Classification of Digital Images using topological signatures – A Case Study

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Abstract: Topological Data Analysis (TDA) is relatively new filed of Applied Mathematics that emerged rapidly last years. The main tool of Topological Data Analysis is Persistent Homology. Persistent Homology provides some topological characteristics of the datasets. In this paper we will discuss classification of digital images using their topological signatures computed with Persistent Homology. We will experiment on the Fashion-MNIST dataset. Using Topological Data Analysis, the classification was improved.
***Keywords**: TOPOLOGICAL DATA ANALYSIS, PERSISTENT HOMOLOGY, MACHINE LEARNING, COMPUTATIONAL TOPOLOGY*

1. Introduction

Image classification has been one of the most trending topics in both research areas machine learning and computer vision. Daily usage of high-quality cameras and social media platforms have provided many pictures that can be used in machine learning. There are machine learning models, especially deep learning models, that have shown state of art performance in image classification. In this paper, we investigate how persistent homology can improve image classification using neural networks with simple architectures.

Homology is a mathematical concept which associates sequences of algebraic objects with topological spaces. One way to study a topological space is to find and compute its homology groups. The motivation behind defining homology groups was that two shapes can be distinguished by examining their holes. For example, a disk is different from a circle, or a disk is not a circle, because the disk is solid while the circle has a hole through it. Homology groups are set of invariants of a topological space. Homology groups characterize the topological space. The main idea of Persistent Homology is to track topological characteristics of a reconstructed from a dataset.

The advantages of using Persistent Homology are the robustness and invariance of the topological signatures which can be traced by Persistent Homology. Topological signatures are global and more resistant to local deformations and computations of these signatures do not depend on the scale of data. The Persistent Homology has found applications in gene expression, cancer detection, chemoinformatics, natural language processing, sensor networks, complex networks, noise detection, signal processing, bioinformatics and many other research areas [1-5].

In this paper we will discuss how persistent homology may improve image classification that using topological signatures combined with original digital images as a dataset for classification task.

2. Persistent Homology of digital image

In this work it is applied Persistent Homology on digital images. CW-complexes are generalization of simplicial complexes that allow cells that are not necessarily simplices, homeomorphic to balls or open discs [6,7]. For example, cubes instead of tetrahedra. In this work it is used CW-complexes that are regular and in this paper for such complexes we will use the term cell complex or simple complex. We will use cell complexes instead of simplicial complexes because digital images are matrices. By the nature they are some kind of cell, grid structures.

Definition 1. A filtered cell complex is (X, F) is a cell complex X together with a monotonic function $f: X \to \mathbb{R}$. A linear ordering $\sigma_0, \sigma_1, \sigma_2, \dots, \sigma_n$ of the cells in X, such that $\sigma_i \preccurlyeq \sigma_j$ implies $i \leq j$, is compatible with the function f when

$$f(\sigma_0) \leq f(\sigma_1) \leq f(\sigma_2) \leq \cdots \leq f(\sigma_n)$$

We will construct cell complexes from the images and then for the complexes, we will compute Persistent Homology to detect topological signatures of the images.

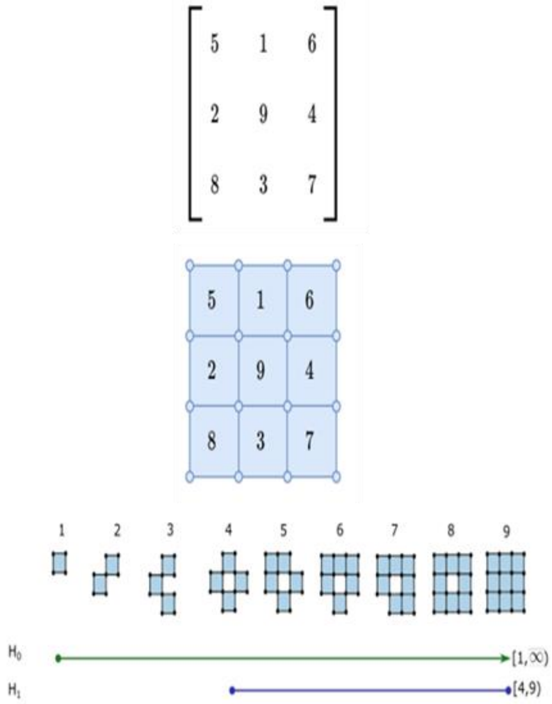

Figure 1. An example of constructing cell complex from two dimensional image together with persistent homology.

In the *Figure 1*, there is an example of construction of cell complex from a digital image together with persistent homology More information about construction of cell complexes from digital images and computing Persistent Homology of cell complexes can be found in [7].

3. Combining Topological Signatures with Original Images

The main idea in this work is to combine original images with topological signatures of the images and do a classification task. We start with a dataset consisted of digital images. In the experiments for this paper grey-scale digital images are used. Every image in the dataset is represented with two-dimensional matrix. Firstly, every image of the dataset is preprocessed. After preprocessing, two different datasets are constructed. The first dataset is constructed in a way that every image is represented as a sequence of three two dimensional matrices, we will call them three channels, and every matrix in this sequence is the matrix that represent original image. In other words, on every image of the new constructed dataset three channel matrix and on every channel there is the original matrix of the grey-scale image.

Persistent Homology can be represented with persistence diagrams [9]. In Figure 2, upper right picture, it is shown synthetic generated dataset of points in a two dimensional plane which represent two noisy circles. For this dataset Persistent Homology is computed and it is constructed persistent diagram. The diagram is shown on the left upper picture in Figure 2. The black circles in the persistent diagrams are signatures with dimension 0 and the red triangles are the signatures with dimension 1. More for persistence diagram, as a tool in TDA, can be found in [9]. Then from the persistent diagrams we compute persistent image for dimension 0 and persistent image for dimension 1 that are shown on the two down pictures in Figure 2. Persistent images are stable representation of persistence diagrams. More about persistent images can be found in [10].

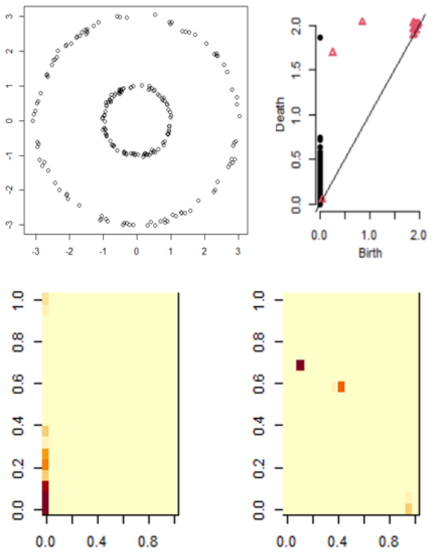

Figure 2 A noisy circles dataset with persistence diagram and persistent images for the dataset

For the construction of the second dataset, there is one additional step. In this step, the persistent homology of the images is computed to track the topological signatures of the images. For each image in the dataset persistent homology is computed and represented with a persistence diagram. After this, persistent image for dimension 0 and persistent image for dimension 1 are computed from the persistence diagram. The dimension of persistent image is the same as the dimension of the matrix of the original images. After these computations, we create a sequence of three matrices. At first place, there is the original image matrix. At second place there is the matrix which represents the persistent image for dimension 0 of the original matrix and, as a third element in this sequence, there is the persistent image for dimension 0 of the original matrix. Finally, the second dataset is constructed of such three element sequences for every original image.

In the experiment we will classify both dataset, the first dataset which is without topological signatures, and the second which is with topological signatures.

4. Experiments

Dataset

For the purpose of this paper, the Fashion-MNIST dataset [11] is used. Fashion-MNIST is well known dataset and is a good starting point to evaluate new model for classification. It has 70000 grey-scale digital images which represent article clothing. The dataset is divided in train set consisted of 60000 images and test set of 10000 images. Each image is 28 pixels in height and 28 pixels in width, for a total of 784 pixels in total. Each pixel has a single pixel-value associated with it, indicating the lightness or darkness of that pixel, with higher numbers meaning darker. This pixel-value is an integer between 0 and 255. The training and test data sets have 785 columns. The first column consists of the class labels, and represents the article of clothing. Class labels are 0:T-shirt, 1:Throusers, 2: Pullover, 3: Dress, 4: Coat, 5: Sandal, 6: Shirt, 7: Sneaker and 9: Ankle boot, see *Figure 3*. The rest of the columns contain the pixel-values of the associated image.

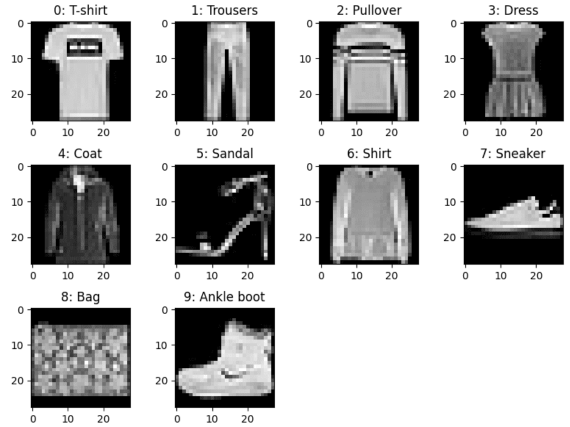

Figure 3 Images from the Fashion-MNIST with their classes

In Figure 3 ten pictures from Fashion-MNIST. Each of them represents one class of the dataset.

Preprocessing of data

In the preprocessing of data, the original dataset's digital images were scale such that the value of the pixels will be in the range from 0 to 1. This process was done using R.

Constructing the persistent images

For each image in the dataset, persistent image for dimension 0 and persistent image for dimension 1 were constructed. The dimension of the persistent images was 28x28. It is the same dimension as the dimension of original images. Then the concatenating these persistent images with the original images, as it is explained in previous section of this paper, was done. And finally the second dataset consisted of original images and topological signatures was created.

Classification

Simple neural network is used as a classifier. First layer transforms the input from three matrices to a row of 2352 elements. The neural network has two hidden layers which are fully connected. The last layer, the output of the network has 10 neurons. "Softmax" is used as an activation function and „adam" as an optimizer. "Sparse_categorical_crosentropy" is used as a loss function. The accuracy converges after 30^{th} epoch in the training process in both tasks without topological signatures and with topological signatures. The codding is done in R for computing topological signatures, using RCPP library, and for the classification are used Keras models.

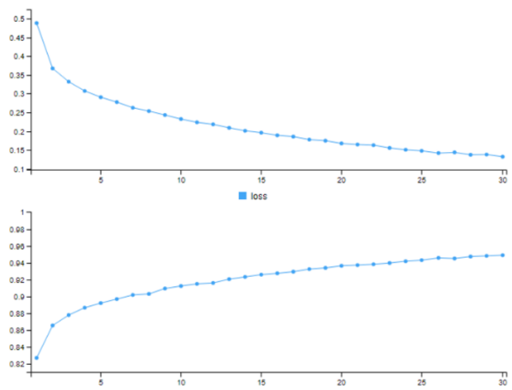

Figure 4. Loss and accuracy of training process of the model without topological signatures.

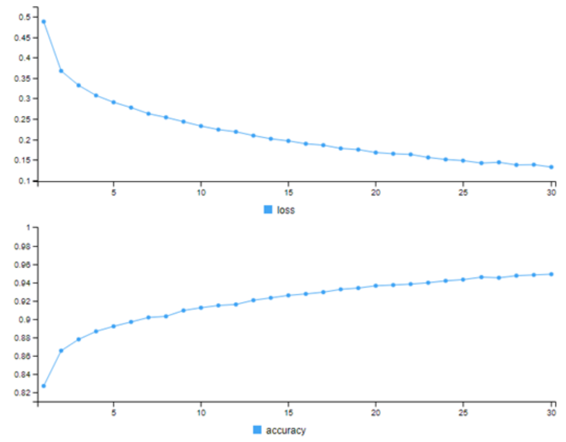

Figure 5. Loss and accuracy of training process of the model with topological signatures.

The loss and accuracy of training process of the model without topological signatures and the model with topological signatures are given in Figure 4 and Figure 5, respectively.

Hardware and time consumption

As it is mentioned, for the classification task it is used neural network with simple architecture. So, we used cheap hardware uncharacteristic for training neural networks. The training process for both the dataset without topological signatures and the dataset with topological signatures ended in less than an hour. Before that the process of computing topological signatures ended in 1 hour. So experiments ended in less than 3 hours.

5. Results

In this section, the results of two models, one without topological signatures and second with topological signatures, are given. For evaluation of this models on the test dataset we used four metrics: accuracy, precision and f1-score. The accuracy for the model without topological signatures is 0.879 and the loss is 0.573. The values of other metrics are given in *Table 1.*

Table 1. Metrics of testing of the model without topological signatures.

CLASS	PRECISSION	RECALL	F1-SCORE
0	0.8534031	0.815	0.83376
1	0.9928058	0.966	0.97922
2	0.8658854	0.665	0.752262
3	0.8957055	0.876	0.885743
4	0.7150974	0.881	0.789427
5	0.9835391	0.956	0.969574
6	0.6666667	0.746	0.704106
7	0.9412341	0.961	0.951014
8	0.9876923	0.963	0.97519
9	0.9543198	0.961	0.957648
AVERAGE	0.8856349	0.879	0.879794

The accuracy for the model without topological signatures is 0.889 and the loss is 0.4. The values of other metrics are given in *Table 2.*

Table 2. Metrics of testing of the model with topological signatures.

CLASS	PRECISSION	RECALL	F1-SCORE
0	0.844898	0.828	0.836364
1	0.98491	0.979	0.981946
2	0.811695	0.819	0.815331
3	0.865275	0.912	0.888023
4	0.822497	0.797	0.809548
5	0.988566	0.951	0.969419
6	0.714141	0.707	0.710553
7	0.949744	0.926	0.937722
8	0.975928	0.973	0.974462
9	0.914019	0.978	0.944928
AVERAGE	0.887167	0.887	0.886829

We can say that two models have good results on this dataset. The accuracy of the model with topological signature is improved. We should accent that we have less loss in the model with topological signatures than in the model without topological signatures

6. Discussion and further work

In this paper, classification model with topological signatures and without topological signatures are evaluated on digital image dataset. From the results of the experiment, two of the models are good. Also, the model with topological signatures has improved results for the metrics. Maybe, the improvements are not significant because this dataset is classified as "easy" for classification task. The results of the model with topological signatures are comparative with state of art models, which are using complex deep network architectures combined with transfer learning.

As, a further work we will define loss function based on the topological signatures of a dataset. Also, we will construct more complex neural network combining convolutional networks, which are concentrating on the local features of the images and topological signatures, which give insight for global features of the images, to investigate the improvements of classification task.

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8. References

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