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RESEARCH PAPER

Numerical modelling of saturated boundless media with infinite elements

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Abstract

Based on the elastic theory assumptions and averaging theories, an infinite element boundary which is frequency independent is derived for saturated soil media. The infinite element development is based on mapping functions and viscous layers for damping propagating waves of both solid and water phases. The newly developed infinite element is able to simulate the boundaries of fully saturated soil medium considering both soil displacement and fluid pressures. The main point is that the fluid pressure gradient at infinity is taken to equal to zero thus enabling the realistic consideration of the fluid pressures in numerical simulations. In numerical modelling, the general finite element software ANSYS using its User Programmable Features is used. Related comparisons are done with references. In simulation of propagating waves, the numerical approach is verified considering different models. The verification models include wave propagation through a saturated soil column. The implementation of the newly developed infinite element is done by simulation of an earth fill dam boundaries while the dam body is simulated as a three-phase medium. The obtained results from simulations show promising results and are thoroughly discussed.

Keywords Infinite elements · Numerical analysis · Saturated soil medium

1 Introduction

Seismic soil–structure interaction analysis of massive engineering structures such as dams, power plants, high rise buildings is a very complex issue, which has gained an increasing importance for the last decades. The interaction effects considerably influence the seismic performance of these structures. Lessons from recent strong earthquakes (Christchurch 2010–2011, Great Tohoku 2011, Sichuan 2008, 2013, etc.) undoubtedly show that soil–foundation–structure interaction and local site effects can significantly increase total damage if they are neglected or not treated appropriately. The present study focuses on numerical simulation of underlying saturated soil media which are unbounded and extend to infinity. In many engineering applications, the numerical treatment of unbounded domains is of considerable interest especially when fully saturated conditions are the point of interest.

The first attempts to numerically treat infinite domains involved applying the finite element method directly by simply truncating the outer region. Although this method worked well for static cases, in dynamic analysis, the results diverged enormously due to the reflected waves on the artificially introduced boundaries. Artificial boundaries simulating energy radiation towards infinity were proposed by many researchers. In the work of Kausel [11], a layered half space was considered by introducing viscous stress boundaries. Liao [12] developed a system for non-reflecting boundary conditions. Manolis [14] used a boundary element method in time domain enabling usage in transient elastodynamics.

One alternative for the simulation of the boundary conditions is the infinite element (IE) attempting to simulate the behaviour of the unbounded domain. The development of infinite elements dates back to a more recent period. The concept is very similar to that of finite elements

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(FE) including the concept of infinity to the element domain. The use of infinite elements together with the well-known finite elements is a promising choice for the investigation of such unbounded domains. An infinite element is an element that represents the behaviour of unbounded domains. One of the first publications introducing mapping infinite elements was that of Zienkiewicz and Bettess [22].

There are mainly two types of infinite elements. The first type uses the decay function together with a shape function, which approaches zero at infinity. In the case of the second one, the geometry is mapped from a finite to an infinite domain. The mapping infinite elements have an advantage in the sense that application of standard Gauss integration formulas is possible. Bettess [6] showed that mapped infinite elements work very well for static analysis of elastic media. The application of infinite elements in wave propagation requires more attention to be paid to outwardly propagating waves. The application of infinite elements is extensive and can be used in many fields of engineering. Application of infinite elements in mass transport is explained in the work of Zhao [21]. In the work of Askar [3], ground freezing problems are considered using the infinite elements. A comprehensive overview for the acoustic case is given by Astley [4]. Medina and Penzien [15] proposed a different type of infinite element considering both P-wave and S-wave propagation although shape functions appeared to be extremely complicated for application. In the work of Haeggblad et al. [9], static infinite elements are combined with an absorbing layer, leading to good results. In the work of Edip et al. [7], infinite elements are upgraded by absorbing properties at each node and validation is done accordingly. On the other hand, the fluid field is assumed at side nodes of the infinite element allowing the nodes at the right side to simulate the infinity as given in Fig. 1. The shape and mapping functions are selected accordingly in order to allow linear distribution of the fluid field inside the infinite elements. The correctness of fluid field

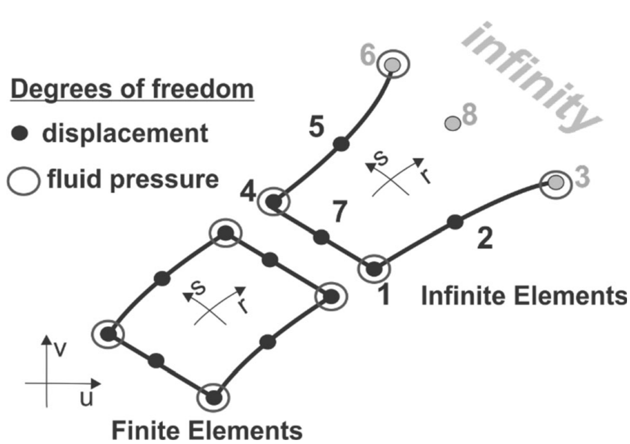

Fig. 1 Coupling of finite and infinite elements

simulation using the infinite elements depends directly on number of integration points. The optimum performance of the infinite elements considering both nodal displacements and fluid fields is obtained by using four to five integration points in the direction of extending to infinity. In this work, the infinite element is further developed such that the number of nodes is increased and the mapping functions are arranged accordingly.

The basic idea of the newly developed infinite element is to consider the assumption that the displacement field approaches zero at infinity, absorbing the outward propagating waves. The application of this type of an infinite element in soil–structure interaction problems is preferable due to the formulation, which is similar to that of finite elements. Thus, the exterior domain is partitioned into a finite number of infinite elements, which are directly connected with the finite element mesh of the interior domain.

2 Soil modelling and infinite elements in saturated soil media

In saturated porous media, the behaviour of acoustic waves basically depends on the frequency of the excitation, the hydraulic permeability, and the mechanical properties of the constituent materials [1, 16]. As given in the work of Heider [10], in general, three apparent modes of bulk waves can be observed in biphasic solid–fluid aggregates:

1. The compressional waves are fast with a motion of the solid and fluid constituents. Compressibility of the constituents governs the propagation of this type of waves.
2. The compressional waves are slow with motion of solid and fluid. This highly damped type of waves cannot propagate in the domain under low-frequency excitations.
3. The transverse shear waves are transmitted only in the solid skeleton and are mainly governed by the shear stiffness of the solid phase.

In simulating boundaries in saturated soil elements, numerous approaches have been proposed in the literature to efficiently treat unbounded spatial domains. In the current contribution, the simulation of wave propagation into infinity is realized in the time domain. The near field is discretized with finite elements, whereas the spatial discretisation of the far field is accomplished using the infinite elements. This makes certain the representation of the far-field stiffness in implementing rigid boundaries surrounding the far field. A version of a mapped infinite element has already been successfully applied by Simoni and Schrefler [20] to simulate the isothermal and non-isothermal consolidation of unbounded biphasic porous media. In particular, Schrefler and Simoni [19] have performed a coupled analysis under quasi-static conditions, where infinite elements with different decay functions are applied to the solid displacement, the pore pressure and the temperature fields. Moreover, the numerical results have been calibrated by comparison with respective analytical reference solutions. However, in dynamical applications, some additional considerations must be taken into account. In fact, when body waves approach the interface between the FE and the IE domains, they tend to reflect back to the near field due to the fact that quasi-static infinite elements cannot capture the dynamic wave pattern. To overcome this, the waves are absorbed by adding absorbing properties to the nodes of the infinite elements. The idea of adding viscous damping layer is seen in the work of Lysmer and Kuhlemeyer [13], in which damping forces are introduced to get rid of artificial wave reflections.

In this work the main focus is given to the geotechnical problems, where commonly low-frequency excitations are present. The permeabilities are low and entail very low relative motions between the solid matrix and the pore fluid. Thus, it is accepted that the pore fluid is almost trapped in the solid matrix at far boundaries. Here, only fully saturated poroelastic media with intrinsically incompressible solid and fluid constituents in the low-frequency regime are considered giving rise to only fast compressional and transverse shear waves.

The element displacement in u and v direction is interpolated with the usual shape functions N_u^1, N_u^2, N_u^4, N_u^5 and N_u^7. On the same element, the fluid pressure (water or air pressure) is interpolated with the shape functions N_p^1 and N_p^4.

$$
\begin{aligned}
u &= [N_u^1 \quad N_u^2 \quad 0 \quad N_u^4 \quad N_u^5 \quad 0 \quad N_u^7 \quad 0]\bar{\mathbf{u}} \\
v &= [N_u^1 \quad N_u^2 \quad 0 \quad N_u^4 \quad N_u^5 \quad 0 \quad N_u^7 \quad 0]\bar{\mathbf{v}} \\
p_f &= [N_p^1 \quad 0 \quad 0 \quad N_p^4 \quad 0 \quad 0 \quad 0 \quad 0]\bar{\mathbf{p}}_f
\end{aligned}
\tag{1}
$$

In expression (1), $\bar{\mathbf{u}}$, $\bar{\mathbf{v}}$ and $\bar{\mathbf{p}}_f$ are vectors of nodal point displacements and fluid pressure in global coordinates. Equation (1) implies that displacements and fluid pressures gradients are set to zero at infinity. The shape functions for displacement and fluid pressures are given in expression (2) as:

$$
\begin{aligned}
N_u^1 &= -(r-1)(-1+s)(s+1+r)/4 \\
N_u^2 &= (r-1)(1+r)(-1+s)/2 \\
N_u^4 &= -(r-1)(1+s)(s-1-r)/4 \\
N_u^5 &= -(r-1)(1+r)(1+s)/2 \\
N_u^7 &= (-1+s)(1+s)(r-1)/2 \\
N_p^1 &= (s-1)(r-1)/4 \\
N_p^4 &= -(s+1)(r-1)/4
\end{aligned}
\tag{2}
$$

Based on the isoparametric concept, the infinite element in global coordinate is interpolated onto an element in local coordinate system using the expressions (3) and (4). In the formulation of the infinite element, only the positive r direction extends to infinity. Following Fig. 1 the mapping functions for coordinate interpolation considering displacement degrees of freedom are defined as follows:

$$
\begin{aligned}
r &= [M_u^1 \quad M_u^2 \quad 0 \quad M_u^4 \quad M_u^5 \quad 0 \quad M_u^7 \quad 0]\bar{\mathbf{r}} \\
s &= [M_u^1 \quad M_u^2 \quad 0 \quad M_u^4 \quad M_u^5 \quad 0 \quad M_u^7 \quad 0]\bar{\mathbf{s}}
\end{aligned}
\tag{3}
$$

The mapping functions for coordinate interpolation considering fluid pressures as degrees of freedom are defined as follows:

$$
\begin{aligned}
r &= [M_p^1 \quad 0 \quad 0 \quad M_p^4 \quad 0 \quad 0 \quad 0 \quad 0]\bar{\mathbf{r}} \\
s &= [M_p^1 \quad 0 \quad 0 \quad M_p^4 \quad 0 \quad 0 \quad 0 \quad 0]\bar{\mathbf{s}}
\end{aligned}
\tag{4}
$$

where

$$
\begin{aligned}
M_u^1 &= -\frac{(1-s)rs}{1-r} \\
M_u^2 &= -\frac{(1-s)(1+r)}{2(1-r)} \\
M_u^4 &= -\frac{(1+s)rs}{1-r} \\
M_u^5 &= -\frac{(1+s)(1+r)}{2(1-r)} \\
M_u^7 &= -\frac{2r(1+s)(1-s)}{(1-r)} \\
M_p^1 &= \frac{1-s}{1-r} \\
M_p^4 &= \frac{1+s}{1-r}
\end{aligned}
\tag{5}
$$

In expression (3) and (4), $\bar{\mathbf{r}}$ and $\bar{\mathbf{s}}$ are vectors of nodal point displacements in local coordinates where it is to be mentioned that, on the side of infinity (r = 1), no mappings have been assigned to the nodes as it is taken that displacement in infinity tends to zero while fluid pressures gradients in infinity are zero. The number and location of the nodes connecting finite and infinite elements must coincide to guarantee the continuity condition between the elements. The main advantage of the proposed infinite elements is that the number of nodes for displacement on the infinite element allows coupling with finite elements with eight nodes which are used for displacement-sensitive problems. The difference of fluid pressure and displacement node numbers is in full agreement with the Ladyzhenskaya–Babuška–Brezzi conditions as given in the work of Pastor et al. [18]. Construction of element matrices is done by using the usual procedures as described above for the infinite elements. The developed infinite element has the advantage concerning the fact that is due to the correct assessment of the boundary conditions. In case of finite elements only extending to infinity, the fluid pressure development is influenced at the boundary of domain by the displacement limitation conditions, while considering the newly developed infinite elements the fluid pressure gradient is considered to be zero at boundaries, thus allowing correct fluid pressure distribution in both finite and infinite elements. As pore pressures are very sensitive to the boundary conditions $1/r$ decay function in mapping functions is used while constructing the newly developed infinite elements. The matrix corresponding to the mapped infinite element is very similar to the one used for the standard finite elements.

Construction of element matrices is done by using the usual procedures as described in Bathe [5]. The new coordinate interpolation functions are taken into consideration in the Jacobian matrix as described in Bettess [6]. For the absorbing layer of the infinite element, the Lysmer-Kuhlemeyer approach [13] is used. In all cases, a plane strain two dimensional case is studied. For impact of plane waves on element sides, normal and tangential stresses are derived as follows:

$$\begin{bmatrix} \sigma^n \\ \tau \end{bmatrix} = \begin{bmatrix} a\rho c^p & 0 \\ 0 & b\rho c^s \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}^{\mathbf{n}} \\ \dot{\mathbf{u}}^{\mathbf{t}} \end{bmatrix} \quad (6)$$

where cp and cs indicate the wave velocities for the P wave (compressional) and S wave (shear), respectively. The term ρ stands for density of soil medium. In order to take into account the directions of the incident waves coefficients, a and b are used as multipliers. Transformation from local to global coordinates is done by the software ANSYS [2] in such a way that there is no need of defining the transformation matrices. Time derivatives are approximated by the Newmark's method. The programming part of the infinite element has been performed using the Programmable Features of the ANSYS software. The overall implementation in ANSYS software is shown in Fig. 2:

The starting point for deriving the equations is the mass balance equation for fluid phase with respect to the motion which can be written as follows:

$$\frac{\mathrm{d}^f \rho^f}{\mathrm{d}t} + \rho^f \nabla \cdot v_f = 0 \quad (7)$$

In Eq. 7 the term ρ stands for density while the term υ stands for fluid velocity. On the other hand, the momentum balance equation for fluid phase yields simply the generalized Darcy's law:

$$\dot{u}_i^{sf} = k_f\left(-p_{f,i} + \rho_f(b_i - \ddot{u}_i)\right) \quad (8)$$

where $\dot{u}_i^{sf}$ stands for velocity of fluid phase relative to the moving solid, k_f stands for the relative permeability of the fluid phase, b_i is the body force vector while $\ddot{u}_i$ is the acceleration vector.

Following the work of authors Edip et.al [8] the finite element equations for the numerical model can be summarized as follows:

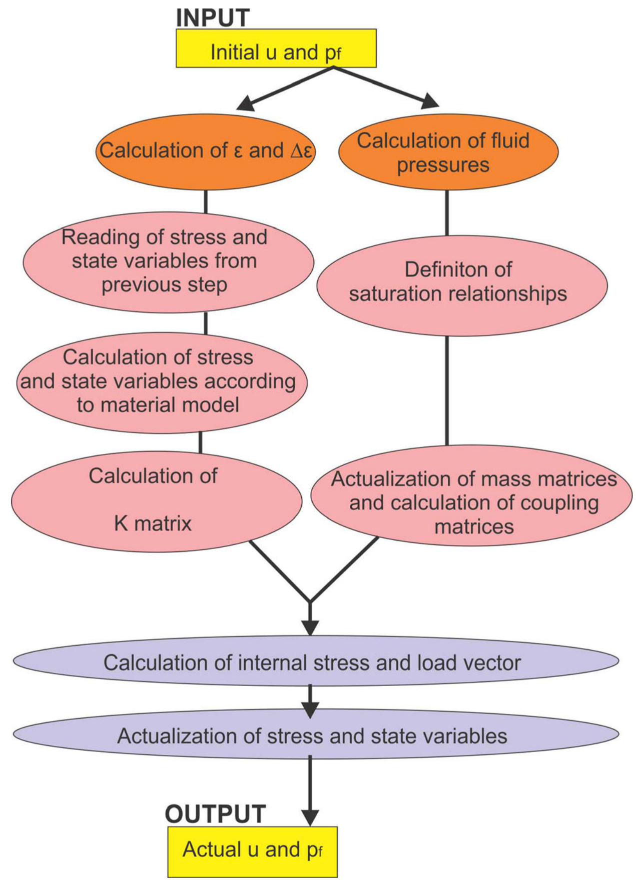

Fig. 2 ANSYS implementation of the proposed numerical model

$$\begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{M}_f & \mathbf{0} \end{pmatrix} \begin{pmatrix} \ddot{\bar{\mathbf{u}}} \\ \mathbf{0} \end{pmatrix} + \begin{pmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{C}_{sf}^{\mathbf{T}} & \mathbf{P}_{ff} \end{pmatrix} \begin{pmatrix} \dot{\bar{\mathbf{u}}} \\ \dot{\bar{\mathbf{p}}}_f \end{pmatrix} + \begin{pmatrix} \mathbf{K} & -\mathbf{C}_{sf} \\ \mathbf{0} & \mathbf{H}_{ff} \end{pmatrix} \begin{pmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{p}}_f \end{pmatrix} = \begin{pmatrix} \mathbf{f}_{\mathbf{u}} \\ \mathbf{f}_f \end{pmatrix} \quad (9)$$

The nodal degrees of freedom for displacement and fluid pressure are taken into consideration as u and p_f. Their first and second time derivatives of solid phase complete the system of equations. The different matrices of the system of equations describe different properties of the numerical model. The definition of saturation relationships enables usage of different saturation models to perform analysis of partially saturated soil media.

The indices provide information about the nature and function of the matrix, which can be interpreted as follows. The coupling matrices $\mathbf{C}_{sf}$ describe the interaction of the solid phase with fluid phase. The compressibility of the various phases and their effects on the entire media is considered by compressibility matrix $\mathbf{P}_{ff}$. The permeability matrix $\mathbf{H}_{ff}$ on the other hand concerns the flow behaviour.

3 Verification of infinite elements in saturated soil media

In order to verify the infinite elements in saturated soil media, a fully saturated soil domain is shown in Fig. 3. The model considers the coupling of finite and infinite elements considering all degrees of freedom. The restraint conditions at the bottom are fixed boundaries as shown in Fig. 3.

As can be seen in Fig. 3, the vertical soil column is modelled in two ways considering the discretization without and with infinite elements. The soil properties are given in the table below (Table 1).

In order to show the applicability of infinite elements in saturated soil model, the infinite elements have been placed at the bottom and two points of interest at depths of 5 m and 50 m have been selected to compare the of results. At the top of the soil layer, a fixed pressure of 1 kPa is applied as a transient load.

In Figs. 4 and 5 the time histories of displacements and water pressure at the depths of 5 m and 45 m for the transient wave propagation problem are presented. It can be observed that there is a good comparison between the analytical solution and the numerical one obtained by using coupled finite-infinite elements. However, when the fixed boundary is used at the bottom of the finite elements, the accuracy of the numerical results becomes significantly worse because spurious reflections take place at the artificial boundary and the reflected waves propagate back to the near field system. The displacement is considerably underestimated in the case using the artificially fixed boundary. This fact indicates that if the artificially fixed boundaries are used in the analysis, the near field of the system should be made large enough to avoid reflections on the artificially truncated boundary within the duration of the analysis. Otherwise, the numerical results will be affected by the reflected wave. Although a small numerical oscillation in the pore fluid pressure exists, in the case of using the absorbing boundaries it decreases quickly as time goes on. Thus, it is concluded that the use of the proposed absorbing boundary is an effective and efficient way of modelling the far field of the system for the transient wave problem. As can be seen from the figures, in the cases where the infinite.

elements are used the results show good correlation with the analytical results for both vertical displacement and water pressures. This verifies the correctness of the proposed infinite absorbing infinite elements.

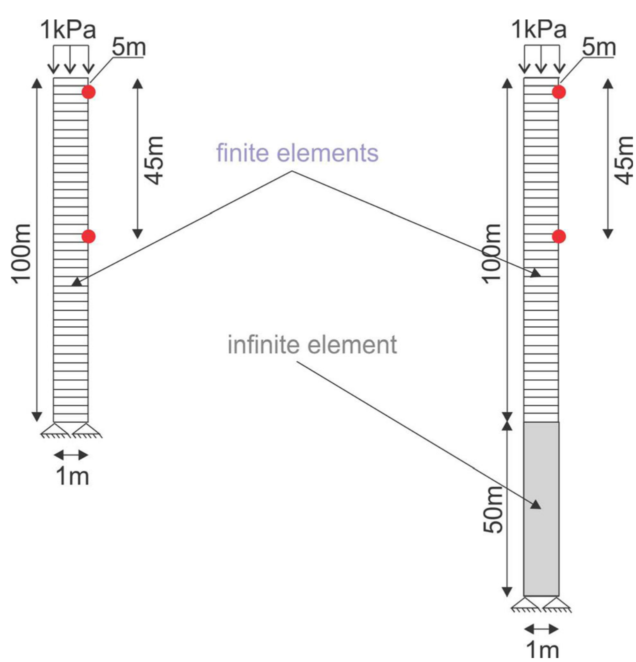

Fig. 3 Fully saturated soil domain discretized by finite and infinite elements

Table 1 Mechanical properties of soil medium

Young's modulus of elasticity	$E = 0.8333$ kPa
Poisson's ratio	$\nu = 0.25$
Solid grain density	$\rho_s = 0.31$ kg/m^3
Bulk modulus of solid grains	$K_s = \infty$
Bulk modulus of water	$K_f = 40$ kPa
Fluid density	$\rho_f = 0.2977$ kg/m^3
Initial porosity	$n = 0.33$
Permeability	$k = 4.883 \times 10^{-3}$ m^2

4 Numerical simulation of an earth dam considering infinite element boundaries

In order to implement the proposed saturated infinite elements, the case study shown in Oettl [17] has been simulated. In this study, water flow through an earth dam is simulated. In this particular example the air pressure is considered to be atmospheric while the suction effects are presented as negative fluid pressures. Our main focus is on the water flow in the body of the dam. Therefore, the analysis of the foundation soil is limited to the quasi static case. The material parameters for the solid phase of the earth dam problem are given in Table 2.

In this work, the same dam geometry as [17] is taken into consideration, whereas the material model for the body is taken as nonlinear (hypoplastic) [8]. The analysis performed focuses on the dynamic analysis case with the inclusion of the surrounding field which is composed of infinite elements as shown in Figs. 6 and 7.

The geometry of the dam is shown in Fig. 6 with the cross section width of 52 m at the base which is reduced to

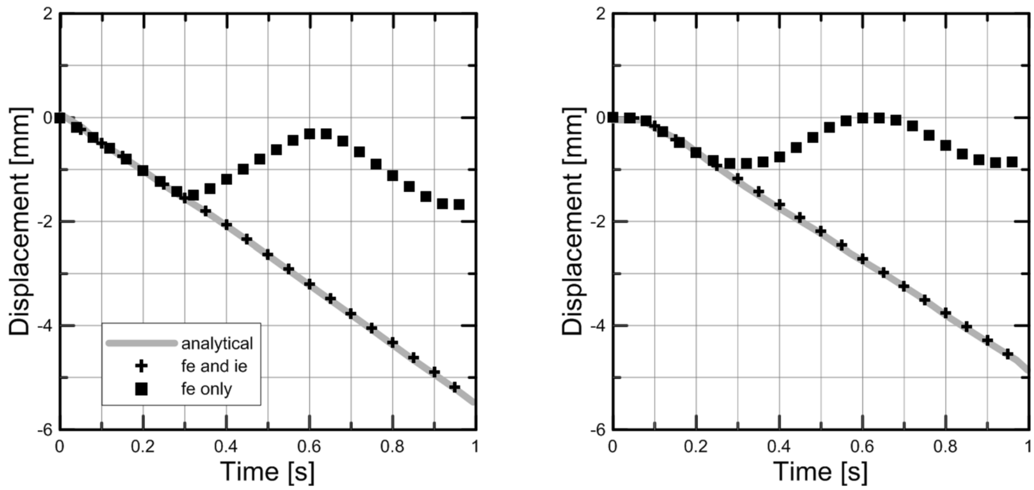

Fig. 4 Time histories of displacement at 5 m and 50 m depth

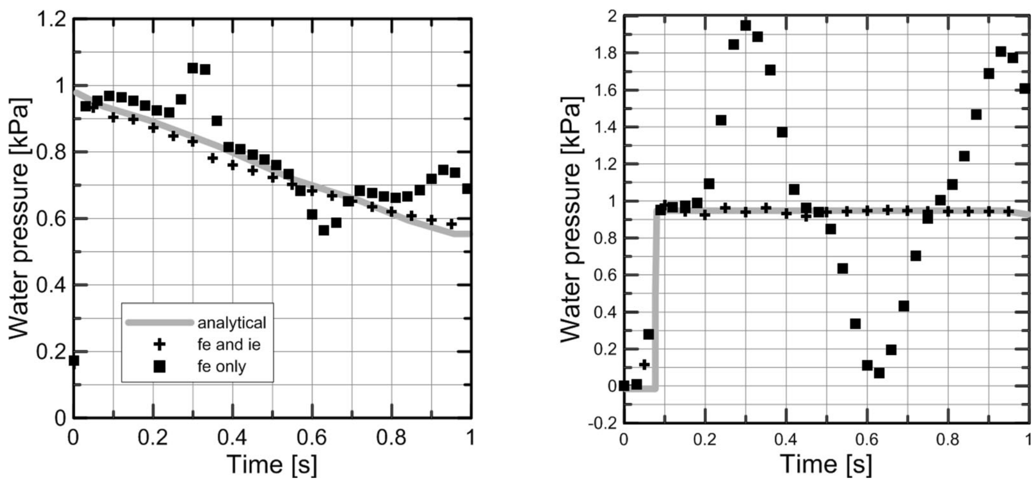

Fig. 5 Time histories of pore water pressure at 5 m and 50 m depth

4 m at the top. The height of dam is 12 m. In order to get a drainage of the leakage occurring through the dam and thus to prevent stability problems, a drainage with a length of 12 m is provided at the base of the downstream slope. The height of the water level at the upstream slope is 10 m. Furthermore, the soil used for the construction of the earth dam is considered to be homogeneous and isotropic with respect to the coefficient of water permeability. In the numerical simulation of the problem, both earth dam and base are taken into consideration. The domain is discretized by quadrilateral elements as given in Fig. 7.

Quadratic interpolation is used for the displacements of soil skeleton, while bilinear interpolation approximates the fluid pressure.

The air stress is taken to be atmospheric. The matric suction is equal to the negative value of the hydrostatic stress in the water phase which also governs the degree of saturation. This explains the reason for the definition of the saturation relations, i.e. to enable the simulation of the partially saturated soil media. It is to be mentioned that, in classical seepage analysis, the soil skeleton is assumed rigid and only the flow problem is investigated. In this case, an earth dam of trapezoidal cross section built to retain reservoir water is considered numerically. The bottom boundary is modelled as a long layer of soil extending to infinite. At the drainage, in the bottom boundary of the dam, in the vicinity of the downstream slope, the initial water stress is set to zero. Therefore, a permeable boundary at the drainage is simulated. At the ends of the soil layer infinite elements are used to absorb the waves.

Similar boundary conditions with respect to the water phase are assumed for the dam at the top and at the upstream slope above the water level in the reservoir. Therefore, the excess water pressure at the upstream boundary linearly increases from zero at the water level to 100 kPa at the base of the dam as shown in Fig. 8. The

Table 2 Material parameters for solid phase of the earth dam problem

Geometry		Dam	Dam	Soil layers	Infinite elements
Material parameters	Symbols	Hypoplastic	Linear	Linear	Linear
Density of solid phase	ρ_s (t/m^3)	2.7	2.7	2.7	2.7
Density of water phase	ρ_w (t/m^3)	1	1	1	1
Permeability	k (m^2)	10^{-7}	10^{-7}	10^{-7}	10^{-7}
Permeability of the drainage filter	k (m^2)	10^{-2}	10^{-2}		
Compression modulus of solid phase	K_s (kPa)	10^9	10^9	10^9	10^9
Compression modulus of water phase	K_w (kPa)	2×10^4	2×10^4	2×10^4	2×10^4
Dynamic viscosity of water	μ_w (kNs/m^2)	1.31×10^{-6}	1.31×10^{-6}	1.31×10^{-6}	1.31×10^{-6}
Elasticity modulus	E (kPa)		7000	9000	9000
Poisson's ratio	v		0.3	0.3	0.3
Porosity	n	0.5	0.5	0.5	0.5
Critical friction angle	φ_c	35			
Granulate hardness	h_s (MPa)	1600			
Exponent	n	0.39			
Minimum void ratio	e_{d0}	0.62			
Critical void ratio	e_{c0}	0.94			
Maximum void ratio	e_{i0}	1.08			
Numerical parameter	α	0.2			
Numerical parameter	β	1			
Intergranular strain	R	0.0001			
Intergranular strain	m_r	2.5			
Intergranular strain	m_t	9.0			
Intergranular strain	β_r	0.25			
Intergranular strain	X	9			

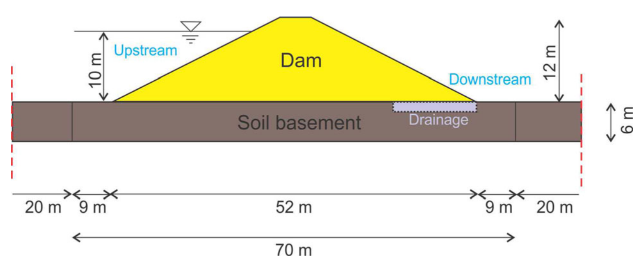

Fig. 6 Coupled soil-dam system

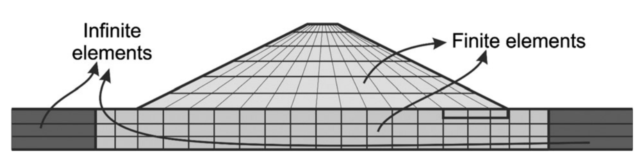

Fig. 7 Element types in the simulation of dam problem

pore pressure at the bottom has different values due to the boundary conditions which are impermeable conditions. On the other hand, the negative values of fluid pressure at the end nodes of infinite elements show the incapability of infinite element to correctly simulate the pressures at the very end nodes. However, the presence of infinite elements does not affect the fluid pressure distribution at the boundary with finite elements.

For the sake of completeness, the same dam body is considered composed only of finite elements in order to compare the pore pressure distribution as given in Fig. 9.

As shown from Fig. 9 the pores pressure field has similar distribution at the boundaries when compared with Fig. 8 in which infinite elements are used. Negative value of fluid pressures at the dam body is for tensile hydrostatic water stress due to the assumed prevailing atmospheric air pressure. In the numerical simulation at the beginning (time $t = 0$) the water level in the reservoir is raised instantaneously, i.e. the water pressure boundary condition at the upstream slope is applied suddenly and held constant in the course of the calculation. The computed distribution of the degree of saturation at steady-state conditions for hypoplastic material models are presented in Fig. 10.

The typical curve indicating the position of a zero hydrostatic water stress, i.e. the so-called phreatic surface, stretches from the position of the water level to the upper end of the drainage region. In the region above this curve, the tensile water stress according to the initial conditions

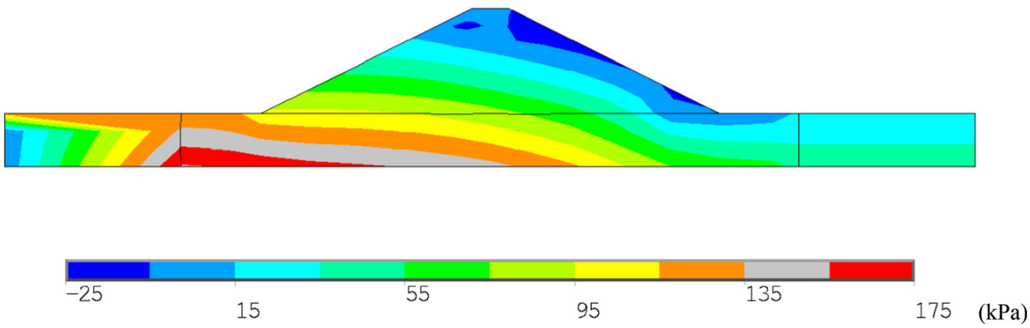

Fig. 8 Pore pressure distribution at the beginning of the problem

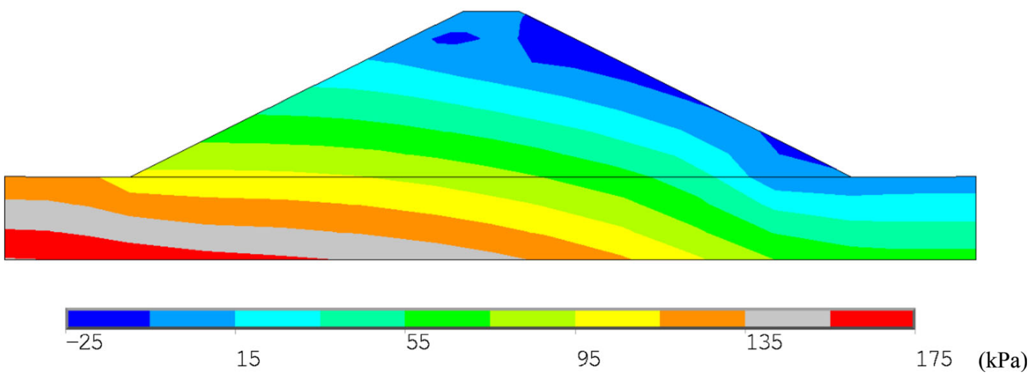

Fig. 9 Pore pressure distribution at the beginning of the problem (discretization only with finite elements)

prevails. Due to the assumed atmospheric air stresses, the development of the degree of water saturation is governed by the negative hydrostatic water stress. In the course of the numerical simulation, the wetting front propagates from the upstream face of the dam to the upper end of the drainage region. Starting from this steady state of saturation, the computation is carried out for a 10 s period of an earthquake input. The fully coupled analyses are performed using the acceleration time history of the El Centro earthquake with a scaled peak ground acceleration of 0.25 g and the results presented. Horizontal displacements and the pore pressure build up are shown in Figs. 11 and 12.

As can be seen from Figs. 11 and 12, the horizontal displacement distribution in the dam body at the end of earthquake input shows the deformation which is mainly due to the effect of the nonlinear hypoplastic modelling of the solid phase of the soil medium. On the other hand, when comparing the pore water pressure distributions, it is observed that there is an increase in the pore water pressure at the upper part of the dam body due to the earthquake loading. In both cases the infinite elements have shown to be stable elements allowing continuation of the spreading fields of deformation and pore pressure to the infinity. Thus, it can be concluded that the infinite elements can be used as boundary conditions in saturated soil media in which the domain of interest lays within the region of the finite elements.

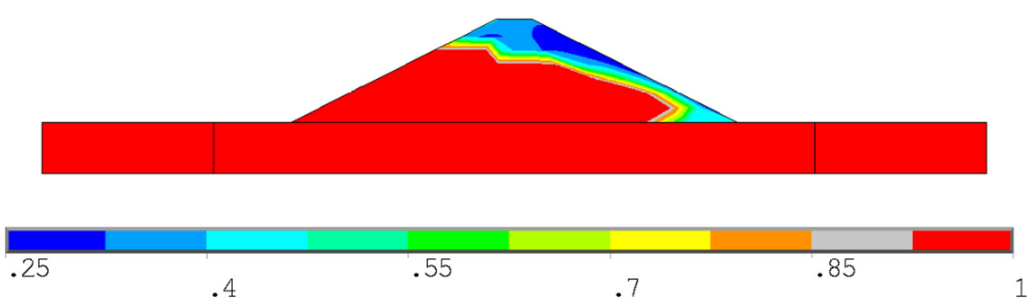

Fig. 10 Water saturation degree at the beginning of the problem

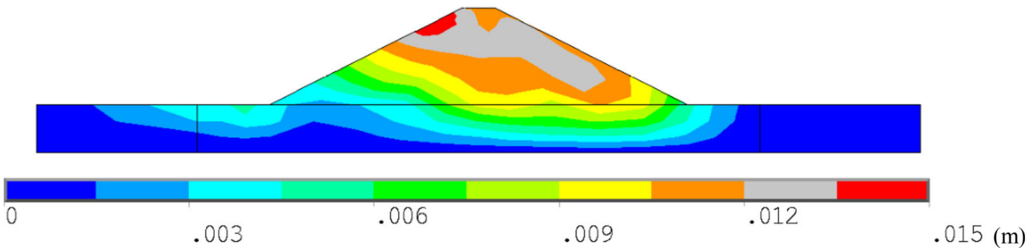

Fig. 11 Horizontal displacement at the end of earthquake input

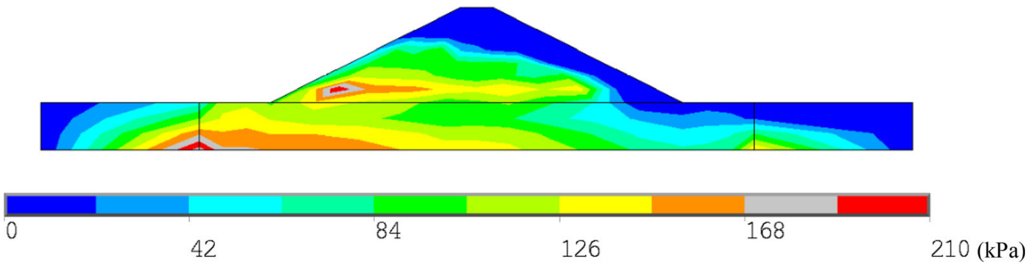

Fig. 12 Pore pressure at the end of earthquake input

5 Conclusion

The proposed infinite element, which is frequency independent, is derived for simulation of unbounded saturated soil media. The usage of infinite elements as boundary conditions together with finite elements in numerical simulations gives promising results. The verification of the newly developed infinite element has been done by simulation of fully saturated soil domain in which the infinite element has proven to hinder the wave reflections back to the domain of finite elements. On the other hand, in case of numerical simulation of an earth dam, the infinite elements have been considered as good replacement for far-field geometry in which the waves are absorbed successfully. The pore pressure build up and horizontal displacements for the earth dam presented, show the continuity between finite and infinite elements. Summarizing, the proposed infinite elements approximate the far field behaviour in an adequate manner. Thus, it can be concluded that the usage of the proposed infinite elements is an efficient and inexpensive way of modelling the far field of the saturated soil domains.

Appendix

The mass matrices are given as follows:

$$\mathbf{M} = \int \mathbf{N_u}[\rho_s(1-n) + nS_w\rho_w]\mathbf{N_u}\, \mathrm{d}\Omega$$

$$\mathbf{M}_f = \int \nabla \mathbf{N}_\mathbf{p}^{\mathrm{T}} \frac{\mathbf{k}k_{rf}}{\eta_f} \mathbf{N_u}\, \mathrm{d}\Omega$$

The coupling matrix follows as:

$$\mathbf{C}_{\mathrm{sf}} = \int \mathbf{N}_p^T \alpha S_f m^T \mathbf{L}\mathbf{N_u}\, \mathrm{d}\Omega$$

The compressibility matrix is given as:

$$\mathbf{P}_{\mathrm{ff}} = \int \mathbf{N}_p^T \left[\frac{S_f}{K_f} + (\alpha - n)\frac{S_f}{K_s}\left(S_f + p_c\frac{\partial S_f}{\partial p_c}\right) - n\frac{\partial S_f}{\partial p_c}\right]\mathbf{N_p}\, \mathrm{d}\Omega$$

The permeability matrix can be written as:

$$\mathbf{H}_{ff} = \int \nabla \mathbf{N}_\mathbf{p}^{\mathrm{T}} \frac{\mathbf{k}k_{rf}}{\eta_f} \nabla \mathbf{N_p}\, \mathrm{d}\Omega$$

The domain forces follow as:

$$f_u = \int \mathbf{N}_\mathrm{u}\left[\rho_s(1-n) + nS_f\rho_f\right]\mathbf{g}\mathbf{N_u}\, \mathrm{d}\Omega$$

$$\boldsymbol{f}_f = \int \mathbf{N}_\mathbf{p}^{\mathrm{T}} \frac{\mathbf{k}k_{rf}}{\eta_f} \rho_f \mathbf{g}\, \mathrm{d}\Omega$$

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