Classification of Digital Images Using Topological Signatures

Petar Sekuloski
Faculty of Computer Science and Engineering
University of Ss. Cyril and Methodius
Skopje, Macedonia
petar.sekuloski@finki.ukim.mk

Vesna Dimitrievska Ristovska
Faculty of Computer Science and Engineering
University of Ss. Cyril and Methodius
Skopje, Macedonia
vesna.dimitrievska.ristovska@finki.ukim.mk

***Abstract*—Topological Data Analysis (TDA) is a new area of Applied Mathematics that has become increasingly popular in recent years. TDA utilizes Persistent Homology, a mathematical tool that analyzes the topology of data sets. The focus of this paper is on using Persistent Homology to extract topological signatures from digital images and investigate how these signatures can improve image classification. In this short paper there are some preliminary results obtained on real world digital image datasets. There are improvement in evaluation metrics from 19% to 37%, using topological signatures.**

***Keywords*—topological data analysis, persistent homology, image classification, computational topology**

I. Introduction

Image classification is a fascinating and rapidly evolving field in both machine learning and computer vision. With the advent of high-quality cameras and social media platforms, an enormous amount of digital images is now available to researchers for machine learning purposes. Deep learning models, particularly convolutional neural networks, have demonstrated state-of-the-art performance in image classification tasks.

However, these models rely solely on pixel intensity values to extract features, and they can struggle when presented with complex or noisy images. Topological Data Analysis (TDA) provides a new approach to image analysis that uses persistent homology to extract topological signatures from digital images. In this paper, we investigate how Persistent Homology can improve image classification using neural networks with simple architectures.

Homology is a mathematical concept that associates algebraic objects with topological spaces. The fundamental idea behind homology is that two shapes can be distinguished by examining their holes. For instance, a disk is different from a circle because the disk is solid while the circle has a hole through it. Homology groups are sets of invariants of a topological space that characterize its topology.

Persistent Homology is a variation of homology that tracks the topological characteristics of a dataset. It does this by reconstructing the dataset into a simplicial complex, which is a geometric structure composed of interconnected triangles, edges, and vertices. Persistent Homology then computes homology groups for each of the simplicial complex's components over a range of filtration values.

The primary advantage of using Persistent Homology for image analysis is the robustness and invariance of the topological signatures that it computes. These signatures are global and more resistant to local deformations than traditional image features extracted by deep learning models. Additionally, computing these signatures does not depend on the scale of the data.

Persistent Homology has found widespread application in diverse research areas, including gene expression, cancer detection, chemoinformatics, natural language processing, sensor networks, complex networks, noise detection, signal processing, bioinformatics, and many others. [1][2][3][4][5][6][7][8][9][10][11][12] In this paper, we focus on how topological signatures computed by Persistent Homology can enhance image classification tasks.

By combining topological signatures with simple neural network architectures, we demonstrate how Persistent Homology can significantly improve image classification accuracy. We evaluate our approach using several benchmark datasets and demonstrate that our method outperforms conventional image analysis techniques. Overall, our results suggest that Persistent Homology can provide a powerful tool for image analysis and may have broad applications in computer vision and machine learning.

II. Methodology and the Model

In this work, it is applied Persistent Homology to digital images using CW-complexes. CW-complexes are a generalization of simplicial complexes that allow for cells that are not necessarily simplices, but are still homeomorphic to balls or open discs [13][14]. This means that more complex shapes, such as cubes, can be used instead of tetrahedra. In this paper, we use regular CW-complexes, which we refer to as cell complexes or simple complexes.

Cell complexes instead of simplicial complexes are used because digital images are represented as matrices, which are essentially grid structures. Thus, cell complexes are more suitable for representing digital images, as they allow for a more natural representation of the grid structure of the images. By using cell complexes, can applied Persistent Homology to extract topological features from digital images and improve their classification accuracy using neural networks.

Definition 1. A filtered cell complex is (X, F) is a cell complex X together with a monotonic function $f: X \to \mathbb{R}$. A linear ordering $\sigma_0, \sigma_1, \sigma_2, \ldots, \sigma_n$ of the cells in X, such that $\sigma_i \preccurlyeq \sigma_j$ implies $i \leq j$, is compatible with the function f when

$$f(\sigma_0) \leq f(\sigma_1) \leq f(\sigma_2) \leq \cdots \leq f(\sigma_n)$$

In our previous work [6], we introduce a novel model classification of digital images based on Persistent Homology. In

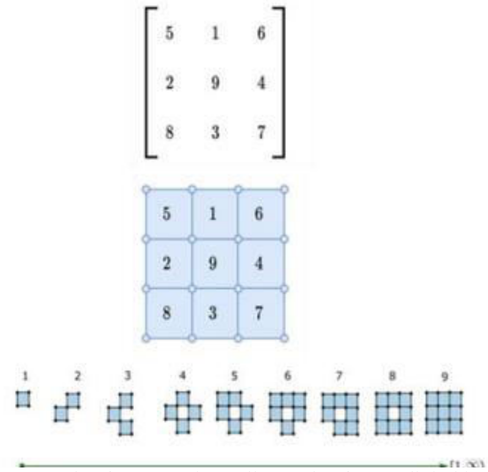

Fig. 1 An example of constructing cell complex from a digital image

this work that model with some modifications is applied to improve classification on two datasets consisted of digital images. In this model's, the main focus is on constructing regular cell structures from digital images and using them to compute persistent images. Figure 1 displays an instance of how to build a cell structure from a digital image and then perform persistent homology on it. If you want to delve deeper into the subject of generating cell structures from digital images and calculating persistent homology, reference [14] can provide you with more details.

The main idea behind this work is to perform image classification by combining the original images with their topological signatures. The dataset used in the experiments consists of digital RGB images, each of which is represented as a two-dimensional matrix. The images undergo preprocessing, after which two separate datasets are created. The first dataset is constructed by representing each image as a sequence of three two-dimensional matrices, which we refer to as "channels". Each matrix in this sequence represents the original RGB image. Therefore, in this new dataset, each image is represented by a three-channel RGB matrix. The modification that we made in the previous our model is to use RGB images instead of grey-scale images.

The key phase of the model, to investigate how topological signatures can improve a classification, is the second phase of the model. Firstly, a cubic complex with a V-construction is constructed for each image in the dataset. Then, homology is calculated on the complex that was obtained from the image. Once homology is computed, the Persistent Images can be determined. More information about Persistence Diagrams can be found in [17].

Persistent Images are related to the persistent groups of a specific dimension. Since the images that were worked with in this study are two-dimensional, the dimension can only be either 0 or 1. Therefore, for each digital image in the dataset, two persistent images are obtained from the persistent diagram. This is actually the main objective of this work, to observe how the topological features extracted from the images will influence the classification process. More about persistent images can be found in [14]. The persistent images are stable vector representations of persistent diagrams. Once these computations are completed, a sequence of three matrices is created. The original image matrix is placed in the first position, followed by the matrix that represents the persistent image for dimension 0 of the original matrix in the second position, and the persistent image for dimension 1 of the three RGB channels of the original digital image on 3rd, 4th and 5th place. This sequence is created for every original image, and the second dataset is constructed from these three-element sequences.

III. Experiments

A. *smallNORB dataset*

The smallNORB dataset [19] is a dataset for 3D object recognition from shape. It contains images of 50 toys belonging to 5 generic categories: four-legged animals, human figures, airplanes, trucks, and cars. The objects were imaged by two cameras under 6 lighting conditions.

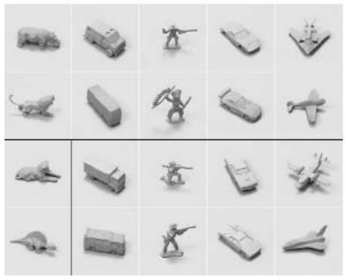

Fig. 2. Each of columns represents one image of the dataset in each lighting condition

Preprocessing of data

In the preprocessing step of the data, the original dataset's digital images were rescaled so that the pixel values would be within the range of 0 to 1. This was achieved using the programming language R.

Constructing Persistent images

To construct the persistent images, a persistent image for dimension 0 and a persistent image for dimension 1 were created for each image in the dataset. The dimension of each persistent image was set to 128x128, which is the same as the dimension of the original images. These persistent images were then concatenated with the original images, as explained in the previous section of the paper. This process resulted in the creation of the second dataset, which consists of the original images along with their corresponding topological signatures.

Classification

In the classification phase, a simple neural network is utilized as the classifier. The first layer of the neural network converts the input of 5 matrices, the persistent images for dimension 0 and 1, and original three channel RGB images into a single row. The neural network consists of two hidden fully connected layers, and the output layer has 5 neurons. The activation function used is "softmax," and the optimizer used is "adam." The loss function employed is "sparse_categorical_crossentropy." The accuracy of the network converges after the 50th epoch in both tasks, with and without topological signatures. The coding is done in R for computing topological signatures, utilizing the RCPP library, while Keras models are used for the classification task.

We will compare the results from the classification with topological signatures and without topological signatures using the same classifier, where we train the network with the original images without the persistent images.

We used a simple neural network because we like to see if such a classier that does not require a high computational power may be improved by topological signatures.

IV. RESULTS

A. *Results of the classification of smallNORB dataset*

Using topological signatures, on the test set we got an accuracy of 0.837, and a loss of 0.32. For the evaluating of our model, precision, recall and f1 – score were used as metrics except accuracy. The values of these metrics are given on Table I.

Class	Precision	Recall	f1-score
0	0.9417	0.9035	0.9223
1	0.8778	0.9201	0.8787
2	0.8657	0.8801	0.8479
3	0.9212	0.9212	0.9011
4	0.9147	0.9325	0.9545
Average	**0.90422**	**0.91148**	**0.9009**

TABLE I Results of the evaluation of the classification with topological signatures

We can say that all of the metrics implicate a good classification model.

On the other hand, for the classification with the same classifier we got an accuracy of 0.691 and a loss of 0.51. From the results we can conclude that classification with topological signatures has improvement of 21.12%. In the Table 2 are shown metrics from the classification without topological signatures. It is obvious that there is an improvement in all metrics.

TABLE II Results of the evaluation of the classification without topological signatures

Class	Precision	Recall	f1-score
0	0.8011	0.7909	0.7512
1	0.7576	0.8316	0.7621
2	0.7321	0.6821	0.3321
3	0.7456	0.6924	0.7012
4	0.7541	0.7901	0.7621
Average	0.7581	0.7574	0.6617

Also, f1-score for the class 2 is very low 0.3321 in for the test in classification without topological signatures. If we use topological signatures this value is 0.8479.

In Table 3, a summary of the improvement of the average value of metrics of the classification using topological signatures compared to the classification without using topological signatures, for the same classifier, is given.

TABLE III Summary results

	Precision	Recall	f-Score
with top. features	0.90422	0.91148	0.9009
without top. features	0.7581	0.7574	0.6617
absolute difference	0.1461	0.1541	0.2392
improvement in percentages	19.26%	20.34%	36.11%

If we compare the rest of the average values of obtained metrics, they are all significantly better if we use topological signatures, and the same improvements expressed in percentages are: 19.26% for precision, 20.34% for recall, and 36.11% for f1 - Score.

B. *Results of the classification of dogVsCat dataset*

The dogVsCat[20] is a dataset that contains 25 000 images of dogs and cats for training and 12 500 images for training. The classification task is to classify if the image is image of dog/s or cat/s. Although this task is a binary, is still hard to solve. We repeat the same process that we make for the dataset A. The accuracy using topological signatures is 0.4732 and the accuracy without topological signatures is 0.3221. We can say that there is some improvement for the classification using topological signatures, but the accuracy that we got is very low. For the evaluation of other metrics were got poor values.

V. CONCLUSION AND FURTHER WORK

To classify images from this problem, there are quite complex neural architectures that mostly use convolutional networks and transfer learning and obtain comparable results with this model, which is simple and includes a neural network with two hidden layers. During our experiments, the calculation of topological signatures phase takes the most time, but even for such a dataset it is measured in minutes, performed on not very powerful hardware machines that do not include GPU and similar technologies. For the dataset used in the experiment, which is a dataset of collected 3D images, and for which the best classifiers, neural networks, have complex architectures, the proposed model that includes topological signatures gave significantly better results than the model that does not include topological features. The improvement is 28.7% in accuracy, and in the other metrics that we used is from 19% to 37%, although it is a very simple model that does not require a lot of processing power.

In the future, firstly, the effects of topological features could be tested in more complex classifiers or models to see if they continue to have a significant impact. Additionally, a loss function could be defined based on topological characteristics to further enhance the effectiveness of the proposed method.

Secondly, the input parameters of selected algorithms in the field of Topological Data Analysis (TDA) could be studied to understand how the choice of parameter values affects the results obtained for specific datasets. This would help to optimize the performance of TDA in image classification and other fields.

ACKNOWLEDGMENT

The research presented in this paper is partly supported by the Faculty of Computer Science and Engineering, at the Ss. Cyril and Methodius University in Skopje.

REFERENCES

[1] M. Pirashvili, L. Steinberg, F. G. Guillamon, M. Niranjan, J. G. Frey, J. Brodzki, “Improved understanding of aqueous solubility modeling through topological data”, Journal of Cheminformatics, (2018)

[2] J. Nicponski, J. H. Jung, “Topological data analysis of vascular disease: A theoretical framework”, BioRxiv, (2019)

[3] P. Sekuloski, V. Dimitrievska Ristovska, “Application of Persistent Homology on Bio - Medical Dataset – A Case Study”, Mathematical Modeling - Proceedings, III International Scientific Conference, Bulgaria, (2019)

[4] V. Dimitrievska Ristovska, P. Sekuloski, “Mapper Algorithm and It’s Applications”, Mathematical Modeling – Proceedings, III International Scientific Conference, Bulgaria, (2019)

[5] P. Sekuloski, V. Dimitrievska Ristovska, „Classification of Digital Images using topological signatures – A Case Study“ Mathematical Modeling – Proceedings “, III International Scientific Conference, Bulgaria, (2022)

[6] P. Sekuloski, V. Dimitrievska Ristovska, “A novel model for image classification based on Persistent Homology “, International Journal of Science and Research (IJSR), (2022)

[7] V. de Silva, R. Ghrist, “Coordinate - free coverage in sensor networks with controlled boundaries via homology”, International Journal of Robotics Research, vol 25, 1205 – 1222, (2006)

[8] Don, A. P. H.; Peters, J. F.; Ramanna, S.; Tozzi, “A. Topological View of Flows Inside the BOLD Spontaneous Activity of the Human Brain”, Front. Comput. Neurosci.2020, 14, 34

[9] Carrière, M.; Chazal, F.; Ike, Y.; Lacombe, T.; Royer, M.; Umeda, Y. Perslay: A neural network layer for persistence diagrams and new graph topological signatures. In Proceedings of the International Conference on Artificial Intelligence and Statistics (PMLR), Online, 26–28 August 2020; pp.2786–2796

[10] Chung, M. K.; Lee, H.; DiChristofano, A.; Ombao, H.; Solo, V. Exact topological inference of the resting - state brain networks in twins. Netw. Neurosci.2019, 3, 674–694. [CrossRef] Axioms 2022

[11] M. Nicolau, A. J. Levine, G. Carlsson, Topology based data analysis identifies a subgroup of breast cancers with a unique mutational profile and excellent survival (2011)

[12] H. Adams, T. Emerson, M. Kirby, R. Neville, Ch. Peterson, P. Shipman, S. Chepushtanova, E. Hanson, F. Motta, L. Ziegelmeier, Persistence Images: A Stable Vector Representation of Persistent Homology”, vol.18, p.1−35, (2017)

[13] I. Hatcher, Alebraic Topology, Cambridge University Press, (2002)

[14] H. Adams, T.Emerson, M. Kirby, R. Neville, Ch. Peterson, P. Shipman, S. Chepushtanova, E. Hanson, F. Motta, L. Ziegelmeier, Persistence Images: A Stable Vector Representation of Persistent Homology”, **vol. 18**, p. 1−35, (2017)

[15] J. R. Munkers, Topology, Upper Saddle River: Prentice Hall, Vol.2, (2000)

[16] B. Bleile, A. Garin, T. Hesis, K. Maggs, V. Robins, “The Persistent Homology of Dual Digital Images Constructions”, arXiv: 2102.11397, (2021)

[17] A. Zomorodian. G. Carlsson. Computing persistent homology, Discrete & Computational Geometry, vol.2, p.249–274, (2005)

[18] H. Adams, T. Emerson, M. Kirby, R. Neville, Ch. Peterson, P. Shipman, S. Chepushtanova, E. Hanson, F. Motta, L. Ziegelmeier, “Persistence Images: A Stable Vector Representation of Persistent Homology”, vol.18, p.1−35, (2017)

[19] Y. LeCun, F. Huang and L. Bottou, "Learning Methods for Generic Object Recognition with Invariance to Pose and Lighting," in Proceedings of the 2004 IEEE Computer Society Conference on Computer Vision and Pattern Recognition, Washington, DC, USA, 2004 pp. 97-104. doi: 10.1109/CVPR.2004.144

[20] https://www.kaggle.com/datasets/biaiscience/dogs-vs-cats [accessed 2.4.2023]