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MEMRISTIVE NETWORKS OF CHUA'S CIRCUITS

Miroslav Mirchev, Igor Mishkovski, and Ljupco Kocarev *†

Abstract. Although envisioned in 1971 by L. Chua, memristors have attracted the attention of the research community recently by the promotion of their feasibility and a vast number of possible applications in non-volatile computer memory, pattern recognition and modelling neural networks. Synchronization is widely studied as a phenomenon in neural networks. This work provides synchronization analyses of two kinds of memristive networks of oscillators. First, we numerically examine networks of Chua's circuits coupled by memristors that adapt according to the local state disagreements. As second, we employ the Master stability function (MSF) approach to study synchronization in networks of memristive Chua's circuits coupled through simple resistors.

Keywords. memristors, nonlinear oscillators, synchronization, stability, Chua's circuit.

1 Introduction

The three basic passive electrical elements, the inductor, the capacitor and the resistor, are well known long time ago and have been thoroughly studied in the literature. In a paper [1] from 1971, Chua logically predicted the existence of a fourth basic element named 'memristor' that relates flux with charge, which was a missing link at that time. Basically, the resistiveness of the memristor is dependent on the history of the voltage across it or the current that has flown through it. Recently, experimental analysis in the laboratories at HP have revealed that such materials with memristive properties actually exist [2].

Even though memristors are still mostly used in research, they could have technological impacts. The possible applications include non-volatile memory, low power circuits, analog computation, circuits mimicking biological systems, programmable logic, and many more.

The electrical elements are often used to design oscillating circuits with certain characteristics either mimicking some natural phenomena or implementing some desired function. One widely studied phenomenon is chaotic behavior and the simplest electronic circuit with such properties is the Chua's circuit [3]. This circuit exhibits different types of behaviors and it can be used for studying various problems including synchronization of electronic oscillators [4]. In a recent paper [5], we studied synchronization in network of Chua's oscillators with and without parameter disturbances using the Master Stability Function (MSF) approach. Several memristive oscillators have been given in [6], derived from the Chua's circuit, while some dynamical behaviors of memristive oscillatory networks have been studied in [7].

In this work we numerically study how memristive coupling affects synchronization of networks of Chua's circuits. Furthermore, we show that this kind of coupling could be used to find the appropriate weights between the Chua's circuits in order to achieve synchronization. The second approach, used in this work, using the MSF approach examines the local stability of resistively coupled memristive Chua's circuit.

In Section 2, we present the Chua's circuit and the basic concept of memristors. Synchronization in networks of Chua's circuits coupled through memristive interactions is numerically studied in Section 3. The local stability of network of memristive Chua's circuits is investigated in Section 4, while concluding words are given in Section 5.

2 Preliminaries on the Chua's circuit and memristors

2.1 The Chua's circuit

The dynamics of the Chua's circuit, shown in Fig. 1, is defined as [3]:

$$\begin{aligned} \frac{dv_1}{dt} &= \frac{1}{C_1}\left(\frac{v_2 - v_1}{R} - f(v_1)\right) \\ \frac{dv_2}{dt} &= \frac{1}{C_2}\left(\frac{v_1 - v_2}{R} - i_3\right) \\ \frac{di_3}{dt} &= \frac{v_2}{L} \end{aligned} \tag{1}$$

where the characteristic of the nonlinear resistor is

$$f(v_1) = G_b v_1 + 0.5(G_a - G_b)(|v_1 + E| - |v_1 - E|) \tag{2}$$

The voltage E is used to switch between the two slopes G_a and G_b.

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It is often more convenient to work with dimensionless form of the circuit, which after a set of transformations can be rewritten as [3, 8]

$$\begin{aligned}\dot{x} &= \alpha[y - x - h(x)]\\ \dot{y} &= x - y + z\\ \dot{z} &= -\beta y\end{aligned} \tag{3}$$

where

$$h(x) = m_b x + 0.5(m_a - m_b)(|x+1| - |x-1|), \tag{4}$$

$\mathbf{x} = [x, y, z] = (1/E)[v_1, v_2, i_3 R]$ is a state vector, $\alpha = C_2/C_1$, $\beta = C_2 R^2/L$, $m_a = RG_a$, $m_b = RG_b$ and $t = \tau|RC_2|$.

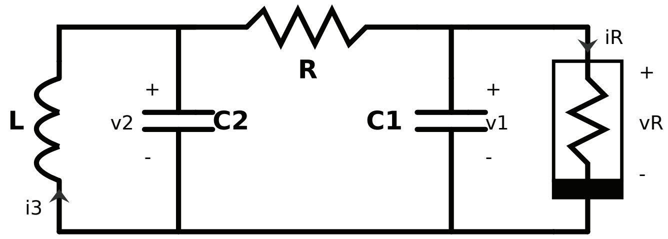

Figure 1: The Chua's circuit

The Chua's circuit can exhibit various types of behavior for different parameter sets, from chaotic attractors to limit cycles.

A network of N symmetrically coupled Chua's circuits can be constructed where the network dynamics can be described using the network's Laplacian matrix $\mathbf{L} = [L_{ij}]_{N\times N}$

$$\dot{\mathbf{x}}_i = F(\mathbf{x}_i) - g\sum_{j=1}^{N} L_{ij} H(\mathbf{x}_j), \tag{5}$$

$F(\cdot)$ is a function defining the dynamics of a single oscillator, $g > 0$ is coupling strength, $H : R^D \to R^D$ is a coupling function and $D = 3$ is the dimension of the oscillators. Standardly, a network Laplacian matrix is denoted as $\mathbf{L} = \mathbf{D} - \mathbf{A}$, with $\mathbf{D}$ being a diagonal node degree matrix and $\mathbf{A}$ being the nodes adjacency matrix.

2.2 Memristors

As in [1], a memristor is typically represented as in Fig. 2 and its dynamics is described as charge controlled

$$v = M(q)i, \tag{6}$$

or flux controlled

$$i = W(\varphi)v, \tag{7}$$

where v is the voltage across the memristor, i is the current through the memristor, q is charge, φ is flux linkage, M is its memristance and W its memductance.

A typical characteristic of memristors is the pinched hysteresis form of their voltage-current curve. As Chua initially described "if it's pinched it is a memristor" and later he expanded this statement to consider an element as memristor if and only if it has a pinched voltage-current curve. The memristor's current-voltage curve is frequency dependent and as the frequency ω increases the pinched hysteresis gets narrower [9], and as $\omega \to \infty$ the curve becomes straight line and the memristor shows properties as a regular resistor. One initial memristor implementation proposed by Chua relied on active circuits [1], while recently memristors have been constructed using nanotechnology, as for example the memrsistor from HP [2].

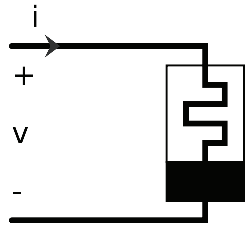

Figure 2: Memristor symbol proposed in [1]

One definition of the memductance, given in [6], is

$$W_{PWL}(\varphi) = \begin{cases} r, & |\varphi| \le 1\\ s, & |\varphi| > 1\end{cases} \tag{8}$$

where

$$\frac{d\varphi}{dt} = v \tag{9}$$

is the dynamics of the internal memristor state and v is the voltage across the memristor.

Another definition given in [10] with a cubic function

$$q(\varphi) = l\varphi + m\varphi^3 \tag{10}$$

provides a memductance function

$$W_Q(\varphi) = \frac{dq}{d\varphi} = l + 3m\varphi^2 \tag{11}$$

where again $d\varphi/dt$ changes as in Eq. (9).

3 Networks of Chua's circuits with memristive interactions

In this section we study synchronization in networks of Chua's oscillators coupled through the first state variable using memristors. The dynamics of each oscillators is given as

$$\dot{\mathbf{x}}_i = F(\mathbf{x}_i) + g\sum_{j\neq i} W_{PWL}(\varphi_{ij})(x_j - x_i), \tag{12}$$

where each memristor state φ_{ij} develops as

$$\frac{d\varphi_{ij}}{dt} = x_i - x_j, \tag{13}$$

and the memductance function $W_{PWL}(\varphi_{ij})$ follows Eq. (8) with $r = 0.1$ and $s = 1$.

Networks with memristive interactions are in a certain way realization of the concept of local adaptive coupling, studied in [11, 12]. In those papers the authors show asymptotic stability of the synchronized state with examples like synchronization in network of Chua's circuits and the network consensus problem.

In this section we simulate networks of four completely connected oscillators with the following two types of behavior

- Periodic Limit Cycle (LC) obtained with $\alpha = 10.2$, $\beta = 21.89$, $m_a = -1.5$, $m_b = -0.81$, and
- Chaotic Double Scroll (DS) obtained with $\alpha = 10.2$, $\beta = 20.27$, $m_a = -1.44$, $m_b = -0.78$.

If the coupling strength g is large enough the network synchronizes and in order to measure the level of synchronization the following error function is used

$$\langle e \rangle = \frac{1}{t_f - t_t} \int_{t_t}^{t_f} \sum_{k=1}^{D} \mathrm{Var}(\mathbf{x}^{(k)}(t))dt, \tag{14}$$

where $\mathbf{x}^{(k)}(t) = [x_i^{(k)}(t), \ldots, x_N^{(k)}(t)]$, with $x_i^{(k)}(t)$ denoting the k-th state variable of the i-th oscillator, the variance function $\mathrm{Var}(\cdot)$ indicates how much dispersed are the node's states, t_t is a time moment after the transient period and t_f is a sufficiently distant moment that provides accurate calculation of the error function.

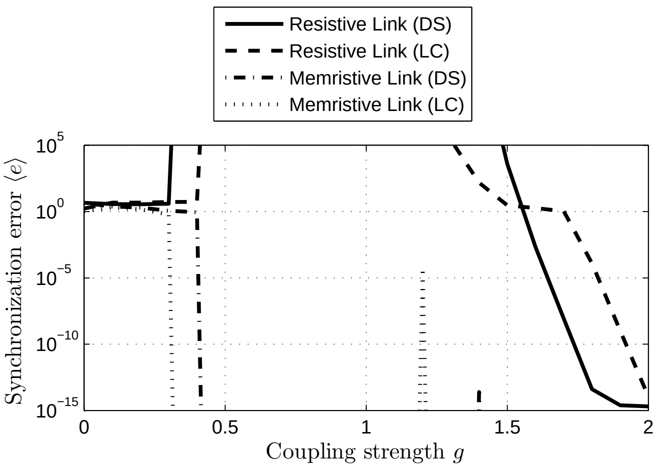

Figure 3: The synchronization error in networks of Chua's circuit with resistive and memristive coupling exhibiting double scroll (DS) and limit cycle (LC) for different coupling strength (g).

It can be seen in Fig. 3 that synchronization occurs in networks with memristive interactions for significantly lower coupling strength. As the maximum possible memductance is $s = 1$ it can be concluded that synchronization is improved only by the adaptability of the interactions, not by increasing the coupling strength. It should be noted that the network synchronization in networks with memristive interactions can result in lower final effective conductance of the links in the synchronized network. Therefore, to further characterize the synchronization properties of memristive networks we calculate the overall memductance W_o in the network as the sum from the memductance function at the last moment of the simulation (finite time)

$$W_o = \sum_i \sum_{j<i} W_{PWL}(\varphi_{ij}) \tag{15}$$

and in Fig. 4 it can be seen that the W_o required for synchronization decreases with the increase of the coupling strength g. In the presented numerical results the required W_o is slightly lower for the limit cycle behavior compared to double scroll.

The analysis of the change of W_o for different coupling strength revealed that for very small coupling strength all the memductances remain high. As the coupling strength increases some of the memductances decrease leading to a kind of star network that is known to have good synchronization properties. Eventually, for large coupling strength all memductances start to take low values. If the memductance function was continuous this transition would be smoother. These observations could be potentially used for designing networks with specified desired synchronization properties.

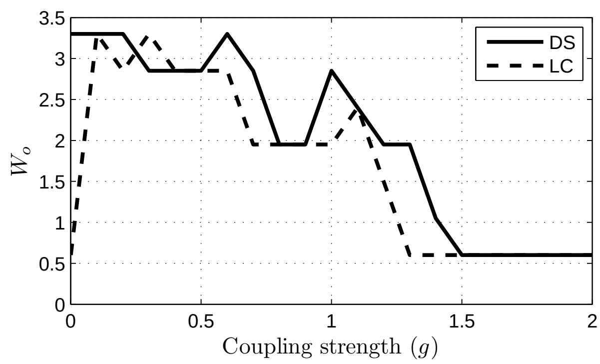

Figure 4: The overall memductance W_o after sufficiently long simulation time in memristively coupled networks of Chua's oscillators exhibiting double scroll (DS) and limit cycle (LC) for different coupling strength (g).

4 Networks of memristive Chua's circuits

In order to build a network of memristive Chua's circuits we have used the Chua circuit from Fig. 1 and we have replaced the Chua diode with a flux-controlled memristor in Fig. 5 [13]. The equations for the memristive version

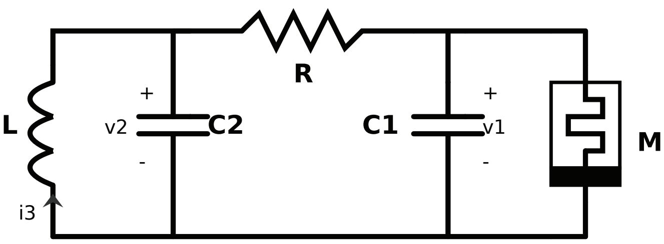

Figure 5: A memristive Chua's circuit

of the Chua circuit, similar to the ones presented in [10], are as follows:

$$\begin{aligned}
\frac{d\varphi}{dt} &= v_1(t) \\
\frac{dv_1(t)}{dt} &= \frac{1}{C_1}\left(\frac{v_2(t)-v_1(t)}{R} - i(t)\right) \\
\frac{dv_2(t)}{dt} &= \frac{1}{C_2}\left(\frac{v_1(t)-v_2(t)}{R} - i_3(t)\right) \qquad (16)\\
\frac{di_3(t)}{dt} &= \frac{v_2(t)}{L}
\end{aligned}$$

where $i = l + 3m\varphi^2$, obtained from Eqs. (7), (9) and the cubic nonlinearity for the $q - \varphi$ function, from Eq. (10). For the circuit parameters we have used: $L = 18mH, C_1 = 6.8nF, C_2 = 68nF$ and to obtain chaotic attractor we have set $R = 2000\Omega$ [14].

By substituting $u = \varphi$, $x = v_1$, $y = v_2$, $z = i_3$ and time-scaling the ODEs for numerical stability with $t = \tau c$ where $c = \sqrt{LC_2}$, the following state variables are obtained for the memristive Chua's circuit [10]:

$$\begin{aligned}
\frac{du}{d\tau} &= cx \\
\frac{dx}{d\tau} &= \frac{c}{C_1}\left(\frac{y-x}{R} - W(u)\cdot x\right) \\
\frac{dy}{d\tau} &= \frac{c}{C_2}\left(\frac{x-y}{R} - z\right) \qquad (17)\\
\frac{dz}{d\tau} &= \frac{cy}{L}
\end{aligned}$$

In addition to this, because of the unrealistic voltage, the author in [10] rescales the set of variable by a rescaling factor $\zeta = 8200\Omega \cdot 47 * 10^{-9}nF$. However, this rescaling was not necessary for our synchronization analysis in this section.

In the following we analyze the synchronization of a network of N identical memristive Chua's circuits coupled symmetrically with resistive elements. For this purpose we are using Eq. (5), where now $F(\cdot)$ is the dynamics of an isolated memristive Chua circuit (see Eq. (16)), the dimension of the oscillators is $D = 4$ and the new state vector is $\mathbf{x} = [u, x, y, z]$.

One systematic approach for estimating local stability of the synchronized state in oscillatory networks was given in [15, 16]. This approach uses the eigenvalues of the network Laplacian matrix to express the dynamics and at the synchronous manifold $\mathbf{x}_1 = \mathbf{x}_2 = \ldots = \mathbf{x}_N = \bar{\mathbf{x}}$, all oscillators evolve according to

$$\dot{\bar{x}} = F(\bar{\mathbf{x}}). \qquad (18)$$

In this approach the variational equations of Eq. (5) are expressed using transversed modes of the form

$$\dot{\delta} = [DF(\bar{\mathbf{x}}) - \sigma DH(\bar{\mathbf{x}})]\delta \qquad (19)$$

where $DF(\bar{\mathbf{x}})$ and $DH(\bar{\mathbf{x}})$ are Jacobians calculated at $\bar{\mathbf{x}}$, having equal values for all modes; $\sigma_n = g\lambda_n, n \in \{2, \ldots, N\}$ are coupling eigenvalues, and $\lambda_1 = 0 < \lambda_2 \leq \ldots \leq \lambda_N$ are eigenvalues of L (all real for a symmetric L).

The master stability function (MSF) $\Lambda(\sigma)$ is the maximum Lyapunov exponent of Eqs. (18) and (19) [15] and it indicates whether the synchronized state is stable (if negative for all transverse modes $\Lambda(g\lambda_n) < 0$) or unstable (if it is nonnegative for any mode).

The linear system, i.e. the Jacobian DF, around the synchronization manifold for the memristor version of the Chua's circuit is as follows:

$$DF = \begin{matrix} 0 & c & 0 & 0 \\ -\frac{c}{C_1}xW'_{PWL}(u) & -\frac{c}{RC_1} - \frac{W_{PWL}(u)}{C1} & \frac{c}{RC_1} & 0 \\ 0 & \frac{c}{RC_2} & -\frac{c}{RC_2} & -\frac{c}{C_2} \\ 0 & 0 & \frac{c}{L1} & 0 \end{matrix}$$

where for the memductance function $W_{PWL}(u)$ we have chosen $l = -0.66710^{-3}$, $m = 0.02910^{-3}$ and $W'_{PWL}(u)$ is the first derivative of the memductance function.

When the memristor Chua's oscillators are coupled only on the x state variable, i.e. $DH = [0000; 0100; 0000; 0000]$ our MSF analysis show that the synchronization manifold is never locally stable no matter how big is the coupling eigenvalue σ, i.e. MSF tends to 0 as $\sigma \to \infty$.

In Fig. 6, we show the MSF for $DH = [1000; 0100; 0000; 0000]$, that is the memristor Chua's circuits are coupled on the w and the x state variable. Using this coupling the synchronization manifold is locally stable only if the coupling eigenvalue, i.e. the product of the smallest non-zero eigenvalue of L and the coupling strength, is larger than 1.25.

Thus, by using simple resistive link between the memristor Chua's circuits the synchronization manifold is never locally stable, this comes from the fact that the memristor is dependent on the history of the voltage across it. However, if the memristances between the circuits are coupled the network synchronization manifold could be stable when $\sigma > 1.25$.

In order to numerically check the MSF approach we simulated in time a network of four fully connected oscillators with network Laplacian matrix

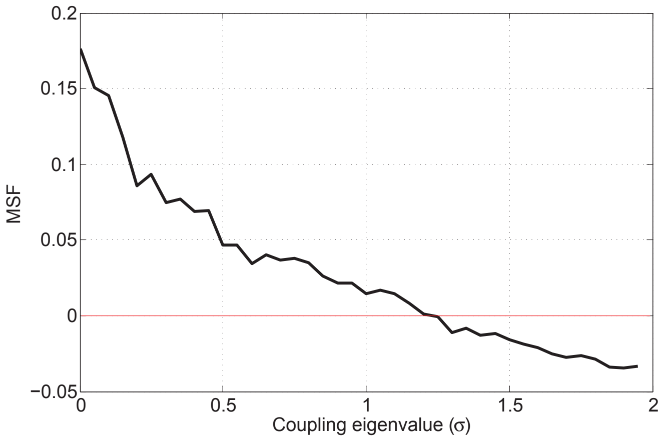

Figure 6: MSF of a network of memristive Chua's circuits exhibiting chaotic behavior coupled on w and x state variables as a function of σ.

$$\mathbf{L} = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & 1 & -3 \end{bmatrix}.$$

The threshold coupling strength at which synchronization occur in the network was well matched with the threshold obtained with MSF.

5 Conclusion

In this paper we have analyzed: i) synchronization in network of Chua's circuits coupled using memristive elements and ii) the local stability of the synchronous state of memristive Chua's circuits coupled with resistors. The numerical simulations for the first analysis showed that in networks with memristive interactions the synchronization occurs for significantly lower coupling values. Moreover, the results showed that the final effective memductance in synchronized networks is quite low. This approach actually provides a network of adaptively coupled Chua's circuits, by using memristive interactions in order to enhance synchronization. Thus, it can be used as a network design method providing optimal network configurations. In the second part, using MSF approach, we showed under which conditions the synchronized state of the network of memristive Chua's circuits with resistive interaction is locally stable.

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