Classification of Finite Groupoids of Order 3 by Using Image Patterns

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Abstract. A groupoid is an algebraic structure (G,*) formed by a non-empty set G and a binary function *: $G^2 \rightarrow G$ defined on the set G. A groupoid which has finite number of elements is called finite groupoid. The number of finite groupoids of not so big order is huge and that implies the needs of their classifications. This paper will give some classifications of groupoids of order 3, the number of them is 19.683, by using suitably defined image patterns.

Keywords: groupoids, classification, *e*- transformations

1 Introduction

Algebraic action is the basic concept of every algebraic object. Algebraic actions can be binary, ternary, n-ary. We say that in the set G we have defined binary algebraic action if in each ordered pair of elements by G corresponds **one and only one** element of G. So, the binary action in G is a function *f*: $G^2 \rightarrow G$. If for (a, b) ϵG^2, we have *f*: (a, b) →c, cϵG, we state:

$$f(a, b) = c \text{ or } a\, f\, b = c.$$

Here, the binary action is denoted by * and is called multiplication. Always action * is associated with the condition of closure. The set G, in which binary action * is given, is called a groupoid without any additional law or axiom.

In this paper, we present the characteristics of groupoids of order 3 and properies of them by using image patterns. In section 2 we define the so called *e*-transformations. In section 2 we define the so-called *e*-transformations. By using these transformations we construct image patterns. In section 3 we classify groupoids of order 3 by using the obtained image patterns. İn the paper [7] it is given a classification of groupoids of order 3 by using a Boolean representation of groupoids and there are defined two classes.Here we give a more complete classification of groupoids of order 3 in in twelve main classes and twelve subclasses. We finish the paper with conclusions.

2 Building Image Patterns

Let G be a set of elements ($|G| \geq 2$). We denote by $G^* = \{a_1a_2 \ldots a_n | a_i \in G, n \geq 1\}$ the set of all finite sequences with elements of G. Assuming that $(G, *)$ is a given groupoid, for a fixed element $l \in G$, called the leader, we define transformations e_l: $G^* \rightarrow G^*$ on the groupoids as follows:

$$e_l\left(a_1a_2 \ldots a_n\right) = \left(b_1b_2 \ldots b_n\right) \Leftrightarrow \begin{cases} b_1 = l * a_1 \\ b_{i+1} = b_i * a_{i+1}, 1 \leq i \leq n-1 \end{cases}. \qquad (1)$$

If we have a string of leaders l_1, l_2 ... l_k, we can apply consecutive e-transformations on a given string, as a composition of e-transformations. This composition of e-transformations we called E−transformation respectively and they are defined as:

$$E_k = e_{l_1} \circ e_{l_2} \circ e_{l_3} \circ \ldots \circ e_{l_k}$$

Further, we will use only one leader $l = l_i$, for $1 \leq i \leq k$.

Example 1. Let the groupid $(G, *)$ be given as below, $l = 1$ be a leader and α =0122222212111121000002001 be the finite sequence of elements of G.

*	0	1	2
0	0	0	1
1	1	2	1
2	2	2	1

If we apply consecutive e-transformation on the sequence α we obtain the following sequences:

Table 1. Application of consecutive e−transformation

	012 222 212111121000002 001	= α

1	121111121222 212222222 222	$= e_l(\alpha)$
1	212222 212111121111 111 111	$= e_l^2(\alpha)$
1	121111121222 212222222222	$= e_l^3(\alpha)$

As we can see on the Table 1, we have the leader *l*=1, and from the multiply table of groupoid, we have 1*0 = 1, 1*1 = 2, 2*2 = 1, 1*2 = 1, 1*2 = 1... In that way from α, we get $e_l(\alpha)$. Also 1*1 = 2, 2*2 = 1, 1*1 = 2, 2*1 = 2 ... and that way from $e_l(\alpha)$ we get $e_l^2(\alpha)$, and so on.

To obtain a pattern we use some enough long sequence and we apply *e*-transformations several times. We choose sequences of length 60 and we apply 60 times an e_l – transformation for some leader *l*. To get interesting patterns the used sequencese should be periodical, for example, 012012012012012012012... or 2210221022102210.... and so on. We can choose different color for each of numbers 0,1,2 (for example 0-yellow, 1-red, 2-blue), and in such way, 60×60 color-scheme will be obtained. We use this patterns for classifications of groupoids of order 3. Examples of different patterns are given on Figure 2.

3 Classification of Groupoids of Order 3 by Random Selection of Their Patterns

Dimitrova and Markovski [1] give a classification of quasigroups of order 4 by using the graphical presentation of quasigroup. The authors start with a periodical sequence s=1234123412341234.....1234 of length 100 and apply 100 times the *E*-transformations and according to the authors, there are two classes, one so-called “fractal” with 192 quasigroups and the others "non-fractal" with 384 quasigroups.
The number of quasigroups of order 4 is 576, but the number of groupoids of order 3 is 19678. Because we have a big number of groupoids of order 3 we have produced estimation by selecting randomly 1000 groupoids and producing 1000 patterns. The starting periodical string was 123123123123….123 and the leader was *l* = 1. We ordered the groupoids lexicographically by giving the lexicographic number a groupoid to be obtained in such a way the first, the second and the third row of its multiplicative table to be concatenated. In that way, the groupoid in Figure 1 has lexicographic order 122232332.

*	1	2	3
1	1	2	2
2	2	3	2
3	3	3	2

Figure 1. The groupoid 122232332

For making patterns of the randomly chosen groupoids we used the periodical sequence *s* = *123123123123123....123123* with length t = 60, leader *l* = *1*, and we applied *k* = 60 times of *e*−transformations. The obtained results are presented on Table 2.

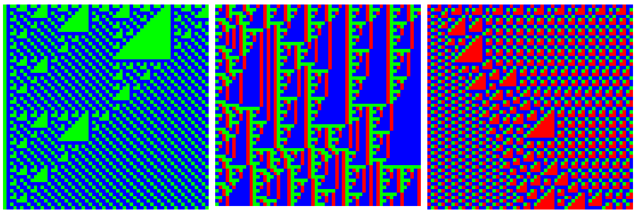
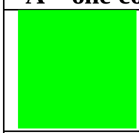
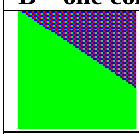
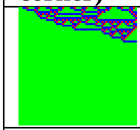
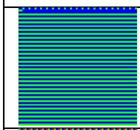
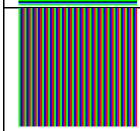
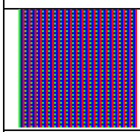
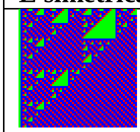
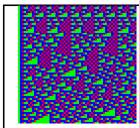
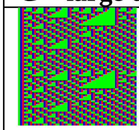
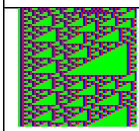
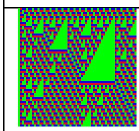
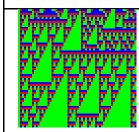
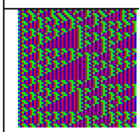
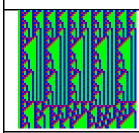


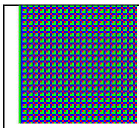
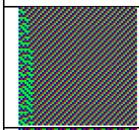
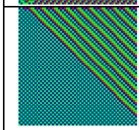
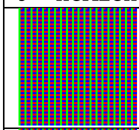
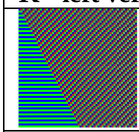
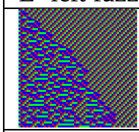
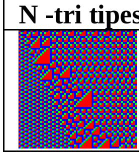
Figure 2. Different patters of groupoids

Table 2. Classification of groupoids of order three

Main type of picture	***Lexicographic order***	***Classes***	***Presentages in 1000 random groupoids***	***Approximately of all groupoids of the same types***
A – one color			**25.1% in 1000**	
	000000000	A	25.1 %	4940.44
B – one color/ 3 color (on corner)			**0.2% in 1000**	
	112231221	B	0.2%	39.366
C- **one color/ 3 color (pattern on**			0.1%	

corner)				
	113113113	C	0.1%	19.683
D – 2 and 3 color in vertical/horizontal/			**53% in 1000**	
	113113331	D1 – 2 color in vertical	19.8%	3897.234
	211211211	D2- hori-zontal 2 color	10.6%	2086.398
	123113123	D3- three color in vertical	20.2%	3975.966
	121323121	D4 - line and point 3 colors	2.4%	472.392
E simetrical triangle with three color			**4.2% in 1000**	
	12123221	E	4.2%	826.686
F - fuzzy triangle			**2.2% in 1000**	
	122221211	F1 - fuzzy tri-angle	1.2%	236.196

	121232212	F2- three colors /fuzzy triangle	1.0%	19.683
G – large triangles			**5.9% in 1000**	
	121313121	G1 – large tri-angles in horizon-tal	1.2%	236.196
	122112122	G2 - big-ger trian-gles in horizon-tal	1.7%	334.611
	132132132	G3 – large tri-angles in vertical	1.5%	295.245
	132211132	G4- big-ger tri-angles in vertical	0.6%	118.098
	212323212	G5- 2 color in triangle	0.9%	177.147
H - horizontal repetition			**0.8% in 1000**	
	122132122	H	0.8%	157.4641
I - square with color			**2.5% in 1000**	

	122231122	I1 - square with three color	1.4%	275.562
	211132122	I2 - Square	0.2%	39.366
	21223 2212	I3- down two color / up three color	0.9%	177.147
J – horizontal	and vertical		**2.8% in 1000**	
	212121212	J1 – hor-izontal - vertical	1.2%	236.196
	132122132	J2 - mixed J	1.6%	314.928
K - left vertical right square			**0.7%in 1000**	
	211231122	K	0.7%	137.781
L- left fuzzy right square			**0.1%in 1000**	
	212222212	L	0.1%	19.683
N -tri tipes			**0.1% in 1000**	
	332233332	N	0.1%	19.683

The table is organized in such a way that the first column contains the pattern of the groupoid with its lexicographic number given on the second column. In the third column we give the name of the classes with short descriptions of the classes. The percentages of the appearances of the classes in 1 000 randomly chosen groupoids are given in the fourth column and the fifth column contains an estimated number of the groupoids of the given type among all 19 683 groupoids of order three.

We classified the groupoids of order 3 according to the graphical representation and we obtained 12 main classes (A, B, C, D, E, F, G, H, I, K, L, N). Here we also put the groupoids in subclasses: 6 for the class D - D1, D2, D3, D4, D5 and D6), 2 for the class F - F1 and F2, 5 for the class G - G1, G2, G3, G4, and G5, and 3 for the class I - I1, I2, and I3. We emphasise that the same result will be obtained if we choose leader l = 2 or l = 3, only the lexicographic numbers of the groupoids will be different.

CONCLUSION

It is useful algebraic structures to be classified according to some of their properties. We classified the set of groupoids of order 3 by using suitable image patterns. The number of groupoids of order 3 is very large and it is not practical to consider all of them. That is why in this paper we considered 1 000 groupoids of order 3 that are randomly chosen, and according to image patterns and the properties of the images they are classified into twelve classes (and twelve sub-classes). We have used the periodical string 123123…123 for producing the patterns, and the same result will be obtained if strings with periods 132, 213, 231, 312 or 321 were used (with a fixed leader l = 1, l = 2 or l = 3). Some other patterns may appear if different periodic strings (like 32231 32231 … 32231, or others) are considered.

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Appendix I.

Program Codes Mathematica modules for graphical presentation of

groupoids of order 3

```
kk=Get["C:/Users/Elissa/Desktop/Glista.txt"];

s=Flatten[Table[{1,2,3},{i,1,20}]];

cryptone[q_,s_,l_]:=Module[{},b[0]=l;

        For[i=0,i≤Length[s]-1,i++,b[i+1]=q[[b[i],s[[i+1]]]]];

t=Table[b[i],{i,1,Length[s]}]];

cryptone[kk[[100,1]],s,1];

cryptntimes[q_,s_,l_,n_]:=Module[{s1=s,l1=l},Transf={};

       For[ik=1,ik≤n,ik++,k=cryptone[q,s1,l1];

       Transf=Join[Transf,{k}]; s1=k];

       Transf];

cryptntimes[kk[[123,1]],s,2,60];

ETransformation[q_,s_,l_,n_]:=Module[{},For[qt=1,qt≤3,qt++,t=cryptntimes[Firs
t[q[[qt]]],s,l,n]; MatrixPlot[Table[Hue[t[[i,j]]]3],{i,1,n},{j,1,n}]]]]

Table[Hue[t[[i,j]]]3],{i,1,60},{j,1,60}];

MatrixPlot[Table[Hue[t[[i,j]]]3],{i,1,60},{j,1,60}]]
```