(8,16,20,25) > (6,12,18,23) \Rightarrow (T_{5,7})_1 = T_{5,7}$$

$$(T_{6,7})_1 = T_{3,6} + t_{6,7} = (4,8,10,15) + (5,7,7,10) = (9,15,17,25)$$

$(T_{6,7})_2 = T_{4,6} + t_{6,7} = (3,6,8,12) + (5,7,7,10) = (8,13,15,22)$

$(9,15,17,25) > (8,13,15,22) \Rightarrow (T_{6,7})_1 = T_{6,7}$

Table 1. Tabular overview of maximal fuzzy time durations for all paths in the fuzzy network diagram, from the event "i" to the event "j".

I \ *J*	2	3	4	5	6	7
1	**(2,4,4,6)**	(1,3,4,7)	(2,3,3,5)			
2				**(6,11,12,16)**		
3					(4,8,10,15)	
4				(4,7,10,14)	(3,6,8,12)	
5						**(8,16,20,25)**
6						(9,15,17,25)

The third step in the tabular method is the determination of fuzzy completion time of the project and the determination of the critical path in the fuzzy network plan. The field with the highest value of fuzzy duration from the last column (j=n) is selected. This value represents the fuzzy time of completion of all activities from the network plan, that is, the project. The number of the row in which the largest value of the last column is located, that is, the last event of the critical path, determines the index of the end event of the penultimate activity of the critical path. That is, if in the last column "n" the highest value is found in row "k", then column "k" is known and the field with the highest value should be selected from it again. This procedure is repeated until the first event of the network plan is reached, thus obtaining the critical path and the critical activities.

From Table 1, applying the described procedure, the critical path and critical activities can be easily and simply determined. The critical path of the given network plan is: 1-2-5-7.

2.1.2 Determination of total fuzzy time reserves

In order to determine the total fuzzy time reserves "$TS_{i,j}$" it is necessary to form a new table consisting of (*n-1*) rows and (*n-1*) columns, where "*n*" indicates the number of events in the fuzzy network plan. The rows are marked with the index "*i*", and the columns with the index "*j*", where: *i=1,2,3....n-1; j=2,3,4.....n*. The total fuzzy time reserves are calculated using the following equation:

$$TS_{i,j} = FS_{i,j} + minTS_{j,k} \quad (2)$$

where:

$TS_{i,j}$ – Total Slack Time; $FS_{i,j}$ – Free Slack Time and $minTS_{j,k}$ – smallest fuzzy value of the raw j

$TS_{6,7} = FS_{6,7} = (-1,1,3,0)$

$TS_{5,7} = FS_{5,7} = (0,0,0,0)$

$TS_{4,6} = FS_{4,6} + minTS_{6,7} = (1,2,2,3) + (-1,1,3,0) = (0,3,5,3)$

$TS_{4,5} = FS_{4,5} + minTS_{5,7} = (2,4,2,2) + (0,0,0,0) = (2,4,2,2)$

$TS_{3,6} = FS_{3,6} + minTS_{6,7} = (0,0,0,0) + (-1,1,3,0) = (-1,1,3,0)$

$TS_{2,5} = FS_{2,5} + minTS_{5,7} = (0,0,0,0) + (0,0,0,0) = (0,0,0,0)$

$$TS_{1,4} = FS_{1,4} + \min(TS_{4,5}; TS_{4,6}) = (0,0,0,0) + (2,4,2,2) = (2,4,2,2)$$
$$TS_{1,3} = FS_{1,3} + \min(TS_{3,6}) = (0,0,0,0) + (-1,1,3,0) = (-1,1,3,0)$$
$$TS_{1,2} = FS_{1,2} + \min(TS_{2,5}) = (0,0,0,0) + (0,0,0,0) = (0,0,0,0)$$

Table 2. Overview of total fuzzy time reserves for activities in fuzzy network diagram.

I \ *J*	**2**	**3**	**4**	**5**	**6**	**7**
1	$(\mathbf{0,0,0,0})$	$(-1,1,3,0)$	$(2,4,2,2)$			
2				$(\mathbf{0,0,0,0})$		
3					$(-1,1,3,0)$	
4				$(2,4,2,2)$	$(0,3,5,3)$	
5						$(\mathbf{0,0,0,0})$
6						$(-1,1,3,0)$

From Table 2 it can be noted that the total fuzzy time reserve of the activities: 1-2, 2-5 and 5-7 is (0,0,0,0), from which the conclusion is that these are the critical activities (according to the rules for determining a critical path in a classic network plan).

Apart from this approach, the principles for defuzzification of fuzzy numbers can also be applied, thus, defuzzification of the total time reserves can be performed, according to one of the following formulas:

$$D(FN)_3 = \frac{(c^2 + d^2 + cd) - (a^2 + b^2 + ab)}{3[(c + d) - (b + a)]} \quad (3)$$

$$D(FN)_4 = \frac{(a + b + c + d)}{4]} \quad (4)$$

Table 3. Total fuzzy time reserves and their defuzzified values.

Activity "A_{ij}"	Start event "i"	Last event "j"	Total fuzzy time reserve "TS_{ij}"	Defuzzified value "$D(TS_{ij})_3$"	Defuzzified value "$D(TS_{ij})_4$"
A	**1**	**2**	$(\mathbf{0,0,0,0})$	**0**	**0**
B	1	3	$(-1,1,3,0)$	0.67	0.75
C	1	4	$(2,4,2,2)$	2.67	2.5
D	**2**	**5**	$(\mathbf{0,0,0,0})$	**0**	**0**
E	3	6	$(-1,1,3,0)$	0.67	0.75
F	4	5	$(2,4,2,2)$	2.67	2.5
G	4	6	$(0,3,5,3)$	2.67	2.75
H	**5**	**7**	$(\mathbf{0,0,0,0})$	**0**	**0**
I	6	7	$(-1,1,3,0)$	0.67	0.75

The activities whose defuzzified value of the total fuzzy time reserve is 0 are called critical activities and they are shown in Table 3.

According to the calculations made by using the tabular method, for the above explained example, it is obtained that the path P_1 (1-2-5-7) is a critical path in the fuzzy network plan, and activities 1-2, 2 -5 and 5-7 are critical activities.

3. CONCLUSIONS

The Critical Path Method (CPM), as a method of network planning, is used in the process of planning and control of complex projects from various fields. For their successful implementation, a clear determination of the duration of each project activity is necessary. However, in real practical situations it is very difficult to fulfill this requirement, because the realization of many activities is accompanied by a certain degree of uncertainty and risk, and also because of the fact that often activities are encountered that have to be performed for the first time.

Since construction projects are characterized by uniqueness and unrepeatability, there are often no available historical data for a clear and precise determination of the duration of project activities. Given that, the duration should be predicted by expert and experienced persons, who, in the conditions of unique influencing factors and facing many inaccuracies, should define their decisions during the whole process of project management. Due to the shortcomings of classical planning methods, related to the calculation of the duration of project activities, which does not offer the possibility of modeling complex construction projects, there was a real need for the definition and implementation of new concepts of planning.

As an alternative way to model uncertainty and risk, the fuzzy concept was introduced in the planning process, with which the durations of project activities are presented through fussy numbers. This fundamental approach, which is based on the application of fuzzy theory, enables advanced and successful modeling of real situations and projects, which overcomes the shortcomings characteristic of classical planning methods.

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ПРОНАОЃАЊЕ НА ФОРМА НА ЛУШПИ СО ГОЛЕМИ РАСПОНИ ПОД ДЕЈСТВО НА СЕИЗМИЧКА СИЛА

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АПСТРАКТ

При дејство на хоризонтални сеизмички сили геометријата на лушпите има клучна улога во нивното однесување, пред се поради нивна помала маса и голема геометриска крутост. Традиционално, техниките за пронаоѓање форми за лушпи се потпираат исклучиво на гравитационите оптоварувања за генерирање на нивниот облик и не го земаат предвид сеизмичкото натоварување. Сепак во современите истражувања се развиваат методи за пронаоѓање форми за комбинирани случаи и на гравитацијата и на сеизмичката сила.

Пристапот за пронаоѓање форма што се разгледува во ова истражување ги зема предвид лушпите направени од материјали кои носат на притисок како што се неармиран бетон, камен, ѕидарија и земја. Техниката се заснова на општо прифатената претпоставка дека конструкциите од вакви материјали не можат преземат напрегања на затегнување. Затоа, за лушпи подложени и на вертикална гравитација и на хоризонтално сеизмичко натоварување, притиснатиот дел од пресекот за пренос на силите (во 2D често се нарекува линија на потисок) треба да биде застапен во самата лушпа за да се избегне активирање на механизам на колапс. Преку примена на модел на инверзен висечки ланец подложен на хоризонтално натоварување преку алгоритам за динамичка релаксација, се генерираат облици на лушпа за кои може да се осигура дека постои таква патека на натоварување.

За да ја илустрира оваа методологија, во виртуелната програмска околина Grasshopper и програмскиот јазик Python изграден е алгоритам кој за зададена форма на основа, дефинирани гранични услови и интензитет на сеизмичка сила генерира двослојна лушпа. Добиените лушпи се анализираат во однос на нивната дебелина, бидејќи одредени форми кои произлегуваат од процесот на пронаоѓање на формата бараат голема конструктивна длабочина, што може да доведе до дебели, а со тоа и тешки лушпи. Поконкретно е анализирано влијанието на три различни параметри врз дебелината на самата лушпа: интензитетот на предвидената сеизмичка сила, сооднос на висината и распонот на лушпата, и бројот на лежишта по периметарот на основата.

Клучни зборови: двослојни лушпи; асеизмичко проектирање; параметарско проектирање; динамичка релаксација

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