Node Density Influence on Link Expiration Time in Ad Hoc Networks

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Abstract — **Because reactive routing protocols act differently on the presence of nodes in the transmission range proximity, the link expiration time depends on the node density in the ad hoc network. We present a model for calculation of the link expiration times that includes the node density influence.**

Keywords — **ad hoc networks, link expiration time, node density, path distribution.**

I. Introduction

MOBILE wireless ad hoc network is an infrastructure free collection of wireless mobile nodes dynamically forming a temporary network without the use of any centralized administration. It is suitable for situations where infrastructure is either not available, not trusted, or should not be relied on in times of emergency.

In existing ad hoc reactive routing protocols, a sending node utilizes the discovered route until it expires or is broken. A problem occurs when a route gets disrupted due to host mobility or poor signal strength. Whenever a node finds that its link to the next hop is broken, it will send a route error packet back to the source node, which will then invoke another route discovery procedure. However, this is costly as route discovery procedures activate a network flooding. Route consisting of multiple hops most frequently breaks because of the failure of a single link caused by the relative movement of only one node. This results in the wastage of scarce wireless bandwidth as well as long delays.

To solve this problem the local route repair (LRR) is used to patch a broken route between the two nodes of the path through some other node placed in the intersection region of the two nodes. The zone in which the route-repair packet propagates is limited, most frequently to two hops as in [1]. The ad hoc network performances can be enhanced if the route maintenance routine of the routing algorithm can create disjoint bypass routes in advance based on the link expiration time (LET).

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To address this problem in [2] statistical estimation of LET in mobile ad hoc networks is reviewed. The authors predict the probability that a link between two nodes exists at time t_2, in case the link existed at the starting time t_0. They use the random waypoint model as a basic movement model. No forecasts about the duration of uninterrupted link can be made, since the link may cease to exists at time t_1 with $t_0 < t_1 < t_2$. In [3] a statistical derivation to forecast the average distance when the routing node is within the scope of the two other nodes is given. With these statistical calculations, the paper investigates the possibility of predictions of the average link expiration times and deviations for different node velocities, independent from the node's radio transmission range and the distances between each other. Sadagopan at al. [4] examine the varying of the statistics of path durations including PDFs with parameters such as mobility model, relative speed, number of hops and radio range by the means of simulations. They suggest that at moderate and high velocities the exponential distribution with appropriate parameterizations is a good approximation of the path duration distribution for a range of mobility models.

In this paper based on the routing nodes path distribution a link reliability model for two hop ad hoc networks is presented. Using this model we can calculate the link mean time to failure (MTTF) which represents the link expiration time (LET). By the means of simulations using the NS-2 simulator [9] we discovered that the number of nodes in the network (node density) has a large influence on LET. This is due to the different reaction of the reactive routing protocols on the presence of nodes in the transmission range proximity. For an example, the LET has a relative change of 16% when the node density in an ad hoc network changes from 100 nodes/km^2 to 25 nodes/km^2. Hence, we developed an improved model for LET calculation that applies to any node density in the network.

II. Ad-hoc Network Model Description

Since real world radio networks are influenced by many factors like irregular terrain, asymmetric radio transmission, and radio interference, we made a few assumptions in order to give a simplified, but reasonable model. We assume that the terrain is perfectly flat while all the mobile nodes have the same fixed transmission power and are equipped with omni directional antenna, thus having equal transmission range *r*. This assumption turns the node radio coverage shape into a circle with radius *r*.

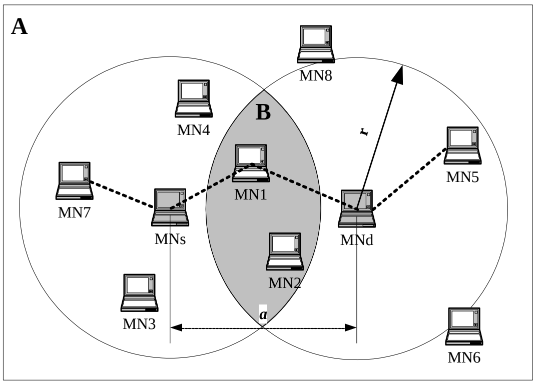

Fig. 1. Ad hoc network model.

Because the local route repair technique is most frequently limited to a two hop zone, we study a two hop connection in an ad hoc network. The network consists of N+2 nodes placed in area A. Two of the nodes are part of a multihop connection one on the source side MNs and one on the destination side MNd, while the rest, N nodes, can enter the intersection region between the two-hop neighbors, and therefore play the part of possible routers. In order to establish a communication between the *two-hop neighbor nodes*, MNs and MNd, (a is the distance between MNs and MNd $r<a<2r$), the communication path has to go through one of the nodes (MN1, MN2) that are currently located in the intersection area B between MNs and MNd (see Fig. 1). While moving around in A, a node can enter the B area and, after a certain period of time, leave B and enter area C defined as A-B. This process is continuously repeated.

III. Routing Nodes Path Distribution

The MN movement is described by a given Mobility Model (MM). There are several MM that are used in performance evaluation simulations for ad hoc networks [6]. One of the most commonly used is Random Walk. In the Random Walk MM (also known as Random Direction MM), a MN moves from its current location to a new location by randomly choosing travel direction and speed. The new speed and direction are both chosen from pre-defined ranges, [speedmin; speedmax] and [0;2π] respectively. Each movement in the Random Walk Mobility Model occurs in either a constant time interval or a constant distance traveled, at the end of which a new direction and speed are calculated. If a MN, which moves according to this model, reaches the simulation boundary, it “bounces” off the simulation border with an angle determined by the incoming direction. The MN then continues along this new path.

In order to get the probability distribution of times that MN spent into intersection region we made series of simulations with TopoSim [7]. TopoSim is a wireless ad hoc network topology simulator created for the purpose of simulating purely topological characteristics of an ad hoc network, while incorporating the dynamical changes caused by the node’s mobility. The simulations scenarios consist of 100 mobile nodes uniformly scattered in 1000m x 1000m area. There are two additional static nodes that represent the source and the destination placed on a given distance. The mobile nodes are moving according to the Random Walk MM with an average speed from 0.5m/s to 5m/s for the different series of simulations. The node transmission radius is set to 250m. During the simulations we measure the path length and the time spent in the intersection region for every mobile node.

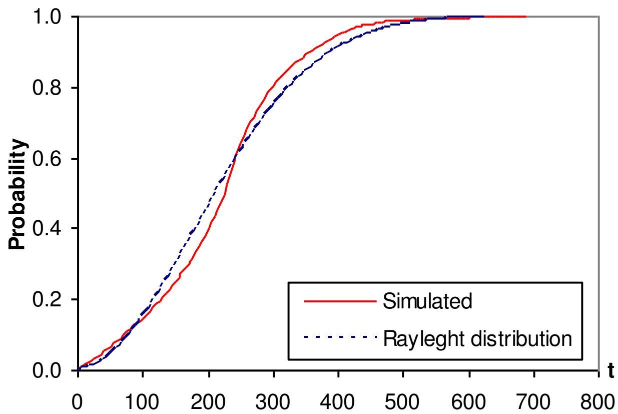

Fig. 2. Link duration time distribution.

The probability distribution of the time that a MN spends in the intersection region is shown on Fig. 2. On the same figure the Rayleight distribution with the same average value is shown. It can be seen that these two distributions are very similar. Thus, in order to obtain a closed form solution, we substituted this distribution with a Rayleight distribution with the same average value.

The Rayleight distribution is given by

$$f(t) = Kte^{\frac{Kt^2}{2}} \tag{1}$$

with average value

$$\bar{t} = \sqrt{\frac{\pi}{2K}} \tag{2}$$

The parameter K depending on the average value is

$$K = \frac{\pi}{2\bar{t}^2} \tag{3}$$

In order to calculate the parameter K we need the average time that a MN spends in the intersection region. This time is given by [5]:

$$\bar{t}_B = \frac{\ln(\bar{v}+\sigma\sqrt{3})-\ln(\bar{v}-\sigma\sqrt{3})}{2\sigma\sqrt{3}}\bar{d} \tag{4}$$

Where $\bar{v}$ is the average speed, σ is the speed’s standard deviation, and $\bar{d}$ is the average value of the eye path, which is given by [5]:

$$\bar{d} = \pi r\left(2-\frac{a\sqrt{4-a^2}}{8\,ArcSec(2/a)}\right) \tag{5}$$

where r is the transmission range and a is the distance between the nodes.

When the routing node leaves the intersection region, the routing protocol reestablishes the connection by choosing a new routing node from the available nodes in the intersection region. Each node spends a given amount of time in the intersection region after which it leaves this region. The probability that a node will be chosen as a new routing node for the connection is proportional to the amount of time the node spends in the intersection region. Accordingly, the probability that a node will be chosen as a new routing node is higher if the path the node travels inside the intersection region is longer.

Hence, the probability that the path length a routing node travels inside the intersection region equals to d is proportional to the length of the traveled path and to the probability that the node will travel on the given path:

$$P(D=d)=\frac{d}{\int_0^\infty x f_R(x)dx} f_R(d) \quad (6)$$

Equation (6) represents the probability density $f_D(d)$ for the overall path lengths that routing nodes pass inside the intersection region. Solving the expression, we get:

$$f_D(d)=d^2 e^{-\frac{Kd^2}{2}} K^{2/3}\sqrt{\frac{2}{\pi}} \quad (7)$$

Accordingly, the probability distribution function is:

$$F_D(d)=Erf\left(\frac{\sqrt{K}}{\sqrt{2}}d\right)-d\,e^{-\frac{Kd^2}{2}}\sqrt{K}\sqrt{\frac{2}{\pi}} \quad (8)$$

where $Erf(z)=\frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt$ is the error function.

The average path that an already chosen routing node will travel in the intersection region is:

$$\overline{d}=\int_0^\infty x\, f_D(x)dx=2\frac{\sqrt{2/\pi}}{\sqrt{K}} \quad (9)$$

In the moment of reestablishing the connection the newly chosen routing node can be anywhere along the path it chose to travel through the intersection region. Therefore, since all of the points inside the intersection region are equally distributed, the exact position of the node is irrelevant.

Let L be a random variable that represents the path length that a given node passes as a router node. Let the full path length that the node passes inside the intersection region be a random variable D with probability density given with expression (7).

In order to have the path length traveled by the router node larger than a given value x, the full path d has to be larger than x, to be precise $P(D>x)=1-F_D(x)$.

If this is the case, than for D=d, the probability that the random variable L will be larger that x is:

$$P(L>x)\,|\,(D=d)=\frac{d-x}{d}P(D>x) \quad (10)$$

Hence, the unconditional probability that the random variable L will be larger than a given value x, is:

$$P(L>x)=\int_x^\infty P(D>x)\frac{d-x}{d}\, f_D(d)dd$$
$$=\left(1-F_D(x)\right)\left(1-F_D(x)-\int_x^\infty \frac{x}{d}\, f_D(d)dd\right) \quad (11)$$

Since $P(L>x)=1-F_L(x)$, we can obtain the probability distribution of the random variable L as:

$$F_L(x)=2F_D(x)+\int_x^\infty \frac{x}{d}\, f_D(d)dd-F_D(x)^2-F_D(x)\int_x^\infty \frac{x}{d}\, f_D(d)dd \quad (12)$$

Solving the above expression, we get:

$$F_L(x)=1-\left(1+x e^{-\frac{Kx^2}{2}}\sqrt{K}\sqrt{\frac{2}{\pi}}-Erf\left(\frac{\sqrt{K}}{\sqrt{2}}x\right)\right)Erfc\left(\frac{\sqrt{K}}{\sqrt{2}}x\right) \quad (13)$$

where $Erfc(z)=1-Erf(z)$ is the complementary error function. Hence, the probability density function for the random variable L is:

$$f_L(x)=\frac{dF_L(x)}{dx}=\frac{1}{\pi}\left(2Kxe^{-Kx^2}+\sqrt{K}\sqrt{2\pi}(1+Kx^2)Erfc(\frac{\sqrt{K}}{\sqrt{2}}x)\right) \quad (14)$$

If the node travels along a straight path with a uniform speed v, the length of the path it is going to pass before it leaves the intersection region is l and the time spent like a router node will be $t=h(l)=\frac{1}{v}l$. Hence, the time T during which the node is a routing node is also a random variable and its probability density is [8]:

$$f_T(t)=v\, f_L(vt)$$

$$f_T(t)=\frac{v}{\pi}\left(2Kvt\,e^{-Kv^2t^2}+\sqrt{K}\sqrt{2\pi}(1+Kv^2t^2)Erfc(\frac{\sqrt{K}}{\sqrt{2}}vt)\right) \quad (15)$$

The random variable T actually represents the expiration time of the connection, since the connection is established when the node is chosen as a router and lasts until the node leaves the intersection region, after which a new node is needed.

The reliability function $R(t)$ represents the probability that the system will work successfully (with no errors) in the time interval from 0 to t. In our case, the reliability is expressed as the probability that there will be a connection in the time interval from 0 to t, which means that the time of utilization of the connection has to be grater than t. Accordingly, the connection reliability will be:

$$R(t)=P(T>t)=1-F_T(t) \quad (16)$$

The mean time to failure (MTTF), equal to LET, is defined as the expected time of system operation before an error occurs:

$$LET=MTTF=\int_0^\infty t\, f_T(t)dt=\int_0^\infty R(t)dt=\frac{3(\sqrt{2}-1)}{v\sqrt{K}\sqrt{\pi}} \quad (17)$$

IV. Improved LET Model

When there are no nodes in the intersection region the routing protocol uses an implementation dependent back-off algorithm. After the first try for route reestablishment there is a pause before the second route request is issued. If the second route request also fails than a bigger pause is made after which a third request follows. The pauses between consequential requests grow larger up to some point after which the process is repeated. This means that the nodes that would enter the empty intersection region would be discovered faster, and their path lengths would be distributed according to (7) and not to (15). In order to incorporate this behavior and get a correct result for ad hoc networks with variable node density, both of the results need to be taken into account.

According to the previous analysis, a weighted function of the average values for the random variables D and L needs to be used:

$$\overline{l_k} = p\bar{d} + (1-p)\bar{l} \qquad (18)$$

where p is the probability that there are no nodes in the intersection region. This probability depends on the number of nodes in the ad hoc network N, the size of the area wherein the nodes are scattered A and the size of the intersection region B:

$$p = (1 - B/A)^N \qquad (19)$$

The size of the intersection region is given by:

$$B = r^2\left(2ArcCos\left(\frac{a}{2r}\right) - \frac{a}{r}\sqrt{1-\frac{a^2}{4r^2}}\right) \qquad (20)$$

where r is the node transmission range and a is the distance between the source and destination node.

For the presumed straightforward movement the average corrected time which the node spends as a routing node is $\overline{t_k} = \frac{\overline{l_k}}{v}$, which also represents the value for the LET:

$$LET = MTTF = \frac{\overline{l_k}}{v} \qquad (21)$$

In order to verify the proposed model a series of simulation using the NS-2 simulator have been made. The simulation scenarios consist of 25, 50 and 100 uniformly distributed mobile nodes in a square area of 1000m x 1000m. In each scenario there are two additional nodes that represent the source and the destination for the connection. The nodes can only communicate by the means of a third routing node that has to be located in the intersection region of the transmission coverage areas for the two nodes. The nodes are moving according to the random walk mobility model with an average speed of 1m/s. During the simulations we measured the time a given link exists and the starting position of the node that is going to be used as a router.

TABLE 1: AVERAGE CONNECTION TIME COMPARISON

Nodes	NS-2 simulations	Calculated LET	Relative error
100	129.7 s	128.9 s	1%
55	132.3 s	130.9 s	1%
25	151.2 s	151.2 s	0%

In Table 1 a comparison of the average connection time obtained from the simulations and calculated with the proposed model is shown. It can be seen that the relative error is very small in the range of 1% which confirms the validity of the model.

V. ANALYSIS

In order to capture the influence of the number of nodes on the LET an analysis that is given on Fig. 3 has been made. The example ad hoc network consists of 100 nodes scattered in an area A=1,000,000m^2. It can be seen that the larger transmission range leads to better LET. It also can be seen that the increasing number of nodes leads to poor LET. However this can not be interpreted in a manner that an ad hoc network with a smaller number of nodes is more resilient to errors. In that case, the probability that there will be no nodes in the intersection region is relatively large, thus making the periods of no connection larger. This situation leads to a decreased connection availability which is a lot smaller when compared to ad hoc networks with a larger number of nodes.

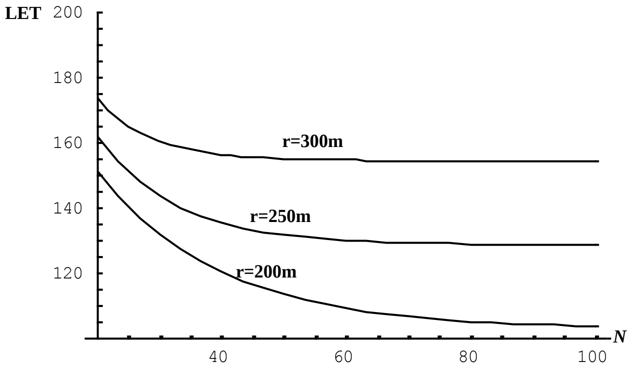

Fig. 3 LET depending on the number of nodes in the network for different transmission range values

VI. CONCLUSION

The proposed LET model can be used to make ad hoc networks analysis.

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