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The Cryptocurrency Market Through the Scope of Volatility **Clustering and Leverage Effects**



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Abstract: In the realm of financial markets, the manifestation of volatility clustering serves as a pivotal element, indicative of the inherent fluctuations characterizing financial instruments. This attribute acquires pronounced relevance within the sphere of cryptocurrencies, a sector renowned for its elevated risk profile. The present analysis, conducted through the Autoregressive Moving Average - Generalized Autoregressive Conditional Heteroskedasticity (ARMA-GARCH) model, seeks to elucidate the enduring nature of volatility clustering and the occurrence of leverage effects within this domain. Over the course of a four-year time frame, it was observed that Bitcoin diverges from the anticipated Autoregressive Conditional Heteroskedasticity (ARCH) effects, in contrast to Ethereum and Cardano, which exhibit marked volatility clustering. Binance Coin, Ripple, and Dogecoin, whilst demonstrating moderate clustering, uniformly reflect the existence of leverage effects. An exception to this pattern was identified in Ripple, where it was discerned that positive market news exerts a disproportionate influence on log returns. The findings of this study illuminate the critical influence of both leverage effects and volatility clustering on the pricing dynamics of cryptocurrencies. It underscores the imperative for a nuanced comprehension of risk management in the context of cryptocurrency investments, given their susceptibility to abrupt price fluctuations. The distinct degrees to which these phenomena are manifested across diverse cryptocurrencies accentuate the necessity for a tailored risk management approach, resonant with the unique attributes of the asset in question. Such strategies, accounting for the potential amplification of losses through leverage, may encompass prudent position sizing, portfolio diversification, and the implementation of stress tests, thereby fortifying the investment against the dual perils of volatility clustering and leverage effects. The implications of this analysis serve to inform investors, providing a foundation upon which to construct risk management tactics that are responsive to the idiosyncrasies of the cryptocurrency market.

Keywords: Cryptocurrency; Volatility clustering; Leverage effects; Autoregressive Moving Average - Generalized Autoregressive Conditional Heteroskedasticity (ARMA-GARCH) model; Risk management; Price dynamics

1 Introduction

Prior to the Great Recession, traditional financial instruments were predominantly recognized, while digital currencies were relegated to the periphery, commonly equated with online banking. The advent of Bitcoin marked the inception of a novel investment paradigm, intertwined with controversy, heightened risk, and the potential for unprecedented yields. In the contemporary financial landscape, cryptocurrencies have become pivotal, meriting significant academic scrutiny. Their emergence is reshaping the realm of finance, offering innovative solutions for transactions. It is of paramount importance that the academic sector investigates cryptocurrencies, given their potential to revolutionize economic exchanges, financial services, and monetary policies. Cryptocurrencies constitute a burgeoning and dynamic area of scholarly pursuit, presenting a spectrum of research opportunities that span economics, finance, computer science, and cryptography. It is incumbent upon the scholarly community to dissect these complexities, gauge their influence on financial markets, and enhance the collective understanding of their global economic ramifications. Furthermore, time series modeling provides a framework for the empirical examination of financial series, pertinent given the temporal nature of financial events. The exploration of phenomena such as volatility clustering and leverage effects within the cryptocurrency domain is essential, as it aids investors in navigating associated risks and in making informed decisions.

This study addresses the volatility of financial series, with a particular focus on the phenomena of volatility clustering and leverage effects. Hypotheses positing the existence of these phenomena are empirically tested against the backdrop of cryptocurrency markets. The analysis encompasses the six foremost cryptocurrencies by market capitalization, expressly omitting so-called stable coins due to their direct pegging to fiat currencies such as the US dollar. Data spanning from the 1st of October, 2019, to the 29th of September, 2023, for Bitcoin (BTC), Ethereum (ETH), Binance Coin (BNB), Ripple (XRP), Cardano (ADA), and Dogecoin (DOGE) serve as the empirical basis for this inquiry. Employing quantitative econometric techniques, the study seeks to evaluate the applicability of the theoretical hypotheses concerning volatility clustering and leverage effects to these select cryptocurrencies. The pertinence of such an inquiry is underscored by its potential to challenge the traditional random walk hypothesis, which posits a non-directional movement of stock and cryptocurrency prices.

In the analysis, three derivatives of GARCH models are applied to the log returns of the variables under scrutiny. The ARMA-GARCH methodology is utilized, where conditional heteroskedasticity is modeled using the GARCH(1,1), GJR (Glosten, Jagannathan and Runkle)-GARCH(1,1), and EGARCH(1,1) variants. The preliminary results do not indicate the presence of conditional heteroskedasticity in Bitcoin's random innovations, thereby excluding it from subsequent modeling within the proposed framework. Conversely, the remaining five cryptocurrencies demonstrate evidence of ARCH effects subsequent to initial ARMA modeling. For Ethereum (ETH) and Cardano (ADA), the hypothesized volatility clustering and leverage effects are substantiated. Binance Coin (BNB), Ripple (XRP), and Dogecoin (DOGE) present weak to moderate indications of volatility clustering. Unique to Ripple is the detection of inverse leverage effects, signifying that log-returns display a more pronounced response to positive stimuli as compared to negative ones.

The structure of the investigation is delineated as follows: Subsequent to the Introduction, Section 2 comprises a comprehensive literature review detailing the theoretical underpinnings and empirical research pertaining to cryptocurrencies. Section 3 explicates the methodology and the rationale undergirding the ARMA-GARCH modeling, coupled with an exploratory analysis of the salient data. The findings from the application of the three-model approach to Ethereum, Binance Coin, Ripple, Cardano, and Dogecoin are elucidated in Section 4. A discussion of these outcomes, along with their implications and the limitations of the research, is reserved for Section 5. The study culminates in Section 6, where a conclusion is drawn in a manner befitting the scope of the research presented.

2 Literature Review

The analysis of price fluctuations in financial assets, encompassing the examination of their historical and prospective behaviors, is frequently necessitated. Variations in price and their magnitude may be attributed to both systematic and idiosyncratic factors, the latter pertaining to individual firm-level influences, rendering the precise underlying relationships elusive. Investors typically resort to technical and selective fundamental analysis, constrained by their limited access to confidential data. Their strategies are predominantly predicated on historical and current data, which suggests that markets are generally characterized by semi-strong efficiency. Malkiel [1] expounds on the irrational behaviors of economic agents within financial markets and how such behaviors contribute to market inconsistencies and anomalies. These anomalies often manifest as observable patterns and trends across various financial instruments, including stocks, bonds, and cryptocurrencies, contributing to the phenomenon referred to as volatility clustering. Within this framework, ARCH and GARCH models are invaluable for elucidating the impact of investor behavior and decision-making on market dynamics and the continuity of specific volatility patterns.

The ARCH methodology was first pioneered by Engle [2] in the context of inflation modeling. It has progressively gained traction, evolving into its generalized form, GARCH [3], which has been established as particularly efficacious for modeling financial time series. This includes applications in financial assets, inflation, interest rates, and exchange rates [4]. Volatility clustering has been recognized as a salient characteristic of financial series [5], warranting ongoing scholarly interest. It is commonly observed that price fluctuations tend to cluster by magnitude; minor changes frequently precede minor changes, and significant fluctuations are often succeeded by similar magnitudes [6–8]. Such patterns in investor sentiment can be attributed to an amalgam of psychological, economic, or other factors, including those of an irrational nature. What might be misconstrued as random volatility is in fact an empirical manifestation of autocorrelation within a brief temporal window, often spanning just a few days. This component is instrumental in engendering clusters of comparable volatility [9]. The incorporation of this concept is crucial in the modeling of financial time series, as it encompasses the behavioral inclinations of investors, which invariably give rise to the notion of leverage.

The rationality of economic agents, as noted by Markowitz [10], is predicated on the concept of 'probability beliefs' in times of uncertainty. This subjective perception of probability is intrinsically linked with expectations of portfolio returns and risks, providing a fundamental premise for this study's progression. It is posited that the rational choices made by individuals in the economic arena, when confronted with uncertainty, are grounded in their interpretations of market event probabilities and consequent return and risk expectations. The investigation herein is predicated on this nexus, specifically within the cryptocurrency context, aiming to empirically examine the

influence of rational decision-making and personal beliefs on investors' risk perception and return anticipation in the cryptocurrency market. Observations of volatility clustering and leverage effects within this market are acknowledged as potential influencers of these perceptions. Yu [11] observes that adverse financial market news precipitates asset price reductions, adversely impacting the balance sheet's debt-to-equity ratio. Such perturbations elicit market responses that further depress the asset's price, engendering prolonged volatility as opposed to the resilience observed in asset price increases of equivalent magnitude. This phenomenon suggests a negative correlation between asset volatility and returns, as elucidated by subsequent studies [12]. The predilection for investors to react more sharply to significant price declines, thereby exacerbating price reductions, undermines passive trading strategies. Conversely, behavioral theories suggest a prevalent belief among investors that substantial price increases are implausible and deviate markedly from fundamental values. Under these circumstances, investors act to preclude extended volatility of securities, in contrast to the converse scenario. The theoretical constructs delineated are anticipated to hold validity for nascent financial assets such as cryptocurrencies.

The scholarly investigation into the volatility of cryptocurrencies, while not a new endeavor, has seen a surge in interest in recent years. The focus of these studies has primarily been on Bitcoin, the most frequently traded cryptocurrency [13], with a broader examination of multiple cryptocurrencies through the lens of various GARCH methodologies also emerging [14]. Katsiampa [13] conducted an analysis of Bitcoin's volatility using several GARCH models for the period between 2010 and 2016, although the study stopped short of delineating the findings' implications for volatility clustering. Chu et al. [14] observed high volatility in cryptocurrencies when considering their inter-daily prices and included a selection of popular cryptocurrencies at the time of their research, such as Bitcoin, Dash, Dogecoin, Litecoin, Maidsafecoin, Monero, and Ripple, confirming volatility clustering and leverage effects across all examined assets. It has been posited that Bitcoin's price is more heavily influenced by overarching demand than by transient price shocks [15], a notion that aligns with the analogy of Bitcoin as the contemporary equivalent of gold. This comparison has been supported by evidence of volatility clustering in such financial assets [14–16]. Yin et al. [17] proffered an intriguing hypothesis regarding the escalating volatility of cryptocurrencies, suggesting a significant influence stemming from oil price shocks and their consequent impact on economic uncertainty. Such conditions facilitate a shift among investors with pessimistic outlooks towards cryptocurrencies as a haven for preserving capital value. Further studies have linked cryptocurrency volatility to liquidity, establishing this relationship for all major cryptocurrencies with the exception of Bitcoin [18]. These findings provide the impetus for extending the research to encompass the remaining nine months of 2021, the entirety of 2022, and the first part of 2023, while also considering shifts in cryptocurrency capitalization rankings.

This paper contributes to the global literature on financial volatility by examining volatility clustering and leverage effects within the realm of the six cryptocurrencies boasting the largest market capitalizations. Its significance is underscored by the specificity of the time interval studied, the selection of assets under consideration, and the employment of the Autoregressive Integrated Moving Average - General Autoregressive Conditional Heteroskedasticity (ARIMA-GARCH) approach for modeling these phenomena.

3 Methodology

This section provides an overview of the dataset's characteristics and the theoretical underpinnings of the applied models. Secondary daily data for the six leading cryptocurrencies were sourced from the Yahoo Finance database, spanning from the 1st of October, 2019, to the 29th of September, 2023, resulting in 1460 observations per asset. The data procured are originally derived from the CoinMarketCap web database. The dataset was scrutinized for completeness, and no instances of missing values were detected. Although a more extensive observation period is accessible for assets such as Bitcoin, Ethereum, Binance Coin, and Ripple, the same breadth of data is not available for the remaining cryptocurrencies, thereby justifying the selected time frame to ensure comparability. An expansion to include the top ten cryptocurrencies by market capitalization was considered; however, the inclusion of four so-called stable coins was deemed inadvisable due to their valuation being intrinsically linked to US government-backed assets, deposits, and gold.

The methodology also entails observing the traded volume alongside the adjusted closing price as a preliminary indicator of potential clustering. Prior to modeling, preprocessing of the data is requisite, as the nominal price values provided do not directly offer insights into the asset returns of interest. Consequently, log returns are employed owing to their advantageous properties, such as additivity and the facilitation of compounding, which aid in achieving stationarity in the series for the level ARIMA equation. Additionally, due to significant price variances among assets, a transformation to a common-based scale is applied [6]. The computation of log-returns is executed as follows:

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right) = lnP_t - lnP_{t-1} \tag{1}$$

where, for each period t = 1, 2, 3, ..., T, the return (r) is computed as the natural logarithm of the ratio of the current period's price to that of the preceding period. The selected six cryptocurrencies represent 73.04% of the total market

capitalization in 2023, substantiating their relevance for this study. This is especially notable given that the myriad of other crypto-assets constitute the remaining 26.96% of the market's total capitalization. With a combined market capitalization of 797.04 billion USD, these cryptocurrencies underscore their indubitable significance within the current investment landscape.

Two primary hypotheses are posited in this research: (a) the cryptocurrencies under study exhibit volatility clustering, and (b) they are characterized by leverage effects. These hypotheses will facilitate a differentiation of each asset from an investment perspective. To achieve this, the analysis will focus on the estimated results and significance of the GARCH term (β) as an indicator of volatility clustering, and the asymmetric term (γ) as an indicator of leverage effects.

3.1 Exploratory Data Analysis

Figure 1 illustrates a significant appreciation in the value of the six cryptocurrencies during 2021. This surge can be ascribed to a confluence of events, including Bitcoin's market value exceeding 1 trillion USD, the burgeoning interest in NFTs (non-fungible tokens), the adoption of Bitcoin as legal tender in El Salvador, and public endorsements by prominent figures such as Elon Musk for Dogecoin. Peak values were observed with Bitcoin reaching 67,566.83 USD, Ethereum at 4,812.09 USD, Binance Coin at 675.68 USD, and Ripple, Cardano, and Dogecoin peaking at 1.84, 2.97, and 0.68 USD respectively. Subsequent to these events, a decline in value was observed throughout 2022 and 2023, with the assets' valuations reverting to levels akin to those prior to the events of 2021.

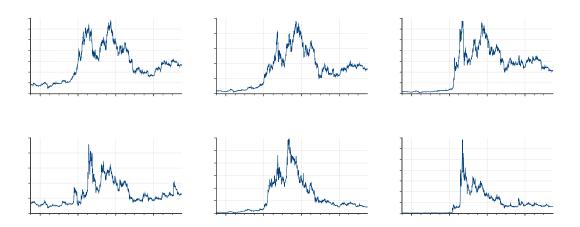


Figure 1. Time series of Bitcoin, Etherium, Binance Coin, Ripple, Cardano, and Dogecoin, in USD

The significance of traded volume is acknowledged as a pivotal factor in deciphering market dynamics, including the identification of clustering and leverage effects in returns. Traded volume is instrumental in gauging the liquidity of a financial asset, with higher volumes typically indicative of greater liquidity. It is also reflective of investor sentiment and interest, which can subsequently impact market efficiency, the accuracy of asset valuation, and the management of portfolio risk. As delineated in Figure 2, an increase in market activity is discernibly correlated with periods of pronounced price volatility, and conversely, market activity diminishes during phases of lesser price fluctuations.

Log returns are predominantly observed to oscillate within a -10% to 10% range, with certain intervals experiencing fluctuations extending to 10 percentage points more pronounced in either direction, as depicted in Figure 3. Notably, outlier observations tend to cluster around specific events, particularly in 2020Q1, coinciding with the onset of the COVID-19 pandemic, and the subsequent quarters of 2021Q1 and 2021Q2. These findings suggest an inquiry into the presence of volatility clustering in log returns, along with the potential for leverage effects, which tend to be more pronounced amidst periods of negative investor sentiment. Over the entire period under study, the cumulative log returns are recorded at 117.1%, 224.1%, 260.5%, 73.7%, 186.9%, and 326% for the respective assets, sequenced according to their appearance in the plot.

The descriptive statistics presented in Table 1 delineate the central tendency, variability, and the distributional characteristics of the adjusted closing prices for each cryptocurrency. Additionally, the coefficient of volatility (CV), serving as a relative measure of data variability, has been computed. This is quantified as the ratio of the standard deviation to the mean value, providing an index of relative variability.

$$CV = \frac{\sigma}{M} \cdot 100 \tag{2}$$

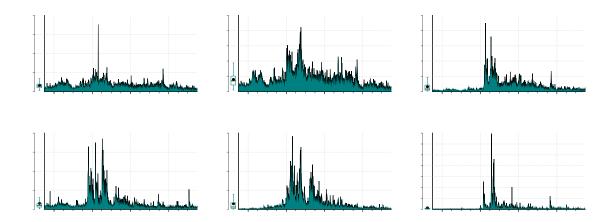


Figure 2. Traded volume of Bitcoin, Etherium, Binance Coin, Ripple, Cardano, and Dogecoin, in millions

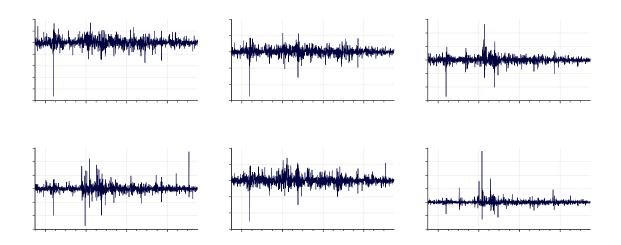


Figure 3. Log-returns of Bitcoin, Etherium, Binance Coin, Ripple, Cardano, and Dogecoin

Table 1. Descriptive statistics of log-returns

	BTC	ETH	BNB	XRP	ADA	DOGE
Mean	0.0008	0.0015	0.0018	0.0005	0.0013	0.0022
Standard Error	0.0009	0.0012	0.0013	0.0015	0.0014	0.0020
Median	0.0001	0.0010	0.0012	0.0005	0.0006	-0.0004
Mode	n.a.	n.a.	n.a.	n.a.	n.a.	0
Standard Deviation	0.0358	0.0466	0.0504	0.0581	0.0534	0.0780
CV	4,456.7%	3,031.18%	2,820.29%	11,497.34%	4,170.44%	3,491.02%
Kurtosis	24.83224	19.85797	26.47621	23.30567	11.24586	111.3568
Skewness	-1.4875	-1.3651	-0.2557	0.4437	-0.2387	5.9478
Range	0.6366	0.7814	1.072	1.099	0.7831	2.032
Minimum	-0.4647	-0.5507	-0.5431	-0.5505	-0.5036	-0.5151
Maximum	0.1718	0.2307	0.5292	0.5486	0.2794	1.5163
Count	1,459	1,459	1,459	1,459	1,459	1,459

Smaller values of CV are indicative of reduced variability within the dataset. It has been observed that all assets exhibit exceptionally high levels of variability, with coefficients exceeding 100%. This suggests that investments in these assets are associated with high levels of risk. Furthermore, the distribution of data for each cryptocurrency

does not conform to normality. Bitcoin and Ethereum are characterized by pronounced negative skewness, Binance Coin and Cardano display slight negative skewness, Ripple demonstrates moderate positive skewness, and Dogecoin is distinguished by substantial positive skewness. The log returns for all cryptocurrencies under analysis are characterized by leptokurtic distributions, reflecting a higher propensity for extreme values as compared to a normal distribution.

Table 2 presents the correlations of adjusted closing prices among the assets, as depicted in the lower triangle of the matrix, revealing a conspicuously high degree of correlation across all assets. The least correlation, at 0.732, is between Bitcoin and Dogecoin, representing the extremities of the pricing spectrum. These elevated correlation coefficients elucidate the phenomenon of market contagion evident during specific events, accounting for the tendency of cryptocurrency assets to exhibit similar trajectories over time. Conversely, the upper triangle of the matrix displays the correlations between log returns, where moderate correlations are generally noted. The minimum correlation observed is 0.3595 between Ripple and Dogecoin's log returns, while the maximum correlation, at 0.8396, is between Bitcoin and Ethereum.

Table 2. Correlation matrix of adjusted closing prices (lower triangular) and log-returns (upper triangular)

	BTC	ETH	BNB	XRP	ADA	DOGE
BTC	1	0.8396	0.7146	0.5939	0.6990	0.4630
ETH	0.902	1	0.7404	0.6361	0.7517	0.4529
BNB	0.833	0.943	1	0.5744	0.6601	0.3595
XRP	0.853	0.861	0.833	1	0.6177	0.3685
ADA	0.855	0.861	0.785	0.872	1	0.4371
DOGE	0.732	0.815	0.814	0.880	0.855	1

3.2 The ARIMA-GARCH Model

The primary econometric method employed in this study is the GARCH model, which integrates aspects of autoregression with models of conditional heteroskedasticity. Prior to the estimation of the GARCH model, an assessment of the key independent variables is conducted. This involves a bifurcated approach based on linear modeling for the expected value and nonlinear modeling for the conditional variance. The linear component may be estimated using any form suited to delineating changes in the dependent variable. A foundational principle of time series analysis in the context of stocks and cryptocurrencies is the random walk hypothesis, which posits that asset prices evolve in a stochastic manner, independent of historical prices. Fama [19] asserts that the random walk hypothesis renders the prediction of stock prices based on past information futile. Nonetheless, subsequent analyses indicate that the hypothesis does not fully encapsulate market behavior, thus allowing for modeling predicated on historical price movements and volatility.

An ARIMA methodology has been adopted for the linear element of the final model. This selection is underpinned by the recognition that financial time series intrinsically rely on their antecedent values and innovations, consonant with the principles of weak-form market efficiency. It is posited that this model more aptly mirrors actual market behavior than a random walk model by acknowledging the presence of conditional dependence. Furthermore, the adoption of an ARMA framework is advantageous over solely autoregressive or moving average models. By amalgamating autoregressive and moving average components within a singular equation, the ARMA model is capable of encapsulating a more extensive array of patterns, encompassing long-term dependencies and acute shocks that may elude individual AR or MA models. This fusion affords a broader spectrum of model combinations for (p,q), enhancing the model's robustness. ARMA models are also characterized by their generation of residuals with diminished autocorrelation, thereby approximating the desirable characteristic of white noise within the residuals. The general ARIMA(p,d,q) model is articulated as follows:

$$y_t = \mu_0 + \sum_{i=1}^p \phi_i y_{t-i} + \sum_{i=1}^q \theta_i \varepsilon_{t-i}$$
 (3)

In the specified ARIMA(p,d,q) model, t=0,1,2,...,T denotes the time period, μ represents the intercept, and ϕ and θ are the parameters subject to estimation. The terms p and q correspond to the lag order for the autoregressive and moving average components, respectively. The differentiation order d in ARIMA(p,d,q) refers to the level of differencing required to attain stationarity, a prerequisite for model estimation, as indicated by the integration order I(d) [8]. The error term ε varies over time, following a normal distribution with $\varepsilon \sim N(0,\sigma^2)$. To verify the stationarity of the series, both the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are employed. The null hypothesis for these tests posits the existence of a unit root, implying non-stationarity within the series.

Central to both tests is the assumption of constant mean and variance over time. While the ADF test employs an autoregressive model and may be influenced by autocorrelation, the PP test utilizes a non-parametric correction that circumvents the explicit modelling of autocorrelation, thus providing robustness in the face of autocorrelation concerns. Despite methodological differences, both tests are anticipated to yield commensurate conclusions.

The determination of the optimal lag length is imperative and may be informed by established theoretical relationships. The appropriate number of lags for the ARIMA model can be ascertained through the classical Box-Jenkins methodology, utilizing autocorrelation and partial autocorrelation plots, or via model optimization, which typically involves the minimization of an information criterion. Given that the log returns of the six cryptocurrencies under consideration exhibit stationarity but do not provide clear indications for lag length on the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots, the latter approach is adopted to ascertain the most suitable model. The Akaike Information Criterion (AIC) is utilized as a benchmark for model adequacy. Subsequently, the Ljung-Box test is applied to ascertain the thoroughness with which the model accounts for autocorrelation within the data. This test scrutinizes the residual autocorrelation to assess the model's fit. The null hypothesis posits an absence of autocorrelation within the residuals up to a specified number of lags, L, implying that they are independently distributed.

Prior to the estimation of the conditional variance, an examination for ARCH effects is conducted for each model under consideration. The critical element in GARCH modeling is the identification of heteroskedasticity within the random innovations, signifying a non-stable variance in log-returns. The principles underpinning GARCH models were established by Bollerslev [3], and subsequent iterations have introduced various adaptations. This model encapsulates the risk component of investments more effectively by integrating conditional variance [3] and is commonly applied in the analysis of financial time series known for volatility clustering [20]. In contrast to traditional linear models, which focus on the behavior of the conditional mean, or the average change in the dependent variable in response to unit shifts in the independent variable, GARCH models adopt a non-linear framework, solely addressing the conditional variance. The GARCH model represents an evolution of ARCH models, paralleling the relationship between AR and ARMA methodologies. The presence of non-constant variance in the standard errors contravenes the homoskedasticity assumption inherent in linear models, resulting in elevated standard errors in the estimated parameters [8]. Given the marked changes in volatility within the selected data, the application of the GARCH methodology is justified as being particularly apt. The present study utilizes the ubiquitous GARCH(1,1) model for its simplicity and effectiveness in modeling conditional variance. This model is expressed as an amalgamation of the long-term variance (GARCH component) and the ARCH components, culminating in the GARCH(q, p) model.

$$\sigma_t^2 = \omega + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2$$

$$\tag{4}$$

where, σ_t^2 is the conditional variance in period t=1,2,3,...,T, $\omega=\gamma V_L$ is the long-term variance V_L where γ is the weight given, σ_{t-j}^2 is the conditional variance for the previous j periods, and ε_{t-i}^2 represents the squared residuals (random innovations) for the previous i periods. The key approach in the GARCH(1,1) process is that the conditional variance σ_t^2 is more efficiently explained when the model is expanded with the estimated conditional variance from the previous period alongside the squared errors, having in mind that not only the innovations generated by the non-modeled factors have an impact but also the process volatility from the previous period. Besides the basic approach, we estimate two additional variants, i.e., the asymmetric derivatives, the GJR-GARCH(1,1), and the EGARCH(1,1) to find any significant differences in their prediction capabilities. These models allow the variance to react differently, depending on the size and the sign of the shock [8]. The first is also called a threshold GARCH and is an upgrade to the classical model, measuring the presence of the leverage effects in returns of the financial assets. We can identify the GJR-GARCH as

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \gamma_i \varepsilon_{t-i}^2 I_{t-i}$$
 (5)

$$I_{t-i} = \begin{cases} 1, & \text{if } \varepsilon_{t-i} < 0\\ 0, & \text{otherwise.} \end{cases}$$
 (6)

where, γ is the asymmetric parameter, while I is a binary variable containing the persistence restriction in cases of negative shocks. The model gives higher importance to large negative shocks as the leading origin of leverage effects. In cases of price decreasing, or a negative return, the financial asset additionally enlarges its volatility, compared to the contrary case [21–23]. On the other hand, for the specific need of finding possible volatility clustering, we can

use the exponential GARCH. Unlike the previous two, it does not require a strictly positive restriction for the α , β , and ω parameters. It can be represented as

$$log(\sigma_t^2) = \omega + \sum_{j=1}^q \beta_j log(\sigma_{t-j}^2) + \sum_{j=1}^q \gamma_i \frac{\varepsilon_{t-i}}{\sqrt{\sigma_{t-j}^2}} + \sum_{i=1}^p \alpha_i \left[\frac{|\varepsilon_{t-i}|}{\sqrt{\sigma_{t-j}^2}} - \sqrt{\frac{2}{\pi}} \right]$$
(7)

Subsequent to model estimation, a comparison is conducted between the realized variance of the log returns and the conditional variance forecasted by the model. Although realized variance is typically computed using intraday data to capture high-frequency fluctuations, the absence of such detailed data necessitates reliance on daily series. The realized variance is thus computed as the aggregate of the squared intraday log returns.

$$\sigma_{rlzd}^2 = \sum_{t=1}^T r_t^2 \tag{8}$$

where, t denotes the period under observation. In this study, the realized variance is derived solely from the squared daily log returns, attributing to the unavailability of higher-frequency data.

4 Results

Initially, the stationarity of log returns for each cryptocurrency was ascertained through the ADF and PP tests. The null hypotheses of these tests postulate the existence of a unit root within the data. The tests uniformly corroborated the stationarity of the log returns for BTC, ETH, BNB, XRP, ADA, and DOGE at a 1% significance level, affirming their appropriateness for ARMA modeling. The data were partitioned into training and testing samples, with the latter utilized to evaluate model efficacy over the final 60 days of the study period, from 1 August 2023 to 29 September 2023.

Subsequent to the automatic estimation process, over 500 ARMA(p,q) model permutations were examined for each cryptocurrency, limiting the lag length to 7 for both autoregressive and moving average parameters. No seasonal patterns were detected within the finalized models. The ARMA(6,2) model, featuring a zero mean, emerged as the optimal model for Bitcoin based on the lowest AIC of -5,312.84 and a Ljung-Box p-value of 0.9061, signifying an absence of residual autocorrelation. Ethereum's optimal model mirrored this configuration, presenting an AIC of 4,573.29 and a Ljung-Box p-value of 0.8785. The ARMA(6,7) model was found most suitable for Binance Coin, with an AIC of -4,371.97 and a Ljung-Box p-value of 0.7893, indicating no residual autocorrelation. Ripple's log returns were best represented by an ARMA model with a single moving average term, yielding an AIC of 3,947.03 and a Ljung-Box p-value of 0.0359, evidencing no serial autocorrelation at the 1% level. For Cardano, the ARMA(2,7) model with an AIC of -4,187.36 marginally rejected the Ljung-Box null hypothesis at the 5% significance level, indicated by a p-value of 0.0583. Lastly, Dogecoin's optimal model was identified as the ARMA(5,2), with an AIC of -3,144.19 and a Ljung-Box test p-value of 0.0868.

Additionally, the ARCH-LM test was conducted on the residuals of each model to verify the presence of ARCH effects and further substantiate the modeling approach. Bitcoin was excluded from subsequent analyses of volatility clustering and leverage effects due to the absence of ARCH effects in initial models. However, for ETH, BNB, XRP, ADA, and DOGE, the presence of ARCH effects was affirmed, prompting the progression to GARCH modeling. The results for the five remaining cryptocurrencies are systematically detailed in separate subsections for clarity.

4.1 Ethereum (ETH)

The estimation of the ARMA(6,2)-GARCH(1,1) model, as delineated in Table A1, reveals the influence of autoregressive components on Ethereum's log-returns, with a positive association for all but the AR(5) term, and a negative relationship with the moving average components. Nonetheless, the estimated coefficients ϕ and δ do not attain statistical significance at the 10 threshold. Conversely, in the modeling of conditional variance, the coefficients ω , α , and β are statistically significant at the 1 level. The sum of the squared residuals and the conditional variance, approximating 0.9889, suggests that variances induced by shocks are enduring over time, supporting the notion of volatility clustering [8]. This indicates that volatility induced by market events—positive or negative—tends to have prolonged effects. A key predictor of future variances in the model is the GARCH parameter $\beta = 0.8696$, overshadowing the influence of both the long-term variance $\omega \leq 0.1$ and the impact of prior period innovations α , which denote the model's sensitivity to shocks. The presence of leverage effects is intimated by the parameter γ , as indicated within the GJR-GARCH model. Given its statistical significance and positive value, it is inferred that substantial negative returns amplify volatility more than equivalently sized positive fluctuations, affirming the existence of leverage effects for Ethereum. This assertion is corroborated by the EGARCH model, which reports a statistically significant negative coefficient for the asymmetrical term γ . Hence, it is substantiated that Ethereum

experiences persistent volatility clustering and leverage effects, with the latter being more pronounced during periods of declining returns.

Figure 4 illustrates that the implemented GARCH model proficiently captures the overall trend in volatility, as reflected by the realized variance for the year 2023. It is pertinent to note, however, that accurately forecasting dramatic shifts in volatility presents a challenge due to their inherently stochastic nature. The initial five months of 2023 witnessed considerable volatility spikes, indicative of moderate clustering, which subsequently exhibited a diminishing pattern over the following four months. On the right side of Figure 4, the conditional variance predictions rendered by the three models—GARCH, GJR-GARCH, and EGARCH—are displayed for the test period, encompassing the final 60 observations. Among these, the threshold GARCH model consistently forecasts higher conditional variances over time, although all three models demonstrate comparable trajectories. Collectively, the trio of models agree on the rapid transitions in volatility, rendering them suitable for both forecasting volatility and deducing insights into volatility clustering and leverage effects.

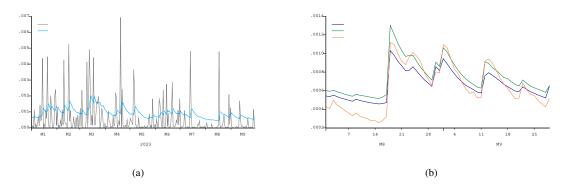


Figure 4. Conditional variance behaviour of Etherium: (a) Realized and conditional variance of Etherium; (b) Conditional variance comparison for the last 60 observations

4.2 Binance Coin (BNB)

In the modeling of Binance Coin (BNB), the ARMA(6,7)-GARCH(1,1) was identified as the optimal model, with nearly all AR and MA terms achieving statistical significance at the 1% level, barring the fifth autoregressive and moving average lags. All estimated coefficients, ω , α and β , attain statistical significance at the 1% level, as detailed in Table A2. With the sum of the ARCH and GARCH terms amounting to 1.0386, previous shocks exert a substantive influence on future volatility, though they marginally breach the weak stationarity assumption of the model as the variability does not wane over time.

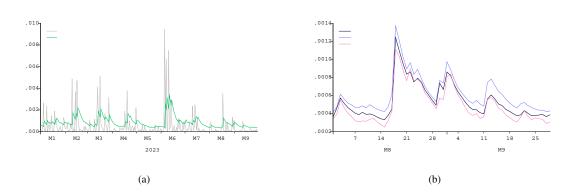


Figure 5. Conditional variance behaviour of Binance Coin: (a) Realized and conditional variance of Binance Coin; (b) Conditional variance comparison for the last 60 observations

Similar to the Ethereum model, Binance Coin's conditional variance demonstrates a significant and dominant influence on log-returns ($\beta=0.8171$) relative to the impact of historical innovations. The threshold GARCH model's γ parameter signals the existence of leverage effects in response to negative returns, corroborated by its high statistical significance and positive value, thereby affirming the second hypothesis for BNB. The exponential GARCH model reinforces this finding, with all parameters reaching statistical significance. Nonetheless, given the

sum of the ARCH and GARCH terms, along with the model's overall stability, the evidence for volatility clustering, as posited in the first hypothesis, is only partially substantiated.

As depicted in Figure 5, the realized variance for Binance Coin presents a contrasting pattern to that of Ethereum, with pronounced clustering observed in June 2023, which subsequently abates. On the right side of the figure, marginal disparities are discernible in the modeling of conditional variance, with the GARCH model serving as a median between the threshold GARCH and exponential models. All three models reflect a commensurate decrease in volatility towards September, mirroring the scale of the realized variance. Notably, both the ETH and BNB cryptocurrencies experience a significant upsurge in volatility on August 18th.

4.3 Ripple (XRP)

When modeling Ripple, a relatively more straightforward model with a single moving average term is estimated, attaining statistical significance at both the 5% and 1% levels, as evidenced by the models presented in Table A3. Contrary to the cases of Ethereum and Binance Coin, Ripple's model attributes diminished weight to past conditional variances, which are indicative of volatility persistence and, by extension, volatility clustering. The sum of the ARCH and GARCH terms, equating to 0.9066, implies that shocks to the system exhibit a declining influence over time, thus maintaining the stationarity of the model. The conditional variance of Ripple, with a moderate yet statistically significant coefficient of 0.4988 at the 1% level, evidences a lesser degree of shock persistence compared to the aforementioned cryptocurrencies, suggesting a more subdued volatility clustering for Ripple. The GARCH model emerges as a marginally superior method for modeling conditional heteroskedasticity relative to the traditional ARCH model in the context of Ripple. Deviating from the previous cryptocurrencies analyzed, the asymmetric term γ in the GJR-GARCH model for Ripple is negative and highly significant, denoting an inverse leverage effect where positive developments exert a more pronounced impact on log-returns than negative ones. However, the leverage effect hypothesis for Ripple is only partially validated, as the γ parameter is not statistically significant in the EGARCH model.

As depicted in Figure 6, the trajectories of both realized and conditional variance for Ripple are notably lower than those of Ethereum and Binance Coin, punctuated by three pronounced spikes in volatility during 2023, particularly in July. This period aligns closely with the litigation outcomes involving the Securities and Exchange Commission, culminating in a favorable verdict for Ripple. Such legal developments affirmed that certain XRP token sales and distributions by Ripple and its executives do not constitute investment contracts, thereby not contravening U.S. securities regulations. During the test period, the projected conditional variances from the GARCH, GJR-GARCH, and EGARCH models are closely aligned, suggesting that a less complex model, such as the general form, is adequate for forecasting Ripple's conditional variance.

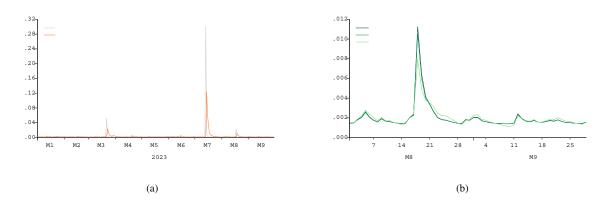


Figure 6. Conditional variance behaviour of Etherium: (a) Realized and conditional variance of Etherium; (b) Conditional variance comparison for the last 60 observations

4.4 Cardano (ADA)

In the modeling of the Cardano cryptocurrency, it is noted that the ARIMA terms exert a significant negative influence on log-returns, in contrast to the moving average terms, which display a positive impact, achieving significance at lags 1, 2, and 4, as delineated in Table A4. A substantial emphasis is placed on the conditional variance ($\beta=0.7855$), with the sum of the ARCH and GARCH terms ($\alpha+\beta=0.9396$) suggesting a persistent influence on volatility over time, indicative of pronounced volatility clustering. The asymmetry parameters within both the GJR-GARCH and EGARCH models affirm the presence of leverage effects in Cardano, with adverse news

exerting a more substantial effect on log-returns than positive developments. The EGARCH model, in particular, evidences a significant and pronounced negative long-term variance (ω).

Figure 7, despite an initial impression of stable and low variance similar to that of Binance Coin and Ripple, actually corresponds with their levels of volatility. Cardano, mirroring the July 2023 volatility spikes, exhibits comparable volatility dynamics. Among the three predictive models evaluated, the exponential GARCH forecasts a lower conditional variance relative to the standard GARCH and GJR-GARCH models. However, it adeptly captures the spike in volatility witnessed towards the end of August and effectively models the subsequent diminution in volatility for the duration of the period under review.

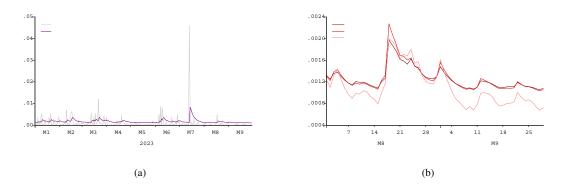


Figure 7. Conditional variance behaviour of Etherium: (a) Realized and conditional variance of Etherium; (b) Conditional variance comparison for the last 60 observations

4.5 Dogecoin (DOGE)

The results for Dogecoin, as presented in Table A5, manifest a degree of ambiguity. The application of the optimal ARMA(5,2)-GARCH(1,1) model to Dogecoin's log-returns yields a sum of α and β totaling 1.5891, surpassing the conventional stationarity constraint of $\alpha+\beta<1$. While instances of $\alpha+\beta>1$ are relatively rare in empirical data, such a configuration in the GARCH model suggests a non-stationary, potentially explosive process, wherein shocks exhibit increasing rather than diminishing effects over time. This contravenes the weak stationarity prerequisite for the model, leading to the conclusion that the ARMA(5,2)-GARCH(1,1) model does not represent an appropriate fit for Dogecoin's log-returns. Despite this, indications of moderate volatility clustering and leverage effects are discernible within the GJR-GARCH framework. Here, the β coefficient is statistically significant at the 1% level, with a value of 0.0954, and the asymmetric term γ , estimated at 0.9966, also reaches statistical significance. The leverage effect discerned is further corroborated by the exponential GARCH model, which evidences a significant β at the 1% level, confirming its presence.

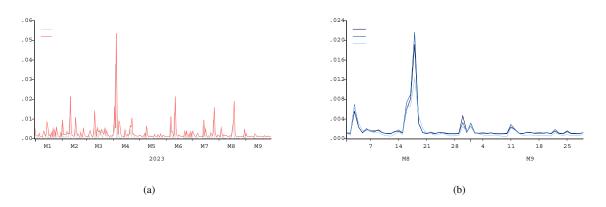


Figure 8. Conditional variance behaviour of Etherium: (a) Realized and conditional variance of Etherium; (b) Conditional variance comparison for the last 60 observations.

The left-hand side of Figure 8 exhibits a heightened variability in Dogecoin's log returns for the year 2023, distinguishing it markedly from the patterns observed in Ethereum, Binance Coin, and Ripple. A notable instance of this volatility was observed at the beginning of April, following a mere social media post by Elon Musk referencing

Dogecoin, preceding changes to the official Twitter logo. This event underscores the sensitivity of Dogecoin's market dynamics to influential external narratives.

5 Discussion

The application of GARCH models to the analysis of log returns within the cryptocurrency sector has attracted considerable academic attention, a consequence of the inherent volatility and the distinct unpredictability characterizing these markets. The results of this study provide nuanced perspectives on the dynamics of cryptocurrency returns and the impact of particular events on their conditional variability. An examination of the established hypotheses concerning volatility clustering and leverage effects has been conducted across the spectrum of assessed assets.

Bitcoin emerges as an outlier wherein neither hypothesis is substantiated, a reflection of the absence of conditional heteroskedasticity in the initial ARMA models. This lack of heteroskedastic anomalies may imply a relative stability and predictability in Bitcoin's volatility, resonating with the findings of Dyhrberg [15]. Furthermore, the analysis suggests an equanimity in Bitcoin's volatility response to both gains and losses, indicating no pronounced asymmetric market reactions to news or events, which can be construed as advantageous from a risk management viewpoint.

Conversely, the investigation into Ethereum indicates a propensity for volatility shocks to exhibit enduring effects. The study also uncovers an asymmetrical volatility response to positive and negative market movements. Notably, negative shifts appear to have a more substantial impact on return volatility than positive changes of a comparable scale. This asymmetry in price volatility is emblematic of the inherent characteristics of Ethereum's market behavior.

In the analysis of Binance Coin, an infraction of the weak stationarity criterion is observed. The study highlights that the variability in the log returns for Binance Coin remains consistent over time, signifying persistent volatility. The statistically robust significance and positive direction of leverage effects, particularly in response to negative returns, underscore the necessity to account for these asymmetrical factors in the modeling of Binance Coin.

Ripple exhibits a contrasting scenario. The research suggests that fluctuations in Ripple's pricing might be subject to less complexity, manifesting a milder persistence in volatility and clustering. A salient aspect of Ripple's behavior is the inverse leverage effect, as demonstrated in the GJR-GARCH model, where positive news exerts a greater effect on log returns as opposed to negative occurrences. However, it is imperative to acknowledge that the EGARCH(1,1) model does not substantiate the leverage effect with significant conviction.

The findings related to Cardano reveal that volatility clusters over time, endorsing the hypothesis of volatility clustering. Additionally, the results support the leverage effect hypothesis; negative market events disproportionately affect Cardano's log returns, suggesting an enhanced sensitivity to unfavorable conditions within its market dynamics.

The analysis of Dogecoin's log returns introduces complexities that question the adequacy of the conventional GARCH model. Like Binance Coin, Dogecoin displays a volatile process where shocks do not attenuate over time, contravening the weak stationarity premise. However, the GJR-GARCH and EGARCH models' suggestion of nascent volatility clustering and leverage effects points towards the need for alternative modeling techniques to more accurately encapsulate Dogecoin's market behavior.

It is in the cases of Ethereum and Cardano that the evidence for volatility clustering and leverage effects is unequivocal, rendering these assets particularly sensitive to news and shifts in investor sentiment. Conversely, for Binance Coin and Ripple—ranked third and fourth in market capitalization—a partial endorsement of volatility clustering is observed, alongside a distinctive divergence in their response to negative news. Dogecoin, despite being the least capitalized among the studied cryptocurrencies, distinctly exhibits an amplification of negative news on log returns. Collectively, the cryptocurrencies under study demonstrate susceptibility to disruption from adverse events, each reflecting the phenomena to varying extents.

This research underscores the applicability of GARCH models in the analysis of log returns among the most capitalized cryptocurrencies. Understanding these dynamic patterns is crucial for risk management and strategic decision-making in the volatile domain of cryptocurrency investment. Times of heightened volatility carry an increased likelihood of substantial price swings and potential losses, thus mandating a cautious approach from investors. It is advisable for investors to refine their risk management strategies, perhaps by employing stop-loss orders, constraining position sizes, or pursuing portfolio diversification. Investors might attempt to time the market based on fluctuations in volatility, increasing their exposure to riskier assets during quiescent periods, and retrenching during tumultuous phases. Nevertheless, such market-timing strategies are not infallible and often do not yield the desired outcomes. Strategies that harness large price movements, like momentum or trend-following strategies, may be more effective during high volatility periods, whereas mean-reversion strategies could be advantageous when volatility is low. The use of leverage can simultaneously elevate the potential for gains and losses. In leveraging scenarios, even minor price movements can significantly affect an investor's returns. While this can augment profits in a rising market, it equally escalates the risk of considerable losses. Furthermore, it may imply that cryptocurrency investors possess a relatively high tolerance for risk. Leverage also intensifies the impact of individual trades, thereby increasing concentration risk. Common trading strategies, such as stop-loss orders, are often recommended to manage the impact of marked price fluctuations. Therefore, the findings of this study not only corroborate but also expand upon existing literature, offering fresh insights into the established knowledge base [14–16, 18].

This study, while affirming the presence of volatility clustering and leverage effects in most instances, is not without its limitations. Expanding the complexity of the models used, such as considering higher-order ARIMA structures, weekly seasonality, or an ARIMAX framework incorporating exogenous variables like gold prices or stock market indices, could significantly refine model precision. Such enhancements are hinted at by the results obtained, although they necessitate substantial computational resources and may not align with the primary focus of our analysis, which does not center exclusively on volatility prediction, thus such detail is deemed ancillary. Academics might also explore a wider array of cryptocurrencies, extend the observational timeframe, and employ intra-day data to yield a disparate set of outcomes—both in model estimations and in realized variance. More sophisticated ARIMA models have the potential to discern subtle volatility patterns that simpler models could overlook. Cryptocurrency markets display patterns of intra-day or weekly seasonality, which could be more accurately captured and analyzed with a granular data approach, allowing for the observation of swift changes in market dynamics. Moreover, the use of intra-day data can sharpen the understanding of event timings and their influence on market volatility and leverage. Advanced modeling and intra-day data utilization not only refine parameter estimates in GARCH models but also improve the detection and understanding of the phenomena under discussion. These avenues are left to be explored by subsequent research.

6 Conclusions

This investigation has illuminated the multifaceted nature of cryptocurrency price behavior through the prism of volatility clustering and leverage effects. It accentuates the aptness of ARMA-GARCH models for delineating the log-returns of cryptocurrencies that lead in market capitalization. The research scrutinizes these digital assets over a recent four-year trajectory, including the period of the COVID-19 pandemic, a time marked by heightened market entry. The findings demonstrate that, with the exception of Bitcoin, the cryptocurrencies examined manifest ARCH, alongside volatility clustering and leverage effects, hence affirming their status as high-risk investments with acute responsiveness to adverse news.

Bitcoin stands apart as it exhibits no ARCH effects, suggesting a lesser tendency toward sharp and sustained price shifts compared to its peers. Ethereum, by contrast, is characterized by pronounced volatility clustering and leverage effects, signaling intricate and unpredictable price movements. Binance Coin presents a departure from the previous patterns, with enduring leverage effects yet without clear signals of volatility clustering, indicating concentrated periods of intense volatility. Ripple's price dynamics emerge as more straightforward, with less pronounced volatility clustering and an inverse leverage effect where positive news has a more significant impact than negative developments. Cardano mirrors Ethereum with prolonged high volatility clustering and leverage effects, highlighting the critical need for vigilant risk management. Dogecoin, while displaying signs of leverage effects and weak volatility clustering, raises questions regarding the adequacy of the adopted model for thorough analysis.

The contribution of this study to the academic field is the provision of novel insights into the behaviors of cryptocurrencies, particularly in the context of volatility clustering and leverage effects, during a time when the cryptocurrency market has seen substantial growth in popularity. It advances our comprehension of the market by underscoring the diverse and asymmetric nature of these phenomena across the six most prominent cryptocurrencies, providing invaluable information for the formulation of risk management and investment strategies. From a broader perspective, the findings emphasize the criticality of risk apprehension and mitigation in cryptocurrency investment, given the propensity of these digital assets for rapid and extensive price movements. The divergent degrees to which volatility clustering and leverage effects are manifested across various cryptocurrencies highlight the imperative for bespoke risk management strategies within this burgeoning asset class. The insights gleaned indicate that cryptocurrencies are assets typified by significant risk, where strategies involving short-term, frequent, and active investment are inadvisable. As the field continues to evolve at a brisk pace, ongoing research will be essential in unraveling the complexities of cryptocurrency price behaviors.

Data Availability

The data used to support the research findings are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest.

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Appendix

Table A1. Etherium modelling results (01/10/2019 - 31/07/2023)

		Linear Model		
ETH	ARMA(6,2)	ARMA(6,2)	ARMA(6,2)	
Parameter	Coefficient	Coefficient	Coefficient	
	0.0565	0.7746***	-0.4838***	
ϕ_1	(1.2522)	(0.0335)	(0.0768)	
4	0.3118	-0.9092***	-0.7593***	
ϕ_2	(0.2784)	(0.0374)	(0.0687)	
4	0.0283	-0.0335	0.0569	
ϕ_3	(0.0781)	(0.0515)	(0.0407)	
Δ.	0.0106	0.0544	0.0956***	
ϕ_4	(0.0437)	(0.0482)	(0.0368)	
4	-0.0186	0.0173	0.0119	
ϕ_5	(0.0481)	(0.0380)	(0.0333)	
4	0.0078	0.0305	0.0702**	
ϕ_6	(0.0351)	(0.0280)	(0.0305)	
$ heta_1$	-0.0919	-0.8104***	0.4806***	
σ_1	(1.2561)	(0.0010)	(0.0663)	
$ heta_2$	-0.2540	0.9981***	0.8381***	
σ_2	(0.2934)	(0.0006)	(0.0591)	
	Conditional Variance Model			
	GARCH(1,1)	GJR-GARCH(1,1)	EGARCH(1,1)	
ω	0.0001***	0.0001***	-0.5322***	
ω	(0.0001)	(0.0000)	(0.0725)	
α	0.1193***	0.0972***	0.2763***	
	(0.0097)	(0.0156)	(0.0253)	
β	0.8696***	0.8527***	0.9464***	
	(0.0118)	(0.0133)	(0.0099)	
-		0.0640***	-0.0725***	
γ		(0.0155)	(0.0124)	
ARCH-LM	0.8199	0.5176	0.6331	

^{***, **, *} indicate 1%, 5%, and 10% statistical significance, respectively

Table A2. Binance Coin modelling results (01/10/2019 - 31/07/2023)

DAID	Linear Model			
BNB	ARMA(6,7)	ARMA(6,7)	ARMA(6,7)	
Parameter	Coefficient	Coefficient	Coefficient	
ϕ_1	-0.7184***	0.1436	-1.0828***	
	(0.0357)	(0.2454)	(0.0216)	
1	-0.1444***	0.1727	0.0718**	
ϕ_2	(0.0506)	(0.1903)	(0.0343)	
1	-0.3508***	0.1167	0.4295***	
ϕ_3	(0.0790)	(0.1796)	(0.0235)	
,	-0.3393***	-0.4289***	0.0652***	
ϕ_4	(0.0415)	(0.1452)	(0.0155)	
1	0.0983	-0.1210	0.3560***	
ϕ_5	(0.0899)	(0.2412)	(0.0237)	
,	0.4654***	0.6951***	00.5397***	
ϕ_6	(0.0481)	(0.1597)	(0.0162)	
0	0.6661***	-0.1949	1.0467***	
$ heta_1$	(0.0000)	(0.2495)	(0.0000)	
	0.1343***	-0.1307	-0.1116***	
$ heta_2$	(0.0233)	(0.2075)	(0.0000)	
	0.4407***	-0.0924	-0.3625***	
θ_3	(0.0628)	(0.2033)	(0.0000)	
	0.4522***	0.4862***	0.0714***	
$ heta_4$	(0.0157)	(0.1518)	(0.0000)	
	-0.0628	0.0791	-0.2723***	
θ_5	(0.0688)	(0.2659)	(0.0075)	
	-0.4252***	-0.6976***	-0.5290***	
θ_6	(0.0214)	(0.1851)	(0.0052)	
	0.0825***	0.0499	0.0078*	
θ_7	(0.0264)	(0.0458)	(0.0042)	
	\ /	nditional Variance M		
	GARCH(1,1)	GJR-GARCH(1,1)	EGARCH(1,1)	
	0.0000***	0.0001***	-0.4649***	
ω	(0.0000)	(0.0000)	(0.0367)	
	0.2215***	0.1679***	0.2975***	
α	(0.0196)	(0.0181)	(0.0144)	
0	0.8171***	0.7865***	0.9601***	
β	(0.0144)	(0.0152)	(0.0048)	
	, ,	0.0946***	-0.0777***	
γ		(0.0202)	(0.0087)	
ARCH-LM	0.3075	0.2899	0.8435	

***,**,* indicate 1%, 5%, and 10% statistical significance, respectively

Table A3. Ripple modelling results (01/10/2019 - 31/07/2023)

	I ingar Madal	
ARMA(0,1)	ARMA(0,1)	ARMA(0,1)
Coefficient	Coefficient	Coefficient
-0.0862**	-0.0920***	-0.0383***
(0.0339)	(0.0343)	(0.0337)
Cor	nditional Variance M	Iodel
GARCH(1,1)	GJR-GARCH(1,1)	EGARCH(1,1)
0.0007***	0.0007***	-1.6844***
(0.0000)	(0.0000)	(0.1107)
0.4078***	0.4515***	0.4533***
(0.0193)	(0.0223)	(0.0218)
0.4988***	0.4921***	0.7611***
(0.0174)	(0.0199)	(0.0164)
	-0.0765***	-0.0158
	(0.0325)	(0.0117)
0.7926	0.7937	0.8473
	Coefficient -0.0862** (0.0339) Col GARCH(1,1) 0.0007*** (0.0000) 0.4078*** (0.0193) 0.4988*** (0.0174)	Coefficient Coefficient -0.0862** -0.0920*** (0.0339) (0.0343) Conditional Variance Machine

***, **, * indicate 1%, 5%, and 10% statistical significance, respectively

Table A4. Cardano modelling results (01/10/2019 - 31/07/2023)

4 D 4		Linear Model		
ADA	ARMA(2,7)	ARMA(2,7)	ARMA(2,7)	
Parameter	Coefficient	Coefficient	Coefficient	
ϕ_1	-0.9316***	-0.9386***	-0.9779***	
	(0.1607)	(0.1519)	(0.1189)	
1	-0.7581***	-0.7592***	-0.7737***	
ϕ_2	(0.1454)	(0.1415)	(0.1157)	
0	0.8651***	0.8781***	0.9108***	
$ heta_1$	(0.1621)	(0.1535)	(0.1226)	
0	0.7372***	0.7482***	0.7749***	
$ heta_2$	(0.1421)	(0.1414)	(0.1181)	
0	0.0054	0.0164	0.0261	
θ_3	(0.0491)	(0.0496)	(0.0461)	
a	0.0771*	0.0819*	0.0872**	
$ heta_4$	(0.0466)	(0.0462)	(0.0436)	
0	0.0249	0.0261	0.0222	
$ heta_5$	(0.0478)	(0.0478)	(0.0468)	
$ heta_6$	0.0499	0.0478	0.0511	
σ_6	(0.0371)	(0.0392)	(0.0374)	
$ heta_7$	0.0268	0.0299	0.0394	
θ_7	(0.0322)	(0.0322)	(0.0294)	
	Conditional Variance Model			
	GARCH(1,1)	GJR-GARCH(1,1)	EGARCH(1,1)	
	0.0002***	0.0002***	-0.7105***	
ω	(0.0000)	(0.0000)	(0.0934)	
α	0.1541***	0.1321***	0.3063***	
α	(0.0181)	(0.0217)	(0.0305)	
β	0.7855***	0.7713***	0.9178***	
	(0.0229)	(0.0238)	(0.0129)	
0/		0.0777***	-0.0346***	
γ		(0.0199)	(0.0115)	
ARCH-LM	0.7124	0.6699	0.7244	

***,**,* indicate 1%, 5%, and 10% statistical significance, respectively

Table A5. Dogecoin modelling results (01/10/2019 - 31/07/2023)

DOCE		Linear Model		
DOGE	ARMA(5,2)	ARMA(5,2)	ARMA(5,2)	
Parameter	Coefficient	Coefficient	Coefficient	
ϕ_1	-0.2084***	-0.1169***	1.1231***	
	(0.0278)	(0.0234)	(0.0304)	
4	-1.0638***	-1.0313***	-0.9565***	
ϕ_2	(0.0239)	(0.0213)	(0.0434)	
4	-0.3677***	-0.2663***	-0.0762	
ϕ_3	(0.0304)	(0.0247)	(0.0473)	
1	-0.0968***	-0.0648***	0.0441	
ϕ_4	(0.0230)	(0.0203)	(0.0326)	
4	-0.1112***	-0.1003***	-0.0788***	
ϕ_5	(0.0094)	(0.0097)	(0.0167)	
0	-0.0402***	-0.0388***	-1.2125***	
$ heta_1$	(0.0045)	(0.0045)	(0.0018)	
0	0.9820***	0.9816***	0.9814***	
$ heta_2$	(0.0044)	(0.0045)	(0.0022)	
	Conditional Variance Model			
	GARCH(1,1)	GJR-GARCH(1,1)	EGARCH(1,1)	
	0.0009***	0.0010***	-2.3766***	
ω	(0.0000)	(0.0000)	(0.1151)	
0.	1.4755***	0.9295***	1.0793***	
α	(0.0718)	(0.0800)	(0.0290)	
β	0.1136***	0.0954***	0.7167***	
	(0.0155)	(0.0168)	(0.0179)	
γ		0.9966***	-0.1454***	
		(0.1028)	(0.0250)	
ARCH-LM	0.4529	0.3655	0.8657	

***,**,* indicate 1%, 5%, and 10% statistical significance, respectively