https://doi.org/10.17221/68/2023-JFS

Modelling the tree height-diameter relationship of Macedonian pine (*Pinus peuce* Gris.) forests in North Macedonia

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Citation: Tanovski V., Matović B., Risteski M., Trajkov P. (2023): Modelling the tree height-diameter relationship of Macedonian pine (*Pinus peuce* Gris.) forests in North Macedonia. J. For. Sci., 69: 497–513.

Abstract: The most crucial individual tree variables in forest management are diameter at breast height (*DBH*) and height of trees (*H*). Projection of stand development over time relies on accurate height-diameter functions. The main aim of this paper is to define the best usable model for tree height prediction from diameter at breast height. We explore the place where the Macedonian pine was discovered in the Baba Mountain in the Pelister National Park in North Macedonia. Thus, we established 48 experimental plots (EP) with a circle shape, a radius of 12.62 m and an area of 500 m^2 each. The EP were established in pure Macedonian pine stands in an elevation gradient between 950 m a.s.l. and 1 700 m a.s.l. Every tree in the EP was attributed with data for diameter at breast height, tree height, and stand characteristics data (elevation, slope, aspect, coordinate) as well. For predicting the tree height, we used 40 models from many authors which are the most usable in forest practice. Also, we prepared evaluations and tests for all models, in order to choose the best responsive model for the Macedonian pine forest. For predicting the best tree height model for Macedonian pine, we decided on Mamoun's equation, with a high correlation value of 0.85 with 73% out of the observed data. Also, this model showed a lower root mean square error of 32.65, a lower model prediction accuracy of 6.77, and a lower mean absolute percent error of 11.73%. Finally, it can be concluded that the nonlinear connection between *DBH* and tree height is the most responsive regression model.

Keywords: connection; diameter at breast height; equations; predictions; tree height

The relationship between the diameter at breast height (*DBH*) and tree height (*H*) is an important factor in forest research and is often used to estimate the forest resources and wood production (Trorey 1932; Stout, Shumway 1982; Fu et al. 2018). Also, these variables are the two most frequently used in forest inventories to estimate stand structure (Spies, Cohen 1992; Álvarez González et al. 2001), volume and biomass (Peng et al. 2004; Gómez-García et al. 2013), carbon (Newton, Amponsah 2007; Van Laar, Akça 2007; Mehtätalo et al. 2015; Mensah et al. 2018), yield estimation (Curtis 1967; Parresol 1992), site index and dominant height estimation (Curtis 1967; Calama, Montero 2004), damage appraisal and stand stability (Parresol 1992; Vospernik et al. 2010), stand growth (Curtis 1967; Burkhart, Strub 1974; Wykoff 1982), and other derived parameters. These parameters are important for preparing forest management

https://doi.org/10.17221/68/2023-JFS

plans and are used as reliable indicators of forest growth and sustainable forest management (Crecente-Campo et al. 2010; Kang et al. 2017; Özçelik et al. 2018). The *H* and *DBH* relation can vary between stands and species, while depending on the variables used, the models from relations can be classified into two types: (*i*) model which can be used locally, and (*ii*) model which can be used generally (Sánchez-González et al. 2007; Lei et al. 2009). The models which can be used locally commonly are only dependent on tree diameter or tree age and applicable only to the data collection stand (Newton, Amponsah 2007). On the other hand, the models which can be used generally, with other stand-level variables (e.g. site condition, competition status, environment, and climatic factors), can be applied to a large area (Trorey 1932; Schumacher, Day 1939). This topic or this problem has been addressed by many authors, e.g. Crecente-Campo et al. (2010); Li et al. (2015); Kearsley et al. (2017); Condés et al. (2018); Özçelik et al. (2018); Zhang et al. (2020). However, so far, the available information about *H-DBH* relationships concerning Macedonian pine is very limited, almost none. Measuring the diameter is much easier than measuring the height of the tree, therefore the precise estimation of the *H-DBH* relationship is essential for creating a clear description of the stand conditions and their variation over time (Arney 1985; Huang et al. 2009). Moreover, *H* and *DBH* relations are required in order to better understand the various relationships that characterise and influence the development of forest ecosystems (Peng et al. 2001).

Macedonian pine (*Pinus peuce* Gris.) presents relict with selective distribution on the Balkan Peninsula which was discovered by German botanist August Grisebah at Baba Mountain in North Macedonia. This pine is the only one from the subgenus *Strobus* native to the Balkan Peninsula (Alexandrov, Andonovski 2011). The native area of this species is on the Baba mountain in the range of the Pelister National Park, as well as the mountains Kozuv and Nidze (Mandžukovski et al. 2009), Shar Planina and Galicica (Em 1969), and Jablanica mountain (Trajkovski 1973, 1977) in North Macedonia. In Bulgaria, the Macedonian pine is located on Pirin mountain, around Blagoevgrad (Pejovski 1971), and on Stara Planina (Horvat 1950). The Macedonian pine is also present in Montenegro and Albania (Koshanin 1924), and Serbia and Greece (Popnikola et al. 1978; Alexandrov, Andonovski 2011). In these regions, Macedonian pine thrives in different conditions and wide elevation amplitude between the northern latitudes of 41° and 43°.

The aim of this research is to detect the best model for predicting the tree height from diameter at breast height, and to explore and research the essential relationship between *H* and *DBH* at the Macedonian pine. The Pelister National Park is the first proclaimed park in North Macedonia due to its being the really native area of some endemic species, with a glacial stone river and other unique characteristics. The Macedonian pine was one of the many reasons for proclaiming the Pelister a national park. Being a Balkan endemic species, the Macedonian pine is a very important species for research and ecology. Besides, the designation of an official national park has a very important economic, ecological, and social meaning at local and global levels. Because of the above-mentioned reasons, and also due to a lack of adequate research data, we considered this topic interesting and important for research, in addition to offering the best model for *H* and *DBH* relations.

MATERIAL AND METHODS

Research site. The research was conducted in *Pinus peuce* Gris. stands at Baba Mountain at the Pelister National Park in the southern part of North Macedonia (41°00'11"N, 21°11'07"E; Figure 1). The research area was located between 950 m a.s.l. and 1 700 m a.s.l., covered with pure even-aged Macedonian pine (*Pinus peuce* Gris.) forests. On this site, the Macedonian pine grows at a silicate geological substrate, although new research from Mandžukovski et al. (2022) presented that it can grow also at a carbon geological substrate. In the research area the forest grows on eutric cambisol and ranker type of soil, on a medium-strong slope (15–30%), and usually at northern, northwestern, and northeastern aspects. The climate data covering the period (1954–2021), including mean annual temperature (°C) and annual precipitation (mm), was used from the nearest weather station in Bitola (12 km), and the data were corrected for the difference in elevation. The research location features a moderately continental climate and mountain climate. The mean annual temperatures and annual precipitations varied from 6.4 °C to 9.1 °C and from 745 mm to 1 245 mm, respectively. According to Lazarevski (1993), the temperature value in this climate region decreases by 0.49 °C, while the precipitation value increases by 50 mm with rising in elevation at every 100 m.

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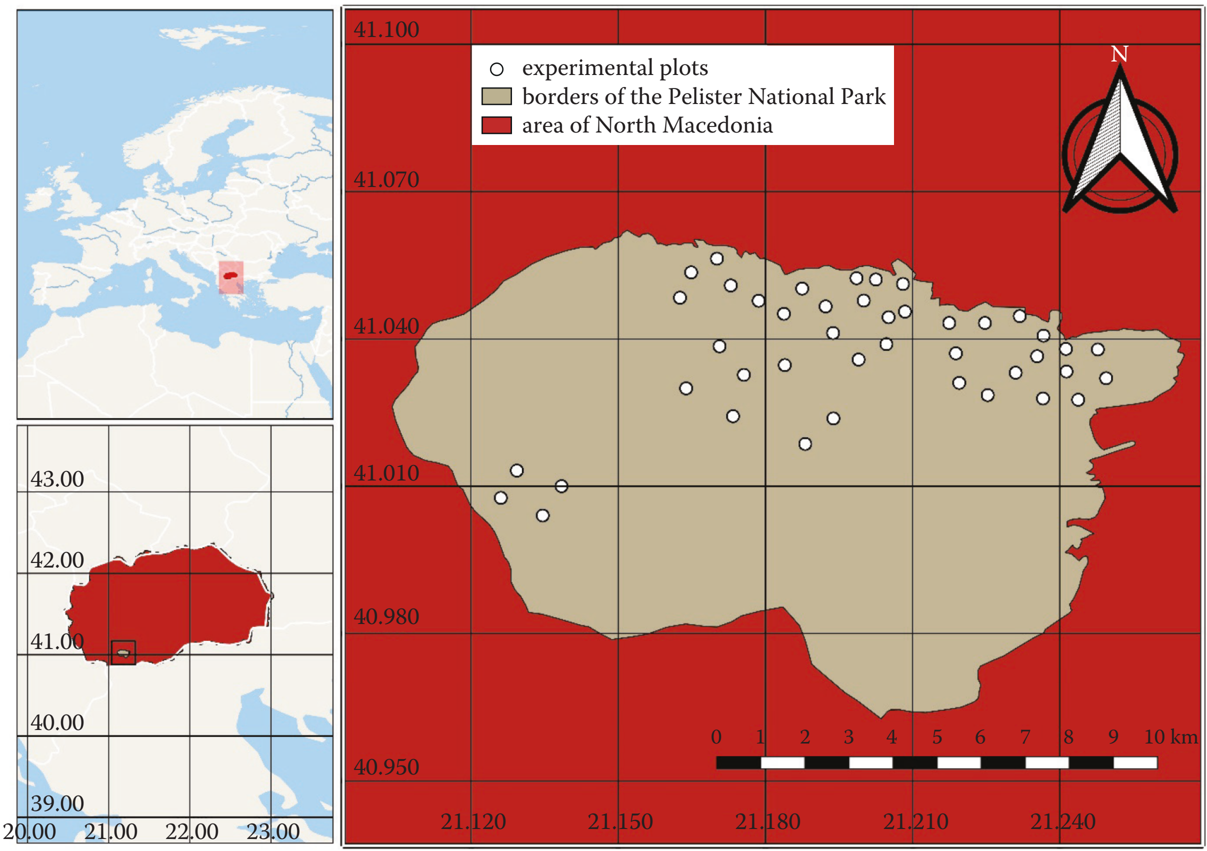

Figure 1. Distribution of the experimental plots at the Pelister National Park, North Macedonia

Sampling method. We intended to calculate the best model for the relationship between the diameter at breast height as an independent variable and the total height of the trees. For this purpose, we randomly established a total of 48 experimental circle plots (EP), with an area of 500 m^2 each. The experimental plots were established in even-aged Macedonian pine height forest. In the stand where the EP were placed, no silvicultural activities were carried out in the past. For all EP we measured the trees with diameter at breast height (*DBH*) > 10 cm, and the total tree height (*H*), and we also collected tree core samples from 10 randomly selected trees at the basal area in each EP for the purpose of calculating the average age of the stands. Also, we attributed each EP with stand characteristics elements such as precise elevation, slope, aspect, management form, and type of reproduction (offspring). For a total of 1 575 trees, the quadratic mean of *DBH*, average Loray height, and density of the stands were obtained and calculated (Table 1).

Table 1. Descriptive statistic for stand elements

Parameters	Descriptive statistic for all EP						
	mean	min	max	*SE*	*CV* (%)	*StE*	*StD*
No. (ha)	656	322	986	43.48	6.11	1.69	–
Qd (cm)	33.9	22.1	51.2	12.56	39.21	0.49	± 12.57
Hl (m)	19.9	13.1	25.2	5.23	26.73	0.12	± 3.96
G ($m^2 \cdot ha^{-1}$)	59.2	38.2	67.1	2.88	4.23	0.11	± 8.82
V (m^3)	598	325	882	0.92	10.23	0.02	± 0.94

EP – experimental plots; No. – number of trees per hectare; *Qd* – mean quadratic diameter; *Hl* – mean Loray height; *G* – basal area; *V* – biomass (volume of trees per hectare); mean – average value; min – minimum value; max – maximum value; *SE* – sampling error; *CV* – coefficient of variation; *StE* – standard error; *StD* – standard deviation

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Data analysis. For the purpose of calculating and defining the best model for predicting the tree height by diameter at breast height, we used 40 models by different authors (Table 2).

Evaluation and testing. In order to develop the new *H-DBH* models for *P. peuce* in North Macedonia, we first selected 40 theoretical functions from the widely used literature in forestry modelling based on their predictive accuracy, simplicity and logic. Using the data for the *H* and *DBH* obtained from 1 565 *P. peuce* trees in North Macedonia, we fitted each of the 40 theoretical functions to this data. We estimated optimum parameters (*a*, *b*, and *c*) and their standard errors. This was per-

Table 2. Used equation models for predicting tree height out of diameter at breast height

Equation model No.	Function	Reference/formula
M1	$H = \frac{a \times DBH}{b + DBH}$	Molto et al. (2014)
M2	$H = a \times DBH^{b}$	Power regression
M3	$H = \left(\frac{DBH}{a + b \times DBH}\right)^2$	Clutter et al. (1983)
M4	$H = \frac{DBH}{\left(a + b \times DBH\right)}$	Prodan (1965)
M5	$H = \frac{1}{a + b \times DBH^{-1}}$	Vanclay (1995)
M6	$H = a \times \frac{DBH}{\left(1 + DBH\right)^{b}}$	Curtis (1967)
M7	$H = \frac{a \times DBH}{DBH + 1 + b \times DBH}$	Bates and Watts (1980)
M8	$H = 10^{a} \times DBH^{b}$	Larson (1986)
M9	$H = e^{\left(a + \frac{b}{DBH}\right)}$	El Mamoun et al. (2013)
M10	$H = e^{\left(a + \frac{b}{DBH + 1}\right)}$	Wykoff (1982)
M11	$H = e^{a + b \times \log\left(DBH\right)}$	Clutter et al. (1983)
M12	$H = a \times e^{\frac{b}{DBH}}$	Schumacher (1939)

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Table 2. To be continued

Equation model No.	Function	Reference/formula
M13	$H \quad e^{a} \times DBH^{b}$	Huang et al. (2000)
M14	$H = a \times DBH \times e^{-b \times DBH}$	Huang et al. (2000)
M15	$H = a \times \left[\ln(1 + DBH) \right]^{b}$	El Mamoun et al. (2013)
M16	$H = \frac{\check{} \quad 2}{a + b \times DBH + c \times DBH^{2}}$	Strand (1959)
M17	$H = \frac{DBH^{a}}{b + c \times DBH^{a}}$	El Mamoun et al. (2013)
M18	$H = a \times DBH^{b \times DBH^{-c}}$	Sibbesen (1981)
M19	$H = a \times DBH^{b} \times e^{-c \times DBH}$	Fast and Ducey (2011)
M20	$H = \frac{a}{\left(1 + b^{-1} \times DBH^{-c}\right)}$	Peschel (1938)
M21	$H = e^{\left(a + \frac{b}{DBH + c}\right)}$	Ratkwosky (1990)
M22	$H = e^{a + b \times DBH^{c}}$	Curtis (1981)
M23	$H = a \times e^{\frac{b}{\left(DBH + c\right)}}$	Ratkwosky (1990)
M24	$H = a \times e^{\left(-b \times DBH^{-c}\right)}$	Lundqvist (1957)
M25	$H = a \times \left(1 - e^{-b \times DBH^{c}}\right)$	Weibull (1951)
M26	$H = \frac{a}{b + e^{-c \times DBH}}$	Pearl and Reed (1920)

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Table 2. To be continued

Equation model No.	Function	Reference/formula
M27	$H = a \times e^{-\frac{k}{DBH}} + 1.3$	Michailoff (1943)
M28	$H = \left(\frac{DBH}{a + b \times DBH}\right)^3 + 1.3$	Pettersson (1955)
M29	$H = \left(\frac{DBH}{a + b \times DBH}\right)^{2.5} + 1.3$	Pettersson (1955)
M30	$H = e^{\ + \frac{\ }{DBH}}$	Gadow and Bredenkamp (1992)
M31	$H = \left(\frac{DBH}{a + b \times DBH}\right)^2 + 1.3$	Näslund (1929)
M32	$H = a \times \left(\frac{DBH}{b + DBH}\right)$	Stanisz (1986)
M33	$H = \frac{DBH^2}{a + b \times DBH + c \times DBH^2} + 1.3$	Prodan (1951)
M34	$H = e^{a + b \times \ln(DBH) + c \times \ln(2 \times DBH)}$	Korsuň (1935)
M35	$H = a + b \times \frac{1}{DBH}$	Rymer-Dudzinska (1974)
M36	$H = a \times \left(\frac{DBH}{1 + DBH}\right)^b + 1.3$	Levaković (1935)
M37	$H = a + b \times DBH + c \times DBH^2$	Binomial
M38	$H = 1.3 + a \times DBH^b$	Stoffels and Van Soest (1953)
M39	$H = \frac{a + b}{(DBH + c)}$	Xifei et al. (2012)
M40	$H = 1.3 + \exp\left[a + \frac{b}{(DBH + 1)}\right]$	El Mamoun et al. (2012)

a, *b*, c – statistical parameters; *DBH* – diameter at breast height; *e* – Euler's number; *H* – predicted tree height

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formed in order to derive and develop models consistent with the North Macedonian data. We then evaluated the performance of each derived model based on (*i*) analyses of fit statistics, i.e. root mean square error (*RMSE*), and (*ii*) the overall model prediction accuracy (*MPA*), which is a measure of model reliability that combines mean bias and the variation of biases. To calculate *RMSE* we first calculated the mean square error (*MSE*). Other calculated values include mean absolute error (*MAE*), *bias*, which evaluated the deviation of the model with respect to the observed results (the smaller, the better), Akaike information criterion (*AIC*; Akaike 1998), coefficient of determination (R^2) and correlation (*R*). We evaluated each of these models for their precision, accuracy, and reliability to predict height for the *P. peuce* in North Macedonia. For this purpose, we used the measured heights of the *P. peuce* trees and compared them with the heights estimated by each of these models by calculating the relative error (*RE*) and the mean relative error (*MRE*) as measures of precision, while the mean absolute percent error (*MAPE*) was used as a measure of accuracy. In fact, any model is highly reliable when *MAPE* is less than 10%, satisfactory when it is 10–20%, and unreliable with acceptable estimated heights when more than 20% (Huang et al. 2003; Sileshi 2014). Furthermore, absolute percent error (*APE*) is calculated. We also plotted the *RE* with the measured diameter to visualise the magnitude and distribution of errors across diameters for each model, but we presented only Model 33 and Model 39 in which a typical distribution of errors can be seen. The *MSE*, *RMSE*, *bias*, *MPA*, *MAE*, *AIC*, R^2, *R*, *RE*, *MRE*, *APE*, and *MAPE* were calculated using Equations (1–12), respectively.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(H_i - \hat{H}_i\right)^2 \quad (1)$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(H_i - \hat{H}_i\right)^2} \quad (2)$$

$$bias = \sum_{i=1}^{n}\frac{\left(H_i - \hat{H}_i\right)}{n} \quad (3)$$

$$MPA = bias^2 + SD^2 \quad (4)$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|H_i - \hat{H}_i\right| \quad (5)$$

$$AIC = n \times ln\left(RMSE\right) + 2p \quad (6)$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(H_i - \hat{H}_i\right)^2}{\sum_{i=1}^{n}\left(H_i - \overline{H_i}\right)^2} \quad (7)$$

$$R = \sqrt{1 - \frac{\sum_{i=1}^{n}\left(H_i - \hat{H}_i\right)^2}{\sum_{i=1}^{n}\left(H_i - \overline{H_i}\right)^2}} \quad (8)$$

$$RE = \frac{\hat{H}_i - H_i}{H_i} \quad (9)$$

$$MRE = \frac{1}{n}\sum_{i=1}^{n}\frac{\left(\hat{H}_i - H_i\right)}{H_i} \quad (10)$$

$$APE = \frac{\left|H_i - \hat{H}_i\right|}{H_i} \times 100 \quad (11)$$

$$MAPE = \frac{100}{n}\sum_{i=1}^{n}\frac{\left|H_i - \hat{H}_i\right|}{H_i} \quad (12)$$

where:
AIC – Akaike information criterion;
APE – absolute percent error;
bias – deviation of the model with respect to the observed results;
H_i – real measured tree height;
$\hat{H}_i$ – predicted calculated tree height;
MAE – mean absolute error;
MAPE – mean absolute percent error;
MPA – model prediction accuracy;
MRE – mean relative error;
MSE – mean square error;
n – number of samples;
p – number of parameters;
R – correlation;
R^2 – coefficient of determination;
RE – relative error;
RMSE – root mean square error;
SD – standard deviation.

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Model development and predictions. Initially, a base model was selected based on the statistics parameters according to the abovementioned 40 worldwide models. The research revealed that the most accurate and consistent results were given using the El Mamoun et al. (2013) model, which was selected as the basic model. After this, we developed a local model for every experimental plot using El Mamoun et al. (2013) model and we fitted the relationship between local model parameters towards some of the stand variables. The below-mentioned independent stand variables (stand age, density, stand basal area, quadratic mean of *DBH*, average Loray height, etc.) were used, nevertheless, the stand basal area provided the best results. Furthermore, the basic simple *H-DBH* model related this connection and developed a modified *H-DBH* model with an independent stand variable (stand basal area), because only stand basal area was showing statistically significant connectivity.

The base model of El Mamount et al. (2013) had the following format of Equation (13):

$$H = \frac{DBH^a}{b + c \times DBH^a} \tag{13}$$

but after adding an independent stand variable, it was changed into the following format of Equation (14):

$$H = \frac{DBH^a + BA}{b + c \times DBH^a} \tag{14}$$

where:

a, *b*, c – statistical parameters;
BA – basal area of total processed trees;
DBH – diameter at breast height;
H – predicted tree height.

We prepared descriptive statistics for parameters diameter at breast height and total tree height. Statistical significance as well as differences between models were determined using analysis of variance (ANOVA). All calculations and statistical analyses were prepared by using Microsoft Excel (Version 16, 2019), StatSoft Statistica (Version 12, 2013), SPSS Software (Version 26, 2018), and Crancod Software (Version 1, 2016).

RESULTS

Different models were developed from the regression statistics where *DBH* and tree height were used as dependent and independent variables at the stands of Macedonian pine. The researched stands have a density of 656 trees per hectare, the quadratic mean diameter is 33 cm, an average height according to Loray of 19.9 m, a basal area of 59.2 $m^2 \cdot ha^{-1}$, and a volume of trees of 598 $m^3 \cdot ha^{-1}$.

The data in the Table 3 point out that the mean values of *DBH* and *H* are 31.5 cm and 18.5 m, respectively, and also that *DBH* is more variable and has a bigger error which can be seen in the standard deviation and coefficient of variation. Figure 2 shows a positive growing regression between diameter and tree height. The diameter has left no asymmetry or negative asymmetry which means that more data is located at thin trees; otherwise, at tree height, the data has a relatively normal position although positive asymmetry can be observed.

The *RMSE* for the models ranged from 32.65 to 37.15 as a mean error at M16, M17, M20, and M39, respectively. Furthermore, model precision accuracy (*MPA*) is in the range of 6.77 in M16, M17, M20 to 8.77 in M35 and M39, which interprets that all models are reliable. Models have a *bias* value lower than 0.007, which means that there is presented a small deviation in the data; also the mean absolute error is very small with a value lower than 0.008. Otherwise, the coefficient of determination for the measured height and calculated height data ranges from 64% to 72%, which means that a significant

Table 3. Descriptive statistic for the *DBH* and *H* parameters

Variable	No.	Mean	Min	Max	Variance	*StD*	*CV* (%)	*StE*
DBH (cm)	1 575	31.51	10	79.9	158.022	12.571	39.89	0.317
H (m)	1 575	18.54	7	34.0	24.744	4.974	26.83	0.125

DBH – diameter at breast height; *H* – height; No. – number of trees per hectare; mean – average value; min – minimum value; max – maximum value; variance – variability among data; *StD* – standard deviation; *CV* – coefficient of variation; *StE* – standard error

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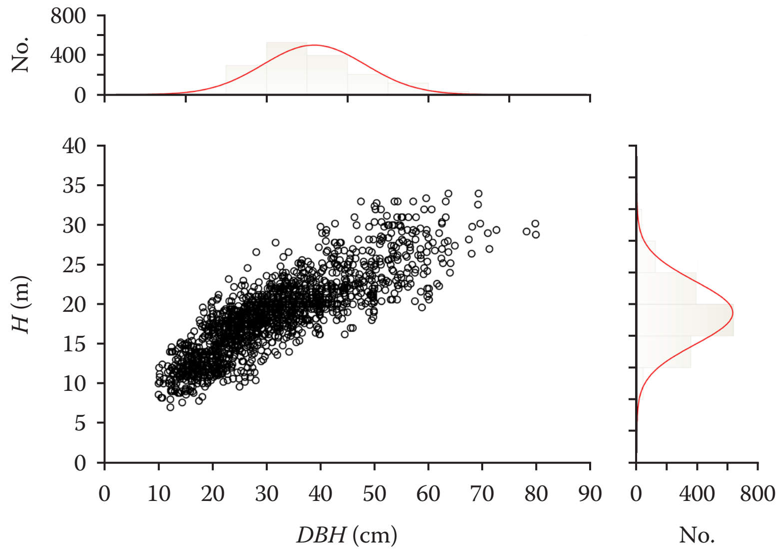

Figure 2. Scatterplot of *DBH* and *H* and histogram with distribution and normal distribution (red line)

DBH – diameter at breast height; *H* – height; No. – number of trees per hectare

amount of data is covered. It is significant that the coefficient of correlation is high and presents a high connection between empirical and calculated data with a range of 0.804 to 0.852, see Table S1 in the Electronic Supplementary Material (ESM). Continuously, the value of the mean relative error is very low within the range of values from 0.014 to 0.023. A large proportion of mean relative errors for models were around zero (*MRE* = 0.0; Table 3), suggesting that these models produced values with small deviations in height compared to the measured values. Otherwise, *RE* in all models has a similar curve as Figure 3, suggesting that the errors increase in *H* estimation for the thinner trees, while for the bigger trees, the errors are around or below zero. In Model 17, the relative error is located equally along the diameter of trees. The accuracy (*MAPE*) of the 3-parameter models ranged from 11.84% (M17) to 13.35% (M39), while the accuracy of the 2-parameter models ranged from 11.69% (M1) to 13.35% (M35).

Out of all 40 models, regarding statistical analysis, we decided that the most accurate model for *H-DBH* connection is Model 17 because it has a *MAPE* of 11.73%, which is very useful. Therefore, this model was used as a modified model for Macedonian pine with stand basal area as an independent variable. However, all the models have

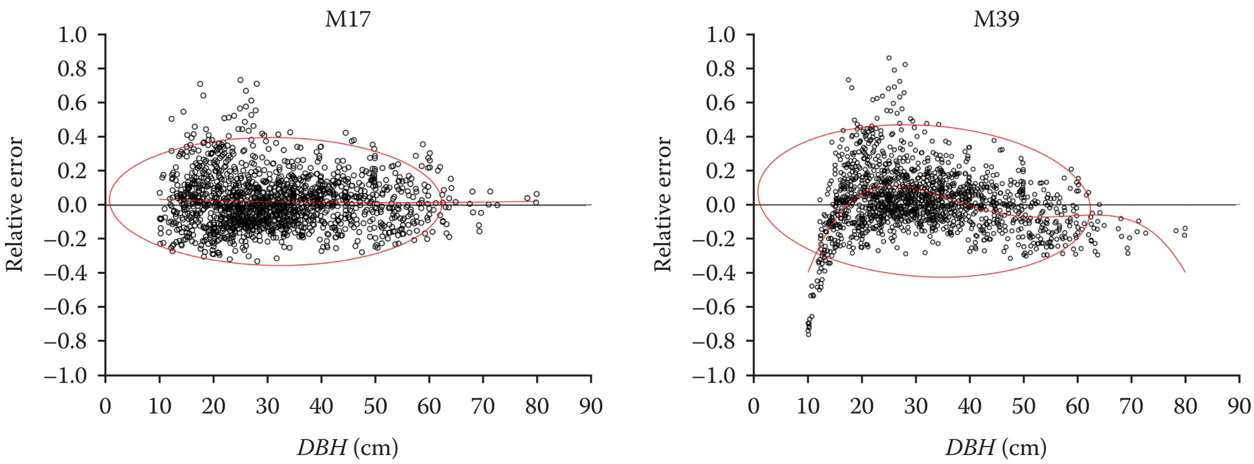

Figure 3. Relative error

DBH – diameter at breast height

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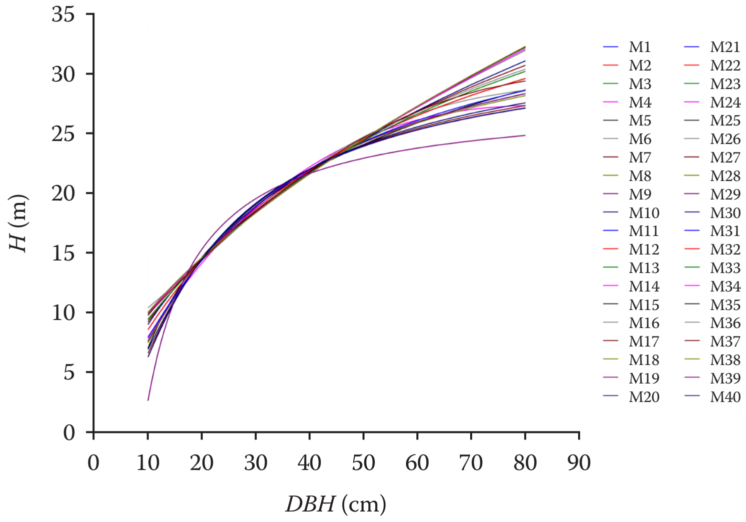

Figure 4. All regression models

DBH – diameter at breast height; *H* – height

a satisfactory percentage of acceptability. It can be seen in Table S2 in the ESM that almost all regression coefficients in all models have a very small standard error, only with the exception of models M18, M22, M25, M34, and M39, where the coefficients are high. This occurrence is a result of numerous data and law dispersion of data from tree height. The statistics used in fitting and evaluating the 40 nonlinear theoretical functions are presented in Table S2 in the ESM. The estimations for the *a*, *b*, and *c* regression coefficient values were statistically significant ($P < 0.05$) for most functions fitted, except for the models M18, M20, M22, M34, M39, and M40, where statistical significance is insignificant with a coefficient greater than 0.05.

Figure 4 shows that almost all models have similar regression lines, only Model 39 has the biggest deviations compared to the rest of the models. Also, a bigger standard error of estimation at the calculation of equation coefficients can be seen in this model. Figure 5A presents the best model of height predictions (Model 17) as well as the best model for the relationship between diameter and tree height. It can certainly be said that the M33 model also has good grades and it can be used in further calculations and projections. Figure 5B presents the modified model M17' with

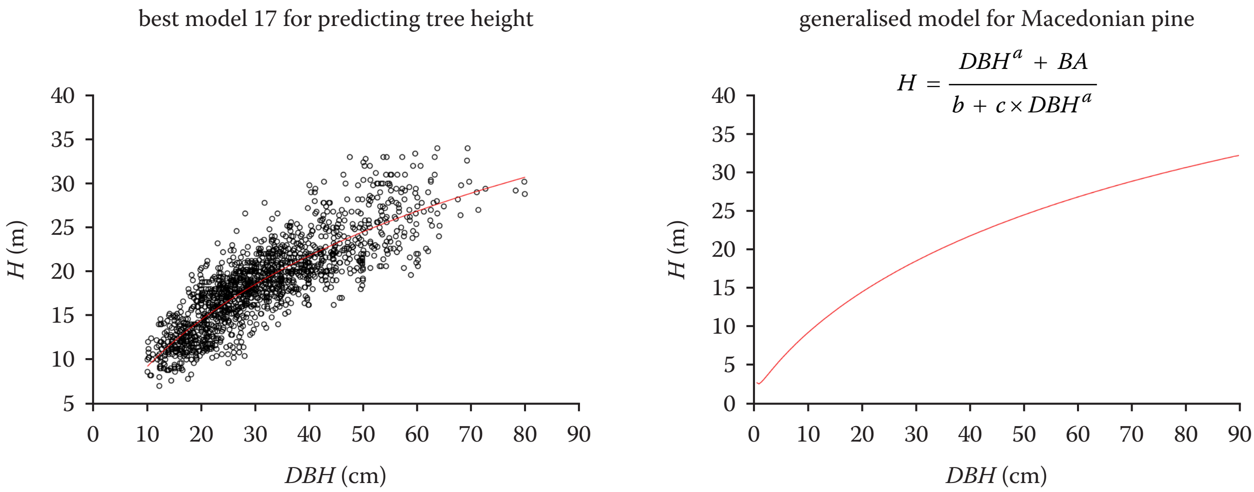

Figure 5. Best model for tree height prediction

a, *b*, *c* – statistical parameters; *BA* – basal area of total processed trees; *DBH* – diameter at breast height; *H* – height

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Table 4. Regression coefficient of modified model for Macedonian pine

Parameter	Regression coefficient of the generalised model M17'			
	value	standard error	*t*-ratio	*P*-value
a	–0.80339	0.068	–11.08	0.00
b	2.09097	0.040	4.67	0.00
c	86.29805	0.000	6.64	0.00

changes in equation (with stand basal area). The statistical calculation (Table 4) showed better results with the input of the stand variable (basal area) in the generalised model (M17).

DISCUSSION

Until today there is no *H-DBH* model for predicting *H* in *P. peuce* in North Macedonia and elsewhere. The choice of the appropriate model is crucial in accurate estimation of *H* in forest plantations for various applications, from commercial exploitation of timber to general forest management. Overall, our study indicates that the newly developed *H-DBH* models based on theoretical function accurately predict the tree height for *P. peuce* and are particularly specific to North Macedonia. According to the statistical analysis, the equation of El Mamoun et al. (2013) gave the best results using M17 model, followed by the modified model with external variable (stand basal area) M17'. The M17' model can be used both locally and more widely because a parameter (*BA*) is added, that is produced by the stand level. The equations M1 Molto et al. (2014), M4 and M33 Prodan (1965), M5 Vanclay (1995), M7 Bates and Watts (1980), and M32 Stanisz (1986) also produce good results. The parabolic equation may be used in specific cases, although in many cases a very good fit was obtained (Näslund 1929; Meixner 1964; Bruchwald 1970). For this reason, this function does not have to be rejected; nevertheless, it should always be verified that results are obtained for values of diameter at breast height in specific cases. If there are negative height values or the maximum value of the function was exceeded and the approximated height decreases with an increase in diameter at breast height, the function has to be rejected (Barzdajn 2017). The Korsuň function in this case has a bigger *MRE* and therefore it is perhaps better to avoid this equation for this species. Also, this function for spruce and beech does not meet the first postulate and its application has to follow similar reservations as those for the application of the second-degree parabola (Barzdajn 2017). Postulates of dendrometry according to Michailoff (1943) in relation to these curves are as follows: (*i*) if $DBH = 0$, it should be $H = 1.3$; (*ii*) if $DBH = \infty$, it should be H = constant, i.e. there should be an upper asymptote; (*iii*) if $DBH = m$ (in the point of inflexion), it should be $H = 0$.

The relationship between height and diameter is also related to species, climate, soil characteristics, region, stand characteristics and even tree diversity (Grubb 1977; Zeide, Vanderschaaf 2002; Feldpausch et al. 2011; Banin et al. 2012; Li et al. 2015). In that context, the significant impact of site and rotation is observed on the prediction error and overall accuracy of the country-level *H-DBH* model suggesting that the country-level model should be refined further to accommodate the effect of the site and rotation (Ng'andwe et al. 2019). Also, the studies Sánchez et al. (2003), Sharma and Yin Zhang (2004), Temesgen and Gadow (2004), Newton and Amponsah (2007) showed that the inclusion of stand characteristics as independent variables in *H-DBH* models improved the prediction accuracy of tree height estimation. Because of this fact, we used stand basal area as an independent variable. The tree age should be considered when choosing models and parameters aimed at predicting tree height more accurately (Li et al. 2015). Sánchez et al. (2003) found that the inclusion of stand age and density contributed to model performance for even-aged *Pinus radiata* stands. Vanclay (2009) also reported that stand density played an important role in even-aged pure plantations. However, the *H-DBH* curve for a shade-tolerant tree species, such as Norway spruce, depends upon the stand average tree size rather than upon the stand age (Mehtätalo 2004). Given that the tree height and the *DBH* usually increased with tree age and are influenced by density, the models predicting the development of the

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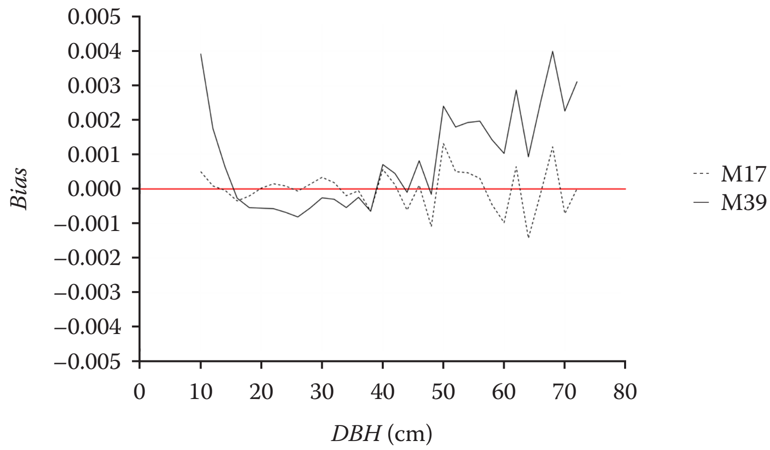

Figure 6. *Bias* in Model 17 and Model 39

Bias – statistics that don't provide an accurate representation of the population; *DBH* – diameter at breast height

DBH and height for a stand have to be used as the variables describing the stand rather than stand age and density. The stand age is not the most important variable in the *H-DBH* relationship, because the tree height increased with the tree age within the same diameter class (Li et al. 2015). Figure 6 presents the distribution of *bias* and it shows the deviations at lower and higher diameters (class) at the best usable modified model (Model 17') and unusable model (Model 39). Here we registered differences in the value of deviations, in M17 the deviations are very small, while in M39 in smaller and higher diameters they are very large (0.004). The same situation is with the mean square errors (Figure 7): the distributions of errors are larger for trees with small diameters and larger diameters. The trees with mid-diameter (20–50 cm) have the same errors. In this case, it is more than clear that Model 39 has a larger error distribution than Model 17. In context with the distribution of *bias*, there is a similar situation in the research of Sánchez et al. (2003), where the value of *bias* is bigger in trees with small diameters, and especially huge oscillation in the distribution of *bias* in trees with larger diameters is observed.

The analysis of predicted height and residuals (Figure 8) shows that in two comparable models (M17 and M39) as best and worst models, the value of residuals is similar with differences in distribution. In that context, in Model 17 the distribution of residuals has a normal circular distribution, which means that errors are located everywhere regarding the tree height. In comparison, in Model 39 the distribution among the

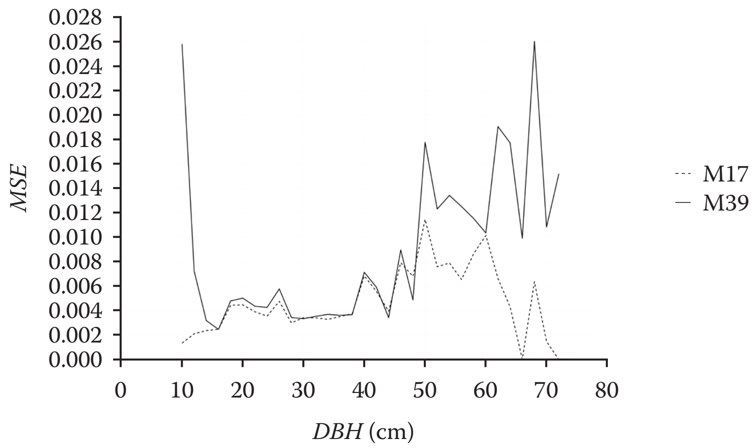

Figure 7. Mean square error in Model 17 and Model 39

DBH – diameter at breast height; *MSE* – mean square error

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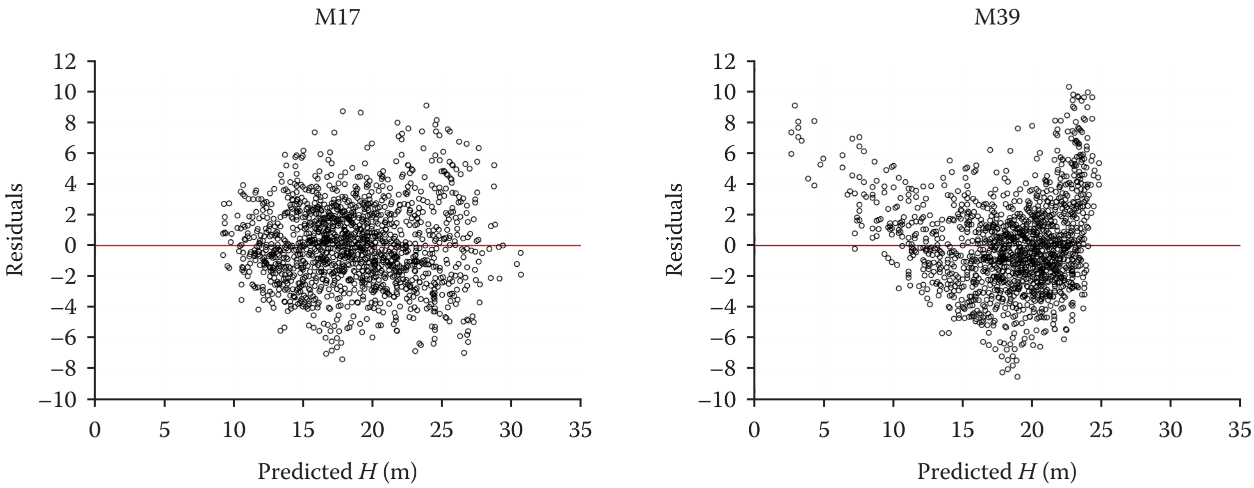

Figure 8. Predicted height and residuals

H – height

lower and the higher trees is unusual, and residual values are positive. Regarding trees with a height of 15–20 m, the residuals are negatively located. The same situation applies to residuals of the predicted tree height regarding *DBH* (Figure 9). It can be said that in Model 39 the positive residual errors were larger for trees with smaller and bigger diameters. Larger residual errors for trees with larger diameters occurred in a study by Kearsley et al. (2017) for height-diameter relationships. This occurrence may be the result of a small number of elaborated trees with large diameters and also may be caused by the large variability in tree heights for trees with large diameters. Of course, thinning has an effect on the relationship between diameter and height as itaffects the diameter and height increment.

From the analysis above, Models 17, 1, 4, 5, 7, and 33 were the best models in this study. There were some similarities between Models 33 and 37 and the fitting statistics of the two models were almost equal. There was no need to estimate too many parameters for prediction models because this study aimed at predicting rather than studying the effects of different factors on the *H-DBH* relationship. The function M17 seems to be the function which may be applied with no reservations as well as in an automated computation, because the curve turned out to be accurate over the entire range of values for diameters at breast height.

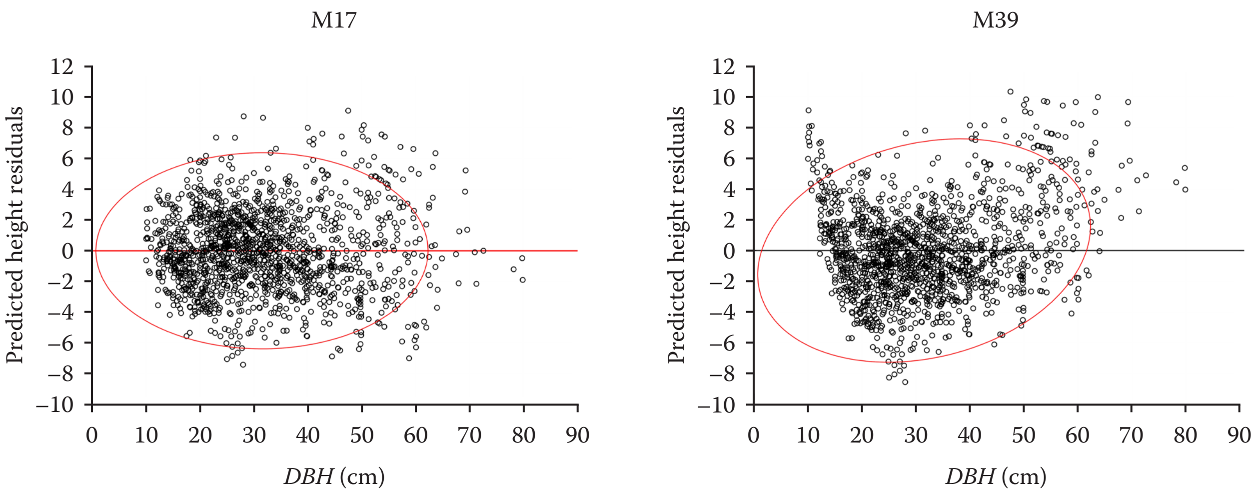

Figure 9. Predicted height residuals and *DBH*

DBH – diameter at breast height

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CONCLUSION

It can be concluded that all the 40 models for predicting the tree height from *DBH* produced useful models, but a few of them are statistically accepted. All candidates for nonlinear growth models allow to describe the *H-DBH* relationship of *P. peuce* in the forest of the Pelister National Park in North Macedonia. According to statistical analysis, the model of El Mamoun et al. (2013) is chosen as the most accurate model that gives the best results and makes the lowest errors in estimations. This model was modified by adding one more external variable (stand basal area) that made it more usable. On the other hand, Model 39 from Xifei et al. (2012) was classified as unusable because it mostly deviates from the measured heights and does not meet the basic conditions to be applied. Also, Models 18, 22, 25, and 33 are unusable because of having big standard errors in the estimation of functional constants. Finally, it can be concluded that the functions are most appropriate to be implemented locally. Although the tested equations can be widely used, there are still numerous factors affecting its accuracy.

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Received: June 15, 2023
Accepted: September 27, 2023