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CONTENT

Sara Aneva, Marija Sterjova and Saso Gelev SCADA SYSTEM SIMULATION FOR A PHOTOVOLTAIC ROOFTOP SYSTEM	7
Elena Karamazova Gelova and Mirjana Kocaleva Vitanova SOLVING TASKS FROM THE TOPIC PLANE EQUATION USING GEOGEBRA	17
Sadri Alija, Alaa Khalaf Hamoud and Fisnik Morina PREDICTING TEXTBOOK MEDIA SELECTION USING DECISION TREE ALGORITHMS	27
Goce Stefanov And Biljana Citkuseva Dimitrovska DESIGN OF TFT SWITCH GRID	35
Angela Tockova, Zoran Zlatev, Saso Koceski GRAPE LEAVES DISEASE RECOGNITION USING AMAZON SAGE MAKER	45
Anastasija Samardziska and Cveta Martinovska Bande NETWORK INTRUSION DETECTION BASED ON CLASIFICATION	57
Aleksandra Risteska-Kamcheski and Vlado Gicev ANALYSIS OF THE DEFORMATION DISTRIBUTION IN THE SYSTEM DEPENDING ON THE YIELD DEFORMATION.....	69
Aleksandra Risteska-Kamcheski and Vlado Gicev DEPENDENCE OF ENERGY ENTERING A BUILDING FROM THE INCIDENT ANGLE, THE LEVEL OF NONLINEARITY IN SOIL, AND THE FOUNDATION STIFFNESS	81
Sijce Miovska, Aleksandar Krstev, Dejan Krstev, Sasko Dimitrov BUSINESS PROCESS MODELING, SYSTEM ENGINEERING AND THEIR APPROACH TO THEIR APPLICATION IN INDUSTRIAL CAPACITY	89
Sasko S. Dimitrov, Dejan Krstev, Aleksandar Krstev MATRIX METHOD FOR LARGE SCALE SYSTEMS ANALYSIS	99
Vasko Gerasimovski and Vlatko Chingoski SMALL MODULAR NUCLEAR REACTORS – NEW PERSPECTIVES IN ENERGY TRANSITION	107
Vesna Dimitrievska Ristovska and Petar Sekuloski TOPOLOGICAL DATA ANALYSIS AS A TOOL FOR THE CLASSIFICATION OF DIGITAL IMAGES	117
Sasko Milev And Darko Tasevski and Blagoja Nestorovski STRESS DISTRIBUTION ALONG THE CROSS SECTION OF THE NARROWEST PART OF THE DIAPHRAGM SPRING FINGERS	127

TOPOLOGICAL DATA ANALYSIS AS A TOOL FOR THE CLASSIFICATION OF DIGITAL IMAGES

VESNA DIMITRIEVSKA RISTOVSKA AND PETAR SEKULOSKI

Abstract. Topological data analysis, as a branch of applied mathematics, is one of newer areas that enable data analysis. The basic tool of this field is persistent homology, the main method of topological data analysis, and it is used to process the data set in this article. Persistent homology is a method that detects topological features of a space reconstructed from a data set. The application is illustrated on simple synthetic generated sets. In this article, we proposed and evaluated a new model that includes topological features into the classification process in real data sets composed of digital images. We got results in which there are some improvements in most of the statistical values for the classification performance over a model that does not include these topological features.

1. Introduction

Topological Data Analysis, for short TDA, is an area of applied mathematics that provides an approach to data analysis using topology techniques [1][5]. Many of the data sets are large, complex, and noisy, which leads to difficulties in understanding and extracting some knowledge from them. Topological data analysis provides a general framework for analyzing such data and offers dimensionality reduction as well as robustness to noise.

The general motivation in TDA is to study the shape of a given set of data and using methods from algebraic topology to determine some of its features and further to interpret what they mean for the data itself.

These methods provide qualitative characteristics for the data space and are based on the following principles:

- 1) Invariance with respect to coordinates
- 2) Invariance with respect to deformations
- 3) Compressed representation.



Figure 1 a) (left) Invariance with respect to coordinates; b) (middle) Invariance with respect to deformations; c) (right) Compressed representation

Figure 1-a) shows an example of invariance with respect to coordinates. Given a cycle, we are interested in how many cycles we have. If the same one is represented in different coordinate systems, it does not change anything, the number of cycles is the same. The same goes for the number of holes.

Figure 1-b) shows an example of invariance with respect to deformations. If we stretch one of the letters as much as we want without “cutting” or “gluing” it, we will have the same number of holes. Topologically speaking, it is the same object. It gives us robustness in terms of noise.

Whereas Figure 1 -c) shows a compressed representation of the same object. That object can be represented by an infinite number of points or by some finite approximation in the form of a graph of several vertices and edges.

This article has a main goal to apply methods from TDA to Machine Learning. As a result, a new classification model is proposed that includes topological features based on neural networks and persistent homology.

2. Basic concepts of topology

What does topology study?

Topology is a field that studies the form, more precisely the form of topological spaces through different types of equivalences. More detailed mathematical foundations can be found in [2] and [3].

Illustratively, let two topological spaces be represented by some physical objects that are elastic. If one of them can be deformed, without cutting it and without gluing it to the other physical object, then for these objects, i.e., topological spaces, we say that they are equivalent. Topologically, both have the same properties. The best known is the example given in Figure 2, (Ceramic model by Keenan Crane and Henry Segerman [23]) the deformation of a coffee mug into a donut. Both objects are topologically equivalent. But all this is not limited to some physical space.



Figure 2. A continuous transformation can turn a coffee mug into a donut

Due to the simplification of model presentation, we will omit here the precise definitions of the basic topological terms used in this paper, such as Abstract simplicial complex, Geometric simplex, Geometric simplicial complex, Cubic complex, Geometric cubic complex, Filtration, Persistent homology, Persistent homology group, Persistence Betty’s number, Persistence barcodes, Persistence diagrams, Persistence images, and Vietoris- Rips complex. Those definitions can be found for example in [4].

3. Persistence Barcodes, Persistence Diagrams, and Persistence Images of synthetic generated Datasets

In this section we will give examples of persistence barcodes, persistence diagrams, and persistence images on synthetic generated simple datasets. Such sets are generated in order to illustrate the functionality of these topological data analysis methods and to visually represent how they work.

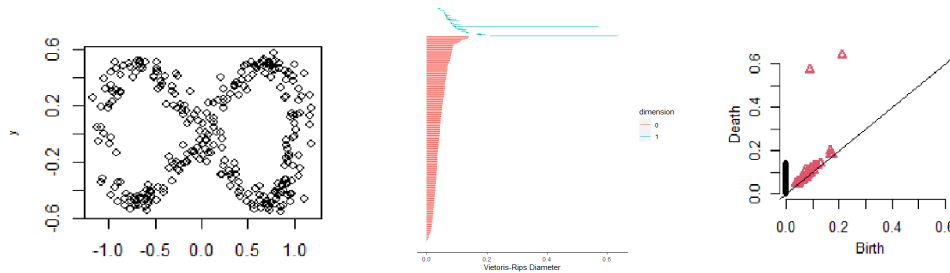


Figure 3. a) data set; b) persistence barcode; c) persistence diagram

Figure 3a) shows a synthetic data set generated from points that lie in a plane and look like a digit eight, but with some noise. After calculating persistent homology using the Vietoris-Rips complex, the barcode shown in Figure 3b) and the persistence diagram shown in Figure 3 c) were calculated.

We can see more red lines in the barcode. They correspond to persistent homology groups of dimension 0. We see that some are shorter, that is, such components existed for a shorter time, and then merged with some other related component, first with dimension 0, which is represented by a longer red line.

We have several blue lines that represent homology groups with dimension 1, of which only two are longer, that is, in this example, they are the holes in the image, the inside of the circular shapes.

In Figure 3c), we show the persistence diagram for the same data set. It offers the same information as a persistence barcode. There, black dots correspond to homology groups with dimension 0, while red triangles correspond to homology groups with dimension 1. The x-coordinate represents the birth, while the y-coordinate represents the death of each of the components. Each point or triangle of this persistence diagram is important, it exists persistently, as far as it is from the diagonal. It means it “lived” longer. We see that there are two red triangles at the top, they indicate the holes in Figure 3a).

In Figure 4, two sets (one circle with noise and two concentric circles with noise) are shown and corresponding persistence images for dimension 0 and dimension 1. Here we are working in a two-dimensional space, so the dimensions we can talk about are 0 and 1. After calculating persistent homology, persistence diagrams are obtained, from them persistence images are obtained, shown in columns 2 and 3. Column 2 gives the persistence images for persistent homology with dimension 0, while column 3 gives the persistence images for persistent homology with dimension 1.

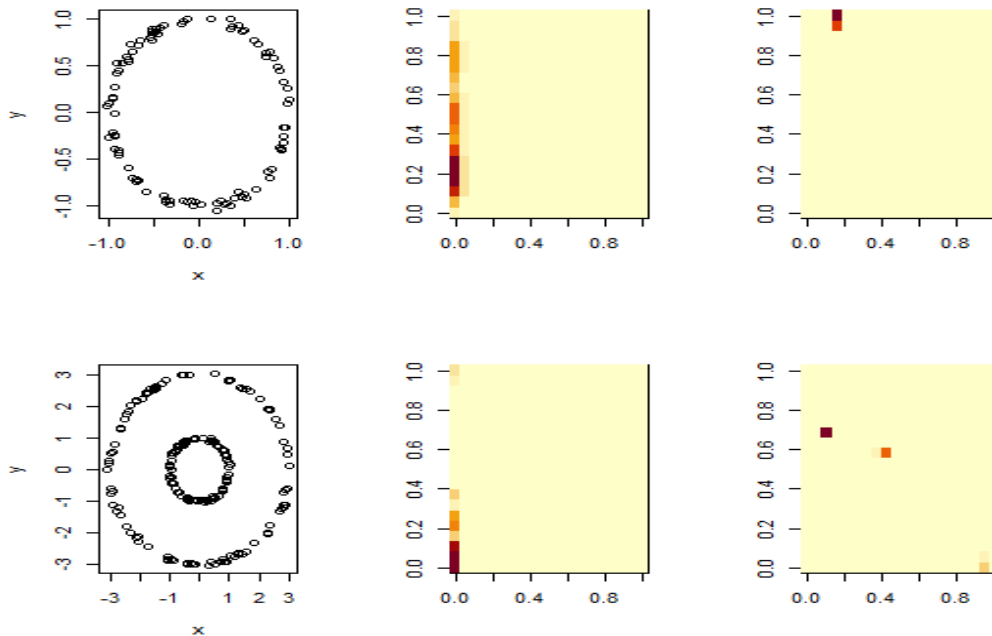


Figure 4. Synthetic generated data sets together with their persistence images

4. Method: new proposed classification model

From the aspect of applying topological data analysis methods, in this article we propose a new model, which we will label as Model 1, and we will explain it in the following.

The main goal was to detect the topological characteristics by reconstructing the space from the data we have, and then use them in the classification process.

As we emphasized earlier, we work on a dataset of digital images. The first stage is the preprocessing of the data, and then the preprocessed data, the images, enter a stage that is key to the topic we are working on and the intermediate steps of this stage are given in Figure 5.

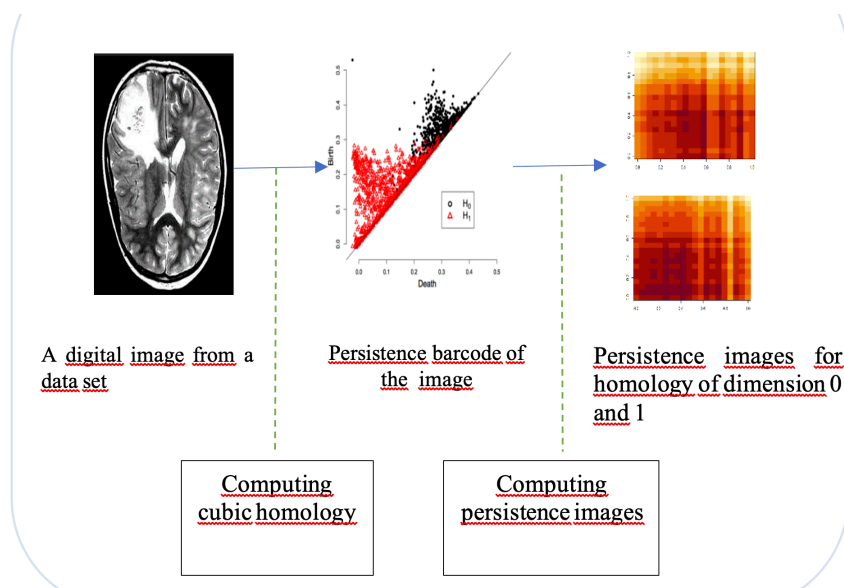


Figure 5. Description of the phase Computing of topological features from Model 1

Each of the images, both those used for training and those used for testing, are processed in this phase.

First, for each of the images, which in the data set are black and white images, a cubic complex is constructed, with a V-construction. Then homology is calculated, on the already obtained cubic complex from the image. Once the homology is computed, the persistence diagrams can be computed.

A given persistence image is related only to the persistence groups of a certain dimension, and since we have two-dimensional images, for the images we worked with, the dimension can be 0 or 1. Therefore, from the persistence diagram, two persistence images are obtained for each of the digital images of the data set.

And, in fact, this is the main benefit - how to extract attributes from the data that will cover the topological characteristics of the data we have, and thus, to check if this will affect the further classification process.

Such obtained persistence images, which correspond to the persistence groups whose dimension is 0, and persistence groups whose dimension is 1, together with the original images enter the Fusion stage. In this phase, from the original image, and the two persistence images, we create a sequence of three matrices, that is, we create a three-channel representation of each of the images that we have in the data set. In the first place in the sequence, or in other words on the first channel, is the original image, and on the second and third positions, i.e., channel, persistence images are obtained.

This model is easily extendable with additional channels, if the original images are like that, but we will not talk about that in this paper.

The data set thus transformed is passed to a classifier of a certain type. For this purpose, we used a neural network.

5. Results and discussion: Application of the Chest X-Ray Images (Pneumonia) Set

Why did we choose this set?

To investigate the applicability of our model in disease diagnosis, we applied it to learn the diagnosis of pediatric pneumonia. According to the World Health Organization (WHO), pneumonia kills about 2 million children under 5 years of age each year and is consistently estimated to be the single leading cause of death in children [12], killing more children than HIV/AIDS, malaria, and smallpox together [10]. WHO reports that almost all cases (95%) of new-onset childhood clinical pneumonia occur in developing countries, particularly in Southeast Asia and Africa. Bacterial and viral pathogens are the two leading causes of pneumonia [11] but require quite different forms of management. Bacterial pneumonia requires immediate referral for emergency antibiotic treatment, while viral pneumonia is treated with supportive care. Therefore, accurate and timely diagnosis is particularly important.



Figure 6. Illustrative examples of chest X-rays in patients with pneumonia [17]

A normal chest X-ray (left panel) shows clear lungs without any areas of abnormal clouding on the image. Bacterial pneumonia (middle) usually shows focal lobar consolidation, in this case in the right upper lobe (white arrows), whereas viral pneumonia (right) presents with a more diffuse “interstitial” pattern in both lungs.

Chest X-rays are a key element of diagnosis (Figure 6), but rapid radiological interpretation of the images is not always possible, especially if available resources are low, and where childhood pneumonia has the highest incidence and mortality rate. For this purpose, we investigated our model in the classification of pediatric chest X-rays to detect pneumonia and to speed up the treatment procedure of children who need immediate intervention.

The Chest X-Ray Images (Pneumonia) dataset [27] contains 5863 images (JPEG) obtained from chest X-rays taken as part of routine clinical care of pediatric patients, from retrospective cohorts of pediatric patients aged one up to five years. The images are divided into two classes, the “Pneumonia” class, and the “Normal” class. The class “Pneumonia” indicates that the image is of a patient who has pneumonia, while the class “Normal” indicates that the image is of a patient who does not have pneumonia. We denote the first class in our processing with 1, while we denote the second class with 0.

The dataset is organized into 3 folders: a folder containing the training images, a folder containing the test images, and a folder containing the validation images (train, test, val) and containing subfolders for each image class (Pneumonia/Normal).

In the training folder, we collected and labeled a total of 5232 chest X-rays from children, including 3883 characterized as pneumonia (2538 bacterial and 1345 viral) and 1349 normal, out of a total of 5856 patients to train the new proposed model. Note that the set is not balanced. The model was then tested with 234 normal images and 390 pneumonia images (242 bacterial and 148 viral) of 624 patients.

The application of deep neural networks in medical images can be seen in [26]. We did this experiment in the way explained in the following.

The images in the dataset are black and white with different dimensions and pixel value from 0 to 255. We also resized all the images to 128x128. During preprocessing we scaled all the images so that the pixel value will be in the range from 0 to 1.

After this stage, we moved on to merging the persistence images with the original image. We have created a three-element array whose elements are matrices of the dimension 128×128 . The first element of the array is the original image, while the second and the third elements are the persistence images obtained in the previous stage and corresponding to the persistence homology groups of dimension 0 and dimension 1. Let us emphasize that we do this transformation on both the training subset and the testing subset.

The second member is the persistence image corresponding to the persistence homology groups of dimension 0 and the third matrix is the persistence image corresponding to dimension 1.

The goal is to check if a classifier to which we give as a dataset the transformed set together with the topological features, which contains such sequences we talked about earlier, will perform better than if we use only the original images.

Next we will specify the classifier we used. We will first describe the model we used for classification without the topological performance calculation phase. We used a simple neural network.

The first layer in the network, `layer_flatten`, transforms the input from a three-element array, whose elements are matrices of the dimension 128×128 , into an array by concatenating the rows of the three matrices into one row, side by side. In this model, in which the attributes obtained from the Computation of Topological Features phase are not used, a three-channel image is given as input and all three channels have the original image. This is how the array of matrices is formed.

In the inner layer of the network there are two dense layers. They are fully connected neuronal layers with 128 neurons each. And in the last layer, which contains two neurons, as many classes as we have for classification, the activation function “softmax” determines the probabilities to which class the image belongs. “adam” optimization and “sparse_categorical_crossentropy” as loss function were used.

We stopped training at the 200th epoch, as both the accuracy and the loss function converged after the 70th epoch. We used a neural network because it is most suitable for working with images.

Figure 7 shows the network architecture.

Layer (type)	Output Shape	Param #
flatten_7 (Flatten)	(None, 49152)	0
dense_23 (Dense)	(None, 128)	6291584
dense_22 (Dense)	(None, 128)	16512
dense_21 (Dense)	(None, 2)	258

Total params:	6,308,354	
Trainable params:	6,308,354	
Non-trainable params:	0	

Figure 7. The neural network used for classification

To classify images from this problem, there are quite complex neural architectures that mostly use convolutional networks, transfer learning, and obtain comparable results with this model, which is simple and includes a neural network with two hidden layers. During our experiments, the Calculation of topological characteristics process takes the most time, but even for such a set it is measured in minutes, performed on not very powerful hardware machines that do not include GPU and similar technologies.

The accuracy of the test set obtained in our experiments without using TDA is 0.7644, while the accuracy of the test set using TDA is 0.7869. As we expected, there is an improvement in our TDA Model 1. If we look at Table 1 and Table 3, we can see that our model classifies those images of patients who do not have pneumonia better, while the model without TDA classifies those images of patients who have pneumonia better.

Table 1. *Confusion matrix from testing the model without topological signatures*

TRUE/PREDICTED	0	1
0	92	142
1	5	385

Table 2. *Metrics of testing of the model without topological signatures*

Class	Precision	Recall	F1-Score
0	0.9484	0.3931	0.5558
1	0.7305	0.9871	0.8396
average	0.8395	0.6901	0.6978

Table 3. *Confusion matrix from testing the model with topological signatures*

TRUE/PREDICTED	0	1
0	194	40
1	93	297

Table 4. *Metrics of testing of the model with topological signatures*

Class	Precision	Recall	F1-Score
0	0.6759	0.8205	0.5558
1	0.8813	0.7615	0.8396
average	0.7786	0.7953	0.7809

The rest of the results for the metrics are given in Table 2 and Table 4. Analyzing them, we can say that for this data set, for the Recall, Accuracy and F1-metric parameters, our model showed better results, while for the Precision, the model without using TDA has slightly better values.

6. Software tools

Preprocessing of the digital images of the experimental part was done using the imager library from R. The cubic homology calculation was implemented in R, using the Rcpp11 and ripser libraries.

The classifier is implemented using the Keras Python library, also using an interface to the R programming language. We developed the main code required for the full functionality of the experiments from this paper in R.

7. Interesting current applications of TDA

Although TDA is a relatively new field, in the last four years there has been a large number of scientific papers involving topological data analysis and it has been successfully applied in a number of fields. Some of the areas covered by the application of TDA are: gene expression, neural network analysis, time series prediction, cancer detection, cyber security, eco-informatics, natural language processing, sound processing, face recognition, analysis and time series prediction, stability of dynamical systems, image segmentation, sensor networks, complex networks, banking, sensor networks, noise detection, signal processing, bioinformatics and many others. [13][14][15][16][22]

8. Conclusion and further work

First, an overview of the field of topological data analysis is given, the basic tools of this field are countered and their application on simple data sets is shown through examples, in order to understand how they work.

A new model is proposed that includes the detection of topological features from the data and as such are added as additional attributes to the original images before the training process of the classifier begins, which is a fully connected neural network - a simple model that does not require a lot of processing power.

In the experimental part, some of the methods for the classification of the data set Chest X-Ray Images (Pneumonia) are applied. For comparison, experiments were made in which the network was trained only on the original images, without the topological features.

In the considered set, with the new proposed model, there is an improvement in a large part of the statistical indicators in machine learning.

In addition, it is possible to develop and study the input parameters of selected algorithms in the field of TDA, in order to consider the influence of the choice of parameter values on the obtained results for specific data sets.

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Vesna Dimitrievska Ristovska

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