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The role of non-ergodicity in asset pricing, wealth dynamics, and income dynamics: Theoretical results and empirical applications

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Main text

In this PhD thesis I present my contribution to a few problems about the role of randomness in economic systems.

While economists were the first among scientists to embrace the impact of randomness on our daily life, they are not the only ones to study this phenomenon.[1,2] Randomness is present everywhere and therefore has a strong interdisciplinary nature. Indeed, a great number of contributions have emerged from the interactions between scientists trained in different fields. Throughout this document, we will use techniques developed in Physics, Biology, Computer Science, and Chemistry to answer questions like: How to predict the price of a financial asset? Who is able to live a wealthy life? How much time does it take for a worker to improve their income status?

Randomness has helped unite different branches of science by implicitly demonstrating that some problems should be worked from many different angles. There are no constraints forbidding the use of theories and methods inspired by a particular field in a different scientific discipline. This creates the need for a field connecting seemingly distant branches of science. The science of randomness, formally known as inferential statistics, has arisen in part to fulfill that particular need. Moreover, statistics has created scientific value which is different from that of the particular fields where its adherents were originally trained. This dual purpose makes the study of randomness attractive from an applied as well as a fundamental perspective.

The bedrock to answering many puzzles besetting the current economic formalism lies in the ergodic hypothesis.[3] Mathematically, the hypothesis tells us that an observable (e.g., the return of an asset or the growth rate of our wealth) is ergodic if its time average is equal to its expectation value. Philosophically, if the hypothesis is valid, it means that randomness does not affect the dynamics of the system. That is, every asset traded on a stock market will exhibit similar prices over time, and on the long run investors will be indifferent about their investment decisions. Also, it will be irrelevant to track economic inequality, as the economy will not discriminate between individuals on the basis of their history: everyone will experience wealth and poverty during their life.

The ergodic hypothesis is often taken as granted in economics. Unfortunately, recent empirical findings have questioned its validity.[4–7] As a result, over the past decade an abundance of economists have been actively searching for methods that go beyond ergodicity assumptions. Yet, despite important advances, it remains unclear how the randomness induced by non-ergodicity is manifested in economic systems.

In this PhD thesis, we bridge the gap in the literature by developing frameworks for studying the role of non-ergodicity in asset pricing, wealth dynamics, and income dynamics. These frameworks unify results from previous research into comprehensive methodologies that can easily address the question of ergodicity. By practically implementing the frameworks, the puzzles besetting the current economic formalism can be resolved in a natural and empirically testable way. Namely, they can be applied in various economics domains: from tailoring optimal investment strategies up to designing essential policy interventions.

The contribution of this document to the economics literature is displayed in four Chapters.

Chapter 1: A unifying framework for pricing non-ergodic financial assets. In the first chapter we unify approaches used to model asset price dynamics by providing a thorough investigation on the properties of a stochastic process called generalized geometric Brownian motion (gGBM).[8] Geometric Brownian motion is the baseline non-ergodic model used to describe asset price dynamics, but is unable to reproduce the real world observation that extreme price events appear more often than what is expected. gGBM resolves this issue by attributing the overabundance of extreme events to prolonged periods in which the price of the asset exhibits approximately constant values. The nature in which these prolonged periods appear is determined by a so-called "memory kernel". By choosing the appropriate kernel, we can recover the standard GBM as well as other more complex models used to describe asset price dynamics. To understand the behavior of gGBM, we perform a detailed mathematical analysis for the properties of its moments and log returns. More importantly, we describe how the model can be used to predict empirical option values and show that the gGBM framework offers a computationally inexpensive and efficiently tractable solution for tracking the non-ergodic dynamics of asset prices. The findings presented in this chapter were published in Ref.[8].

Chapter 2: Economic mobility, non-ergodicity and mixing in wealth dynamics. In the second chapter, we use ergodicity to answer the question about whether everyone is able to live a wealthy life. We do this by introducing mixing as a relevant concept when quantifying the feasibility of every individual in the economy to change their rank. Mixing is a well-known concept in statistical physics. It describes the property of a dynamical system being strongly intertwined. Every system that satisfies the mixing property will also be ergodic. Translated in economic terms, this means that any measure of mixing will evaluate the extent to which every individual in an economy is able to move across the whole steady-state wealth distribution. In practice, however, many economic systems do not satisfy the ergodic hypothesis, and hence are non-mixing.[6] In this case, measures of mixing will be at their lowest value. This is significantly different from economic mobility measures, where any change in the rankings is interpreted as existence of mobility. Thus, if we use measures of mobility to evaluate the possibilities of an individual to move across the wealth rankings, we may conclude misleadingly that everyone is able to move. More details about the results from this chapter can be found in Refs.[9,10].

Chapter 3: The role of non-ergodicity in income dynamics. Next, in Chapter 3 we investigate how ergodicity affects income dynamics by analyzing the properties of a baseline process for modelling this phenomenon called geometric Brownian motion with stochastic resetting (srGBM). We show that the advantage of modelling through srGBM is that its dynamics are regime dependent. That is, and based on the state of the economy, income dynamics may range from a non-ergodic regime where inequality is an increasing phenomenon up to a stable regime where randomness does not impact the inequality and mobility dynamics.[7]

We then utilize United States data for the dynamics of the income from the World Inequality Database, to study the evolution of income under the assumption that it undergoes srGBM dynamics. The economics literature presented above has predominantly focused on modelling the changes in the income dynamics via shocks in the model parameters that induce changes in the stationary income distribution. These shocks do not transform the income dynamics into a non-ergodic regime. Instead, institutional and social characteristics are the major drivers of the rising inequality. Indeed, there is an abundance of studies which suggest that the current income dynamics is best described with parameters that belong to a such regime. However, there is no evidence that the income dynamics will eventually reach the predicted stationary state. Thus, it can also be presumed that the rising inequality might be a result of non-ergodic dynamics. To this end, we indeed find robust evidence which indicates that the United States economy is consistently in a non-ergodic regime. All these results allow us to hypothesise that the economy may be in a situation where inequality is increasing and mobility is decreasing as a result of the high level of randomness that determines the income dynamics. As such, our results serve to expand the knowledge on the possible factors affecting the observed real world income dynamics. The results from these chapter were published in Refs.[11–13].

Chapter 4: Measuring income mobility in the Macedonian economy using ergodicity. In the fourth chapter, we show how the non-ergodicity framework developed in this document can be used to make the first measurements of income mobility and mixing in the Republic of Macedonia. In particular, we exploit the srGBM model together with data on the income distribution and the dynamics of the people who left/changed their jobs to estimate the quantify the mixing properties of the Macedonian economy over the years. In this chapter, we also introduce a measure for the granular representation of mixing of income called Mean First Passage Time (MFPT). MFPT estimates the time required for a worker to reach a certain level of income given their current status.[14] The technical results of this chapter can be read in Refs.[14,15].

But why these frameworks offer more nuanced applications, compared to standard methods?

A shared feature of these frameworks is that they unify results from previous research into comprehensive methodologies that can easily address the question of ergodicity. They quantify the well-being of an individual by what happens over their lifetime instead of averaging across the different possibilities at a given time. This is important from both an individual worker perspective and a policymaker view. In particular, from an individual perspective it is obvious that in this case the right comparison is with their future self: if I am certain that I will be able to experience a wealthy life regardless of my current social class or circumstances of birth, then it is irrelevant who is the richest now and who is the poorest as social structures function as envisaged. Similarly, from a policymaker perspective, if the economy is a non-ergodic state, then it is polarized between the rich and the poor. Yet, standard measures for well-being (as showed for example with the mobility measures) will not account for this, and will indicate that the "average person is fine". The non-ergodic frameworks disaggregate these measures to the level of an individual and show that achieving an ergodic state also maximizes the well-being of the nation as it leads to the largest economic return. Then, the policies implemented for social cohesiveness (e.g., choices on collective investment in infrastructure, education, social programs, taxation etc.), are having their impact. Hence, they offer a natural and empirically testable way approach to various economics domains: from tailoring optimal investment strategies up to designing essential policy interventions.

These frameworks, however, are not without their limitations. First, they are based on the simplest models which can only offer restricted information about the role of non-ergodicity. The frameworks assume that all individuals have the same socio-economic attributes that may affect their decision making, such as education or gender, and do not differentiate across different types.[16–22] In this context, we point out that the frameworks can be easily generalized and applied to more sophisticated situations by assuming that the parameters that model the state of the economy are type dependent. [23] Then, each individual type will have their own features (estimated using the same equations derived in the document) and this may uncover even more detailed information about the role of non-ergodicity. The inability to further disaggregate the frameworks across types stems from the fact that, unfortunately, the datasets produced so far do not allow for a detailed track on the features of every individual type in the economy. Nevertheless, in the absence of a unifying model covering all relevant aspects, the frameworks introduced here can provide the starting point for the development of a more comprehensive understanding for the impact of non-ergodic dynamics on economies. With the development of improved and more granular datasets, the insights obtained from these analysis can influence the development of even more comprehensive and appropriate policy recommendations.[24,25]

Second, here we studied only three aspects of the economy, stock investments, income dynamics, and wealth changes. Non-ergodicity also affects other domains: from the evolution of cooperative behavior[26] up to scientific outcomes[27]. Hence, developing frameworks that investigate the impact of non-ergodicity in other domains of the economy may be a non-trivial future contribution.

Lastly, we emphasize that the applications which we presented here follow a positive approach and, hence, are only descriptive. We invite policy practitioners to delve deeper in the interpretation of the questions arising from our empirical findings. For example: Is the Macedonian Dream alive if it takes around 200 years for a middle-income status US worker to reach the top 1%? Is it reasonable that the economic conditions allow individuals aged 25-30 years currently with an income around 30th percentile to only reach 45th during their working life?

Yet, despite these limitations, the frameworks improve upon the state of the art by providing methods that are more comprehensive, and also more accurate, at investigating how randomness affects the temporal evolution of economic observables. It should motivate new multidisciplinary research focused on creating even more comprehensive frameworks that can be used to explain specific economic phenomena and be applied to economies all across the globe.

References

1. Bernoulli, D. Specimen Theoriae Novae de Mensura Sortis. Translation "Exposition of a New Theory on the Measurement of Risk" by L. Sommer (1954). *Econometrica* **22**, 23–36 (1738).
2. Montmort, P. R. *Essay d'analyse sur les jeux de hazard*. (Jacque Quillau, Paris. Reprinted by the American Mathematical Society, 2006, 1713).
3. Peters, O. The Ergodicity Problem in Economics. *Nature Physics* **15**, 1216–1221 (2019).
4. Peters, O. & Adamou, A. An evolutionary advantage of cooperation. *arXiv:1506.03414* (2018).
5. Adamou, A. & Peters, O. Dynamics of Inequality. *Significance* **13**, 32–35 (2016).
6. Berman, Y., Peters, O. & Adamou, A. Wealth inequality and the ergodic hypothesis: Evidence from the united states. *Forthcoming in Journal of Income Distribution* (2020).

7. Stojkoski, V. *et al.* Income inequality and mobility in geometric Brownian motion with stochastic resetting: theoretical results and empirical evidence of non-ergodicity. *arXiv preprint arXiv:2109.01822* (2021).
8. Stojkoski, V., Sandev, T., Basnarkov, L., Kocarev, L. & Metzler, R. Generalised geometric Brownian motion: Theory and applications to option pricing. *Entropy* **22**, 1432 (2020).
9. Stojkoski, V. Measures of physical mixing evaluate the economic mobility of the typical individual. *arXiv preprint arXiv:2205.02800* (2022).
10. Stojkoski, V. & Karbevski, M. Ergodicity breaking in wealth dynamics: The case of reallocating geometric Brownian motion. *Physical Review E* **105**, 024107 (2022).
11. Stojkoski, V. *et al.* Income inequality and mobility in geometric Brownian motion with stochastic resetting: theoretical results and empirical evidence of non-ergodicity. *Philos. Trans. R. Soc. A* **380**, 20210157 (2022).
12. Stojkoski, V., Sandev, T., Kocarev, L. & Pal, A. Geometric Brownian motion under stochastic resetting: A stationary yet nonergodic process. *Physical Review E* **104**, 014121 (2021).
13. Stojkoski, V., Sandev, T., Kocarev, L. & Pal, A. Autocorrelation functions and ergodicity in diffusion with stochastic resetting. *Journal of Physics A: Mathematical and Theoretical* (2022).
14. Jolakoski, P. *et al.* The fate of the American dream: A first passage under resetting approach to income dynamics. *arXiv preprint arXiv:2212.13176* (2022).
15. Stojkoski, V., Mitikj, S., Trpkova-Nestorovska, M. & Tevdovski, D. *Income Mobility and Mixing in North Macedonia*. https://ideas.repec.org/p/arx/papers/2309.17268.html (2023).
16. Aydemir, A., Chen, W.-H. & Corak, M. Intergenerational education mobility among the children of Canadian immigrants. *Can. Public Policy* **39**, S107–S122 (2013).
17. Corak, M. Do poor children become poor adults? Lessons from a cross-country comparison of generational earnings mobility. in *Dynamics of inequality and poverty* (Emerald Group Publishing Limited, 2006).
18. Haveman, R. & Smeeding, T. The role of higher education in social mobility. *Future Child.* **16**, 125–150 (2006).
19. Jacobs, J. A. Gender inequality and higher education. *Ann. Rev. Sociol.* **22**, 153–185 (1996).

20. Reay, D. Miseducation: Inequality, education and the working classes. *Int. Stud. Sociol. Educ.* **27**, 453–456 (2018).
21. Troyna, B. *Racial inequality in education*. (Taylor & Francis, 2012).
22. Klasen, S. & Lamanna, F. The impact of gender inequality in education and employment on economic growth: new evidence for a panel of countries. *Fem. Econ.* **15**, 91–132 (2009).
23. Gabaix, X., Lasry, J.-M., Lions, P.-L. & Moll, B. The dynamics of inequality. *Econometrica* **84**, 2071–2111 (2016).
24. Ravallion, M. The Luxembourg income study. *J. Econ. Inequal.* **13**, 527–547 (2015).
25. Milanovic, B. Worlds apart. in *Worlds Apart* (Princeton University Press, 2011).
26. Stojkoski, V., Utkovski, Z., Basnarkov, L. & Kocarev, L. Cooperation dynamics in networked geometric Brownian motion. *Physical Review E* **99**, 062312 (2019).
27. Peters, O. & Werner, M. A recipe for irreproducible results. *arXiv preprint arXiv:1706.07773* (2017).