Transactions on Ecology and the Environment vol 25, © 1998 WIT Press, www.witpress.com, ISSN 1743-3541

Mathematical model for the available phosphorus, phytoplankton and zooplankton in the Lake Ohrid

K. Mitreski[1], Z. Koneski[1], D. Davcev[1] and T. Naumoski[2]
[1]Faculty of Electrical Engineering, Karpos II, 91000 Skopje, Macedonia
[2]Hydrobiological Insitute of Ohrid, Naum Ohridski 50, 96000 Ohrid, Macedonia

Abstract

In the paper by Mitreski et al.[1] a dynamic biogeochemical model has been proposed, which tends to describe some of the processes that contribute to the eutrophication of Lake Ohrid. By balancing the complexity between the scope of the model and the available data, we selected state variables and formulated mathematical expressions. With three differential equations, we took into account the change of available phosphorus, as well as time dependent behaviour of phytoplankton and zooplankton concentrations in the lake. By considering both model output and lake-wide observations, the suitability of the model was analysed.

The main goal of this paper is to give experimental validation of the proposed mathematical model. Four groups of phytoplankton and seven zooplankton types were considered in the analysis. The simulated results from the model are validated with the measured values from the lake. We concluded that the model shows the general trends and considers the probabilistic nature of the lake ecosystem.

Transactions on Ecology and the Environment vol 25, © 1998 WIT Press, www.witpress.com, ISSN 1743-3541

1 Introduction

The development of aquatic ecological models can be divided into empirical, dynamic and mixed models.

Empirical models are constructed on the basis of the relationship between different parameters, for example the depth, the rate of phosphorus input, etc., e.g. Vollenweider[2], OECD[3] and Davcev et al.[4].

Dynamic models, derived from a causal analysis of ecological and biological fluxes, are based on calculations using differential equations. Some of these models tend to give an overall picture of the epilimnion, hypolimnion and sediment, by using realistic process equations, e.g. Scavia[5], Janse et al.[6] and Jorgensen[7]. Dynamic models are often difficult to calibrate and validate.

Mixed models combine some advantages of the previously described models in the context of predictive modelling. The most frequently used technique of these models is regression analysis between two or more important parameters for a specific water body, e.g. Hakanson et al.[8].

Our work is related to the ecological state of Lake Ohrid, which represents rare natural ecosystem inhabited by many endemic and relict species. For its outstanding natural surroundings, Lake Ohrid has been placed on the UNESCO world natural heritage list. The lake faces accelerated deterioration of its waters and a change of the trophic state of this aquatic ecosystem, mainly caused by nutrient load. “Man-made” eutrophication, in the absence of control measures, proceeds much faster than the natural phenomenon and is the major reason for pollution of this lake, e.g. World Bank[9].

We proposed in the paper by Mitreski et al.[1] a mathematical model that gives general picture of the level of eutrophication in the Lake Ohrid and shows general trends of the lake behaviour. Our model has been inspired by some of the ecological models proposed by Scavia[5] and Janse et al.[6], and by the analyses conducted on Lake Ohrid. The specific nature of the lake has been taken into consideration and embodied in the model. As suggested in World Bank[9], the nutrient cycling in the sediment is not included. Such dynamic model, which tends to predict future eutrophication and the trophic state lake-wide, is the first attempt undertaken in describing the ecological state of Lake Ohrid.

This paper focuses on the experimeantal validation of the mathematical model over several phytoplankton and zooplankton groups. The section 2 gives a brief overview of the mathematical equations that comprise the ecological model for the Lake Ohrid. In section 3, the

Transactions on Ecology and the Environment vol 25, © 1998 WIT Press, www.witpress.com, ISSN 1743-3541

simulated results from the model are presented. They are validated by the measured values from the lake. Finally, section 4 concludes the paper.

2 The Mathematical Model

On the basis of the biological and chemical processes in the lake, the mathematical model is comprised out of three equations describing the behaviour of the available phosphorus, phytoplankton and zooplankton. Model equations are given below. Mitreski et al.[1] describe the conceptual and mathematical models in more details.

$$\frac{dAvlP}{dt} = \frac{P}{C}\left(D - P_r + R\right) \tag{1}$$

$$\frac{dZ}{dt} = \sum_n C_n a_j - R \tag{2}$$

$$\frac{dPh}{dt} = P_r - R - \sum_j C_j \tag{3}$$

where

AvlP is available phosphorus
P/C is phosphorus to carbon ratio in plankton
D is decomposition
Pr is production
R is respiration
Ph is phytoplankton
Cj is zooplankton consumption
Z is zooplankton
a_j is feeding efficiency of predator j

and where summation indices j = all zooplankton and n = all food.

3 Experimental results

In this section, the experimental validation of the results obtained by the mathematical model will be presented for several phytoplankton and zooplankton groups. The validation of the available phosphorus is given in the paper by Mitreski et al.[1].

Transactions on Ecology and the Environment vol 25, © 1998 WIT Press, www.witpress.com, ISSN 1743-3541

The measured and model output values have been compared for four phytoplankton groups. Each phytoplankton group is given in percent with respect to all four considered groups.

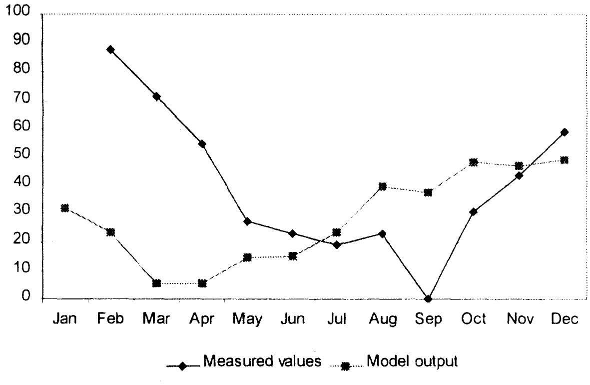

Figure 1: Seasonal dynamics of measured values and model output for Bacillariophyta (given in %)

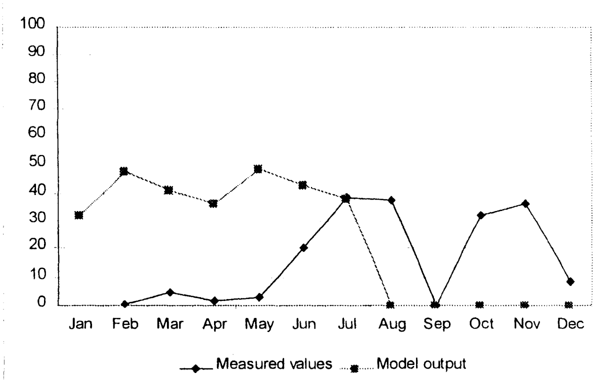

Figure 2: Seasonal dynamics of measured values and model output for Chrysophyta (given in %)

On figures 1 to 4 both measured values and simulated output from the model are given for the phytoplankton groups. As it can be seen from the figures, the simulated curves from the model generally follow the measured values from the lake.

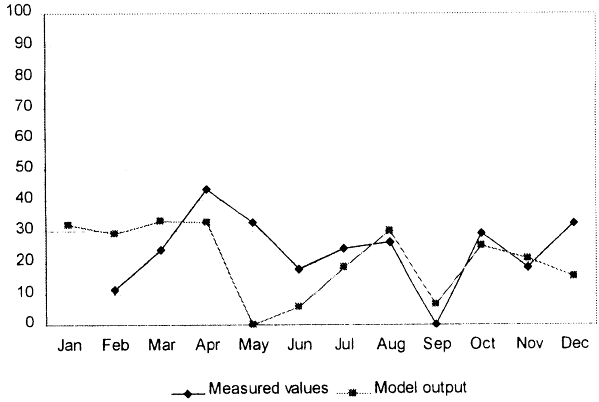

Figure 3: Seasonal dynamics of measured values and model output for Chlorophyta (given in %)

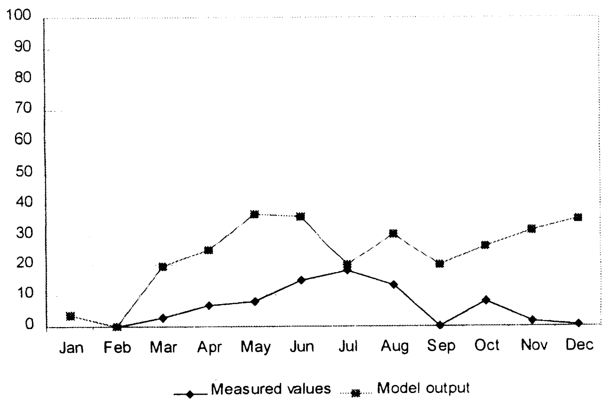

Figure 4: Seasonal dynamics of measured values and model output for Pirrophyta (given in %)

Following figures presents both measured values and simulated output from the model for zooplankton groups Cladocera and Copepoda. Each zooplankton group is given in percent with respect to both considered groups. The simulated curves from the model generally tend to follow the zooplankton dynamics.

Transactions on Ecology and the Environment vol 25, © 1998 WIT Press, www.witpress.com, ISSN 1743-3541

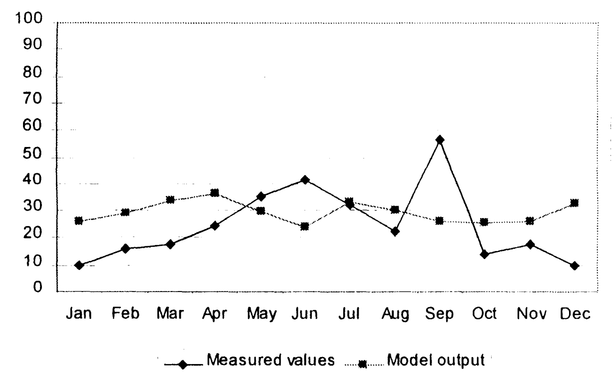

Figure 5: Seasonal dynamics of measured values and model output for Cladocera (given in %)

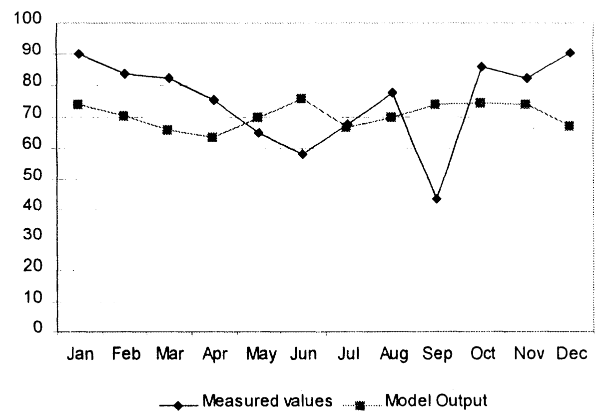

Figure 6: Seasonal dynamics of measured values and model output for Copepoda (given in %)

4 Conclusion

In the paper, we validated the dynamic biogeochemical model of the Lake Ohrid, using four phytoplankton and two zooplankton groups. In this way it was possible to validate the model with more experimental data. We concluded that the proposed mathematical model agrees with most of the simulated cases and gives a real picture of the lake trophic properties.

We plan to improve the model using other phytoplankton and zooplankton groups and involve other relevant parameters into the model.

References

[1] Mitreski, K., Koneski, Z., Davcev, D., Naumoski, T., Mitic, V., Sumka, S. & Guseska, D., An ecological model of Lake Ohrid, *Proceedings of the IASTED Conference, AMS'98*, Honolulu, August, 1998.

[2] Vollenweider, R.A., Input-output models, *Swiss journal for hydrology*, 37, pp. 53-84, 1975.

[3] OECD, *Eutrophication of waters. Monitoring, assessment and control*, OECD Report, Paris, 1982.

[4] Davcev, D., Mitreski, K., Boskovska, V. & Koneski, Z., Management and analysis of the phosphorus data in the lake Ohrid by multimedia presentation, *Proceedings of the IASTED International Conference, Advanced Technology in the Environmental Field*, Editor: M.H.Hamza, Gold Coast, pp. 178-181, 1996.

[5] Scavia, D., An ecological model of Lake Ontario, *Ecological Modelling*, 8, pp. 49-78, 1980.

[6] Janse, J.H., Aldenberg, T. & Kramer, P.R.G., A mathematical model of the phosphorus cycle in lake Loosdrecht and simulation of additional measures, *Hydrobiologia*, 233, pp. 119-136, 1992.

[7] Jorgensen, S.E., *Fundamentals of ecological modelling (2nd Edition), Developments in environmental modelling, 19*, Elsevier, Amsterdam, 1994.

[8] Hakanson, L. & Peters, R.H., *Predictive limnology, methods for predictive modelling*, SPB Academic Publishing, Amsterdam, 1995.

[9] World Bank, *Feasibility study on the lake Ohrid conservation project*, Erhst Basler & Partners, Zollikon, 1995.