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# Ad Hoc Networks Connection Availability Modeling 

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#### Abstract

One of the most important issues for mobile ad hoc networks is to know the availability of the system. In this paper we propose a generalized connection availability model based on real measurable parameters that concern the performances of mobile ad hoc networks. Detailed validation of the connection availability model is presented through simulations. The proposed model can be used for analysis or design of an ad hoc network that needs to satisfy a given connection availability level.


## Categories and Subject Descriptors

C.2.1 [Network Architecture and Design]: Wireless communications I.6.5 [Simulation and Modeling]: Model Development -modeling methodologies.

## General Terms

Performance, Reliability

## Keywords

Ad hoc Network, Connection Availability, Markov Chain.

## 1. INTRODUCTION

Mobile ad hoc networks are suitable for situations wherein infrastructure is either not available, not trusted, or should not be relied on. The network's ability to avoid or cope with failure is measured in three ways: reliability, availability and survivability, all of which have long been important areas of research [1]. The networks availability can be reviewed from different aspects like two-terminal, k-terminal and all-terminal. The two terminal measure measures the ability of the network to satisfy the communications needs of a specific pair of user terminals (connection), thus being a user view of the network fault tolerance. In this paper the connection availability in ad hoc networks is modeled.
The previous work for ad hoc networks availability can be classified in two groups. The first group models link and path availability depending on the node speed, mobility model and transmission radius. However, the influence of the routing protocol, number of nodes in the network, or area size wherein the nodes are scattered, is not included. In [2] path availability model

[^1]for random way point mobility model is given, while in [3] prediction-based link availability estimation is introduced. Link and path availability model using a random walk mobility model is derived in [4] and in [5] prediction about the average link expiration times for a simple network scenario is investigated.
The second group, models ad hoc network connection availability depending on node failures. In [6] the continuous time Markov chain (CTMC) is used to represent a connection availability model for a two hop ad hoc network that incorporates the physical faults like: node, power and link faults. In [7] a generalized model with enlarged number of node failure types is introduced. These models give acceptable results only for static ad hoc networks. In mobile ad hoc networks, the probability that a connection will be broken as a result of the routing node mobility is far greater than the probability that a physical fault will occur at the routing node.
In [8], connection availability model that incorporates the characteristics of commonly used ad hoc routing protocols like the Dynamic Source Routing Protocol (DSR) and Ad hoc On-demand Distance Vector Routing Protocol (AODV) is given. In [9] the impact of mobility on connection availability is studied. The main motivation for this work is to create a connection availability model that will, as much as possible, incorporate the influence of measurable parameters, without additional simplifications of the ad hoc network model used in the referenced papers. Based on the models proposed in [8] and [9], in this paper we propose a generalized connection availability model according to the connection availability model given by the second group, that also incorporates the influence of the parameters from the first group, and is extended with the impact of the routing protocol and size of the area wherein the network participants are scattered.

## 2. CONNECTION AVAILABILITY MODELING

We use a common ad hoc network model where all the mobile nodes have the same transmission power and are equipped with omni directional antenna, thus having equal transmission range $r$. Similarly to [5], [6] and [7] we use a two-hop scenario because of the complexity of an analytical model development for multihop scenario.
We observe $N+2$ nodes placed in area $A$ (see Fig. 1). The two additional nodes represent the source and the destination for the analyzed end-to-end connection. The $N$ nodes play the role of routers in the connection path between the source and the destination. While moving around in $A$, a node can enter the $B$ area and, after a certain period of time, leave $B$ and enter area $C$ defined as $A-B$. This process is continuously repeated. In order to establish a communication between the two nodes, MNs (source) and MNd (destination) the communication path has to go through one of the nodes that are currently located in the intersection area $B$ between MNs and MNd (i.e. MN1, MN2).


Figure 1. Ad hoc network model
When the previously used route becomes unavailable, the routing protocol plays the main role in connection reestablishment, directly influencing the connection availability. The time needed for connection reestablishment depends on the presence of nodes in the intersection region. When there is at least one node in the intersection region the average time needed for connection switching is defined as average switching delay $1 / \delta$. Otherwise, the average time needed for connection reestablishment is defined with the average connection reestablishment delay $1 / \delta_{r}$.
In order to obtain a closed form solution for the connection availability, we use a system model solvable by a Continuous Time Markov Chain (CTMC). Hence, the following assumptions should hold: all entering and leaving events in the intersection region are mutually independent, exponential distribution is assumed for the time of occurrence of each enter and leave event, and the average switching delay is small compared to the average time a routing node spends in the intersection region. Using series of simulations we found that all of these assumptions are valid except for the exponential distribution of the time of occurrence of the leaving events, which can be successfully approximated with an exponential (detailed description is given in section 4).
Our connection availability model is constructed as a parallel system of $N$ components with $N$ repair facilities and depends on the failure rate $\lambda$ (rate of leaving the intersection region $B$ ); the repair rate $\mu$ (rate of returning in $B$ ); number of network participants $N$; average switching delay $1 / \delta$; and connection reestablishment delay $1 / \delta_{r}$. The states of the CTMC model shown on Fig. 2 are labeled with tuple ( $i, j$ ) where $i \in\{0,1,2, \ldots, N\}$ represents the number of nodes currently in the intersection region (the total amount of nodes is $N+2$ ), and $j \in\{0,1,2,3\}$ represents the state of the connection ( 0 -no fault connection is up, 1-route discovery state, 2waiting for route reestablishment, 3 -no routing nodes available).
State ( $N, 0$ ) represents the state where the connection is up, while all of the $N$ nodes are in the intersection region $B$. In this state, any of $N-1$ backup routers may leave $B$ with rate $\lambda$, and bring the system to state ( $N-1,0$ ), representing that the connection is up and the number of backup routers is reduced to $N-2$. The system returns to state $(N, 0)$ if the node returns to the intersection region with rate $\mu$. When the current routing node leaves the intersection region, it brings the system to state ( $N-1,1$ ) wherein the connection is down due to route discovery, with $N-1$ routers available to switch over.

In state ( 1,0 ), after the last routing node leaves the intersection region (there are no nodes that can be used for connection reestablishment) brings the system in state $(0,3)$ wherein the routing protocol tries to find a new routing node. In this state, $N$ backup routers may enter with rate $\mu$ and bring the system to state $(1,2)$, representing that the connection is down because the only availably routing node is not yet discovered. If the node is discovered before it leaves the intersection region, the connection is reestablished and the system returns to state $(1,0)$. If the node leaves the intersection region without being discovered by the routing protocol, it brings the system back to state $(0,3)$.
The steady state connection availability (SSCA) is given by summation of all no faulty states (second index equal to 0 ):

$$
\begin{align*}
A_{s} & =\sum_{k=1}^{N} \frac{N!}{k!(N-k)!}\left(\frac{\mu}{\lambda}\right)^{k}\left(\frac{\delta_{r}}{\lambda+\delta_{r}}\right) \pi_{0,3}  \tag{1}\\
\pi_{0,3} & =\frac{\left(\lambda+\delta_{r}\right) / \delta_{r}}{\left(1+\frac{\lambda}{\delta}\right)\left(1+\frac{\mu}{\lambda}\right)^{N}+\lambda N \mu\left(\frac{1}{\delta_{r}}-\frac{1}{\delta}\right)} \tag{2}
\end{align*}
$$

## 3. CONNECTION AVAILABILITY PARAMETERS

Ad hoc networks connection availability depends on many factors: routing protocol, number of participants in the network, distance between source and destination, nodes velocity, mobility model, transmission range and size of the area wherein the participants in the ad hoc network are scattered. These parameters are defined in the following text.

### 3.1 Routing Protocol

The influence of routing protocol is represented by two parameters: the average link switching delay $1 / \delta$ and average connection reestablishment delay $1 / \delta_{r}$. By the means of series of simulations using the NS-2 [13] simulator the numerical values of those parameters for AODV are obtained. The average switching delay is measured as the average time that passed from the moment when the current route becomes unavailable (due to node movement) till the moment when a new route is established. Using a network scenario consisting of 4 nodes (source, destination and 2 routers), we measured the average time delay needed for connection switching through the second routing node. The measured average delay is 0.0565 s .

When there are no nodes in the intersection area, the routing protocols use a back-off algorithm in order to limit the rate at which new route discoveries for the same destination are initiated. Again, using a test scenario wherein the only routing node leaves the intersection region, it was observed that the AODV NS-2 implementation includes periodical issuing of 4 route requests in exact time intervals $(0.1236 \mathrm{~s}, 0.247 \mathrm{~s}, 0.370 \mathrm{~s}, 10.485 \mathrm{~s})$. The incorporation of this routing protocol behavior in the average connection reestablishment delay $1 / \delta_{r}$ parameter is done as follows. To reestablish the connection, the source node needs to discover a newly entered node. If the node enters the intersection region in

time $t_{l}$ after the previous route request, and the current delay time is $T$, then the probability that the node will be discovered before it leaves the intersection region is equal to the probability that the node will remain in the intersection region for at least $T-t_{l}$ seconds. When a node enters the intersection region, it will stay inside for an average of $1 / \lambda$ seconds. If $P_{\lambda}$ is the probability distribution function for the residence time in the intersection region then the probability of discovering the newly entered node $p_{\text {disc }}$ is:

$$
\begin{equation*}
p_{\text {disc }}\left(T, t_{1}\right)=P_{\lambda}\left(t>T-t_{1}\right)=1-P_{\lambda}\left(t \leq T-t_{1}\right)=e^{-\lambda\left(T-t_{1}\right)} \tag{3}
\end{equation*}
$$

Since the rate of arrival of nodes in the intersection region has a constant value (assumed exponential inter-arrival times), the average probability that the node will be discovered, if it enters the intersection region during the delay period with duration $T$, is:

$$
\begin{equation*}
p_{\text {disc }}(T)=\frac{1}{T} \int_{0}^{T} e^{-\lambda(T-t)} d t=\frac{1-e^{-\lambda T}}{T \lambda} \tag{4}
\end{equation*}
$$

According to the periodically repeated pattern, the average probability that the node will be discovered after it enters the intersection region can be calculated by averaging the probabilities for all delay periods during one periodic pattern:

$$
\begin{equation*}
p_{\text {disc }}=\frac{1}{\sum_{i} T_{i}} \sum_{i} T_{i} p_{\text {disc }}\left(T_{i}\right) \tag{5}
\end{equation*}
$$

Let $\varepsilon$ describe the residence time exponential distribution in state $(1,2)$ of CTMC. Since the probability of entering state $(1,0)$ is given by (5), we have $\lambda=\varepsilon \cdot\left(1-p_{\text {disc }}\right)$ and $\delta_{r}=\varepsilon \cdot p_{\text {disc }}$. Consequently, the average connection reestablishment delay $1 / \delta_{r}$ can be calculated by:

$$
\begin{equation*}
\delta_{r}=\frac{p_{\text {disc }}}{1-p_{\text {disc }}} \lambda \tag{6}
\end{equation*}
$$

### 3.2 Transmission Range and Distance Between Nodes

Both, the transmission range and distance between nodes, affect the size of the intersection area $B$, which is given by:

$$
\begin{equation*}
B=r^{2}\left(2 \operatorname{ArcCos}\left(\frac{a}{2}\right)-a \sqrt{1-\frac{a^{2}}{4}}\right) \tag{7}
\end{equation*}
$$

where $r$ is the transmission radius, $a$ is the relative distance $a=l / r$ and $l$ is the distance between the nodes ( $l \in[r, 2 r], a \in[1,2]$ ).

### 3.3 Leaving Rate and Returning Rate

The mobility of nodes affects the leaving and returning rate. In order to obtain the leaving rate, we must obtain the average time that a MN spends in the intersection region. Due to its shape, the intersection region is called the "eye of coverage" (see Fig 1.).

There are several Mobility Models (MM) that are used in performance evaluation for ad hoc networks. One of most commonly used is Random Walk MM [10] (also known as Random Direction). In this MM a MN moves from its current location to a new location by randomly choosing travel direction and speed, both chosen from pre-defined ranges, [speedmin; speedmax] and $[0 ; 2 \pi]$ respectively. Each movement in the Random Walk MM occurs in either a constant time interval or a constant distance traveled, at the end of which a new direction and speed are calculated. If a MN reaches the simulation boundary, it "bounces" off the border with an angle determined by the incoming direction and continues along this new path. Because the intersection region is very small compared to the area wherein the MN are scattered, a small number of direction changes will occur inside this region. To
simplify the solution, we can presume that the node passes the intersection region in a straight line with constant speed. Presuming these conditions, the time needed to pass the intersection region is given by $t=d / v$, where $d$ is the length of the path that the MN passes in the intersection region. The MN speed $v$ is a uniformly distributed random variable. The eye path $d$ is a random variable and its value depends only on the entry point in the intersection region and the entry angle. Accordingly $d$ and $v$ are statistically independent random variables, and thus their joint probability density function is given by

$$
\begin{equation*}
p_{v D}(v, d)=p_{V}(v) p_{D}(d) \tag{8}
\end{equation*}
$$

The average time, needed for a node to pass through the intersection region is

$$
\begin{equation*}
\bar{t}_{B}=\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{v} d p_{V}(v) p_{D}(d) d v d d \tag{9}
\end{equation*}
$$

Arranging formula (9) we get

$$
\begin{equation*}
\bar{t}_{B}=\int_{-\infty}^{\infty} \frac{1}{v} p_{V}(v)\left(\int_{-\infty}^{\infty} d p_{D}(d) d d\right) d v \tag{10}
\end{equation*}
$$

The expression in parenthesis is the average value of the eye path (detailed derivation is given in the appendix):

$$
\begin{equation*}
\bar{d}=\pi r\left(2-\frac{l \sqrt{4-l^{2}}}{8 \operatorname{ArcSec}(2 / l)}\right) \tag{11}
\end{equation*}
$$

By substituting (11) in (10) together with the density function $p_{v}(v)=1 /\left(v_{\text {max }}-v_{\text {min }}\right)$, where $v_{\text {min }}>0$, we have

$$
\begin{equation*}
\bar{t}_{B}=\int_{v_{\min }}^{v_{\max }} \bar{d} \frac{1}{v} \frac{1}{v_{\max }-v_{\min }} d v=\bar{d} \frac{\ln \left(v_{\max }\right)-\ln \left(v_{\min }\right)}{v_{\max }-v_{\min }} \tag{12}
\end{equation*}
$$

If the node spatial probability density function, for the given MM, is $f(x, y)$ and it is time independent, the probability that a given node will be in area $B$ is given by $p_{B}=\iint_{B} f(x, y) d x d y$. When considering a long enough time interval $T$, the time spent in area $B$ for a given node is $T_{B}=p_{B} T$. If, during $T$, a given node makes $k$ enter-leave cycles in area $B$, then the average time spent in each cycle is $\bar{t}_{B}=T_{B} / k$. Thus,

$$
\begin{equation*}
\bar{k} t_{B}=p_{B} T \tag{13}
\end{equation*}
$$

These relations can also be written for area $C$. Hence, we have

$$
\begin{equation*}
\bar{k} t_{c}=p_{c} T \tag{14}
\end{equation*}
$$

Dividing (13) by (14), we obtain

$$
\begin{equation*}
\frac{\bar{t}_{B}}{\bar{t}_{C}}=\frac{p_{B}}{p_{C}}, \bar{t}_{C}=\frac{p_{C}}{p_{B}} \bar{t}_{B} \tag{15}
\end{equation*}
$$

If the node spatial probability density function is constant as in the case of Random Walk MM [12], then equation (15) becomes:

$$
\begin{equation*}
\frac{\bar{t}_{B}}{\bar{t}_{C}}=\frac{P_{B}}{P_{C}} \tag{16}
\end{equation*}
$$

where $P_{B}$ and $P_{C}$ represent the size of areas $B$ and $C$, respectively. The same procedure can be applied to other mobility models with a known node spatial probability density function. Hence, the leaving rate for intersection area $B$ is

$$
\begin{equation*}
\lambda=1 / \bar{t}_{B} \tag{17}
\end{equation*}
$$

and the leaving rate for area $C$ (returning rate for area $B$ ) is

$$
\begin{equation*}
\mu=1 / \bar{t}_{C} \tag{18}
\end{equation*}
$$



Figure 3. Average times passed in intersection region $B$ and area $C$ ratio and area size $B$ and $C$ ratio.


Figure 4. Probability distributions of inter arrival times for nodes that arrive in the intersection region.

## 4. MODEL VALIDATION

The proposed connection availability model is based on several assumptions: average time passed in the intersection region $B$ and the area $C$ ratio equals the area size $B$ and $C$ ratio (equation (16)) and assumed exponential distribution for both, entering and leaving events. At the end of this section the complete model validation using simulations in NS-2 is given.
In order to verify the theoretical results that consider the topological characteristics, we use the ad hoc network topology simulator TopoSim [11], created for the purpose of simulating purely topological ad hoc network characteristics. The simulations scenarios consist of 100 MN uniformly scattered in $1000 \mathrm{~m} \times 1000 \mathrm{~m}$ area. There are two additional nodes that represent the source and the destination placed on a given distance. The mobile nodes are moving according to the Random Walk MM with an average speed from $0.5 \mathrm{~m} / \mathrm{s}$ to $5 \mathrm{~m} / \mathrm{s}$. The transmission radius is set to 250 m . During the simulations we measure the path length and the time spent in the intersection region for every mobile node.
On Fig. 3 the average time passed in the intersection region $B$ and the area $C$ ratio is shown for Random Walk MM. This MM has a uniform node spatial distribution [12], and thus equation (16) holds. On the same figure the area size $B$ and $C$ ratio is also shown. It can be concluded that they do not differ significantly. This verifies equation (16).
In order to confirm the assumed exponential distribution of inter arrival times for the nodes entering the intersection region we made several simulations. In each simulation there is one mobile node for which the times of entering and leaving the intersection region are recorded. The obtained probability distribution of inter arrival times is shown on Fig. 4. On the same figure an exponential distribution with parameter calculated according to (17) is shown. The maximum relative error between the calculated and simulated distributions is smaller than $1 \%$. The probability distribution of the inter-leaving times is shown on Fig. 5. An exponential distribution


Figure 5. Probability distributions of inter leaving times for nodes that leave the intersection region.


Figure 6. Simulated and calculated SSCA with $N=55$ nodes, average speed $1 \mathrm{~m} / \mathrm{s}$ for different source destination distances
with parameter calculated according to (18) is given on same figure. Although this distribution is not exponential, in order to obtain a closed form solution, we substituted this distribution with an exponential distribution with parameter given by (18). Both distributions have same average values.
In order to validate the complete model we made a series of simulations using NS-2 network simulator. In each simulation scenario, a given number of MN are moving according to the Random Walk MM from [10]. The connection availability is measured as received/sent packets ratio. The connection is established using a constant bit rate, wherein the sending interval is adjusted to its maximum value achieved when there is always available connection (the router node is static). Queue is not used in order to avoid its influence on the number of dropped packets. AODV was adjusted to two-hop environment by limiting the time_to_live parameter.

On Fig. 6 the simulated and calculated SSCA for ad hoc network using AODV routing protocol is shown. It can be seen that the calculated SSCA has larger values than SSCA obtained through simulations. The relative error is less than $5 \%$ in all cases. It is larger for smaller SSCA, but then its absolute value is very small. There are two main reasons for obtaining slightly larger values for the calculated SSCA. First reason is the method used for measuring SSCA in simulations as the received/sent packets ratio. There are two cases when some packets are dropped but the connection between source and destination is available (the packet is partially transferred): If a routing node leaves the intersection region while a packet is being forwarded, the packet will be lost. A packet will also be lost, if a routing node leaves the intersection region while the source node is sending a packet. Second, the proposed model does not cover all cases when route request and route reply packets are sent from the source and the routing nodes, especially in route reestablishing phase. The control packets use the same medium through which the data packets are sent, and, thus, they reduce the useful bandwidth.


Figure 7. Calculation of average eye path length.

## 5. CONCLUSION

In this paper a closed form solution for connection availability of two hop ad hoc networks is presented. Analytical expression for the leaving rate and the returning rate in the intersection region are also presented. All parts of the proposed model are validated trough simulations. The model gives good results compared to the connection availability obtained using NS-2 simulator.
The proposed connection availability model can be used for analysis of the connection availability, or design of an ad hoc network that needs to satisfy a given connection availability level.
Our future research along this line includes: solving the connection availability model using the real probability distribution of the inter-leaving times, application of the proposed model on ad hoc networks that use different mobility models, and routing protocol optimizations using the closed form solution in order to achieve greater connection availability level.

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Figure 8. Calculation of average eye path length.
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## 7. APPENDIX

The following text describes the deriving of the average eye path. Let's suppose that the source node resides in the center of Cartesian coordinate system, while the destination lies on the $y$ axis of the system, see Fig. 7 and Fig 8. Firstly, we will find the average eye path length when a MN enters the intersection region at an angle $\alpha$ relative to the $x$-axis. Note that we consider only one quarter of the possible angles, due to the horizontal and vertical symmetry, so $\alpha \in[0, \pi / 2]$. The average value of this path $d(\alpha)$ can be obtained by

$$
\begin{equation*}
\bar{d}(\alpha)=\frac{B}{w(\alpha)} \tag{19}
\end{equation*}
$$

where $B$ is the eye area size and $w(\alpha)$ is the "width" that corresponds to the given entry angle. Depending of the angle $\theta$ given by

$$
\begin{equation*}
\theta=\operatorname{ArcCos}\left(\frac{l}{2 r}\right) \tag{20}
\end{equation*}
$$

$w(\alpha)$ can be calculated using the following equation (see Fig. 7 and Fig. 8):

$$
w(\alpha)=\left\{\begin{array}{cl}
2 r-l r \sin \left(\frac{\pi}{2}-\alpha\right) & 0<\alpha<\theta  \tag{21}\\
\sqrt{4 r^{2}-l r^{2}} \cos \left(\frac{\pi}{2}-\alpha\right) & \theta<\alpha<\frac{\pi}{2}
\end{array}\right.
$$

The probability that the MN will enter the eye under a given entry angle $\alpha$ is

$$
\begin{equation*}
p_{a}(\alpha)=\frac{w(\alpha)}{\int_{0}^{\pi / 2} w(\alpha) d \alpha} \tag{22}
\end{equation*}
$$

Using (19), (21) and (22) we obtain the average path as:

$$
\begin{equation*}
\bar{d}=\int_{0}^{\frac{\pi}{2}} \bar{d}(\alpha) p_{a}(\alpha) d \alpha=\pi r\left(2-\frac{l \sqrt{4-l^{2}}}{8 \operatorname{ArcSec}(2 / l)}\right) \tag{23}
\end{equation*}
$$


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