

## MDS-based Algorithm for Nodes Localization in 3D Surface Sensor Networks

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**Abstract**— As Wireless Sensor Network (WSN) has become a key technology for different types of smart environment, nodes localization in WSN has arisen as a very challenging problem in the research community. Most of the applications for WSNs necessitate a priori known nodes positions. In this paper, we propose an algorithm for three dimensional (3D) nodes localization in surface WSN based on multidimensional scaling (MDS) technique. Using extensive simulations, we investigated in details our approach regarding different network topologies, various network parameters and performance issues. The results from simulations show that our algorithm produces small localization error and outperforms MDS-MAP in terms of accuracy.

**Keywords**— wireless sensor networks; multidimensional scaling; 3D surface localization; nodes positioning.

### I. INTRODUCTION

A wireless sensor network (WSN) is a network of autonomous distributed sensor devices that obtain various measurements of different real-life occurrences [1][2]. After taking samples of physical or environmental conditions at different locations (light level, air temperature, humidity, etc.), each sensor sends data to its closest neighbor responsible for retransmitting the packets [3]. The final destination is the sink node responsible for storing data or for further processing. Although initially developed for military applications, today, WSNs are used in many industrial and civilian application areas, habitat monitoring, healthcare applications and traffic control [4].

Nodes localization is the basis for many applications of WSN, such as event detection and target tracking. A manual disposition is impossible not only for large scale WSNs, but also when a WSN is deployed on inaccessible terrain. The most straightforward solution to the localization problem is to apply Global Positioning System (GPS) receivers to each node [5]. But it is an expensive solution and inapplicable for indoor environments [6][7].

Finding out accurate positions of the WSN nodes without GPS support has been studied for many years. Many different techniques [6][7] have been proposed for solving this problem, but most of them consider only two-dimensional (2D) network. In this paper, we investigate multidimensional scaling technique [8] for nodes localization in three dimensional surface WSNs. We also propose a heuristic approach in distance matrix calculation that improves the accuracy compared with well known MDS-

MAP [9]. Henceforth, we will refer to our approach as Improved Multidimensional Scaling Algorithm (IMDS).

The rest of this paper is organized as follows. The second section refers to the multidimensional scaling technique for nodes localization in 3D-WSN. The third section gives a detailed explanation of our IMDS algorithm. Section four presents the results provided from our simulations. Finally, we conclude this paper in section five.

### II. THREE DIMENSIONAL MDS

Multidimensional scaling (MDS) is a set of analytical techniques that has been used for reducing the dimensionality of the data (objects), showing multidimensional data as points in 2D or 3D space [8]. MDS algorithm uses the distances between each pair of object as input and generates 2D-points or 3D-points as output. The input required by MDS should be presented as distance matrix, representing the distances between the objects that should be analyzed. The purpose of this method is to visualize dissimilarity data in order to better understand and comprehend it.

MDS can be easily translated into WSN domain if the sensor network and distances between neighboring nodes are represented as a graph with its edges respectively. In WSNs, MDS performs as centralized, range-based localization algorithm. Distance measurements between each pair of neighboring nodes will be collected at the sink node. There, all available information will be used in order to obtain the unknown distances between non-neighboring nodes.

There are a few well-known techniques for distance measurement between neighboring nodes [6][7][10], like Received Signal Strength Indicator (RSSI), Time of Arrival, Time Difference of Arrival (TDoA) and Angle of Arrival (AoA). RSSI [10] measurement of distances is often preferred as it does not require additional hardware. RSSI is based on the phenomenon that the intensity of emitted signal decreases as the distance from the signal source increases. If the function of the attenuation in dependence on a distance is known in advance, the distance between the emission source and the receiver can be easily estimated. The time needed for a message to travel from one node to another is used to provide range information in ToA and TDoA techniques, while AOA is defined as the angle between the propagation direction of the wave and some reference direction.

The main advantage of using MDS is its ability to reconstruct the relative map of the network even when there

are no anchor nodes (nodes with a priori known location). If given sufficient portion of anchor nodes, MDS performs very accurate position estimation enabling local map to be transformed into an absolute map [9][11].

There are different versions of MDS for nodes localization in a two dimensional WSN. The most popular is MDS-MAP, proposed by Yi Shang and Wheeler Ruml [9], where Dijkstra algorithm is used to calculate the unknown distances from the distance matrix. In [9] it is shown that MDS-MAP outperforms other techniques, especially when applied on density networks. Other approaches based on MDS-MAP exist [12], but most of them are complex and thus more computationally dependent. In [13], the authors introduce MDS-MAP(P), which is a decentralized version of the MDS-MAP. MDS-MAP(P) outperforms MDS-MAP on irregular network topologies, but requires intensive computational resources at each node. It computes local maps at each node in the network and then merges local maps into a global map. Using absolute positions of the anchors, this global map can be easily transformed into an absolute map.

Although a lot of research has been carried out regarding MDS-MAP for WSN localization, all of the algorithms proposed in the literature based on MDS-MAP consider only two dimensional networks. To the extent of our knowledge, this is the first research that extensively investigates three dimensional surface WSN localization based on MDS.

#### A. Multidimensional scaling (MDS) for 3D-WSN

MDS-MAP for 3D WSN consists of 3 steps:

- Step 1: Calculate shortest distances between every pair of nodes (using either Dijkstra's or Floyd's all pairs shortest path algorithm). This is the distance matrix that serves as input to the multidimensional scaling in step 2.
- Step 2: Apply classical multidimensional scaling to the distance matrix. The first 3 largest eigenvalues and eigenvectors give a relative map with relative location for each node.
- Step 3: Transform the relative map into absolute map using sufficient number of anchor nodes (at least 4).

#### B. Finding optimal rotation and translation between corresponding 3D nodes

Generating an absolute map (step 3) of the WSN requires at least four anchor nodes.

Let  $P = \{p_1, p_2, \dots, p_N\}$  and  $Q = \{q_1, q_2, \dots, q_N\}$  be two sets of corresponding nodes, where  $N$  is the number of anchor nodes in the WSN. We wish to find a transformation that optimally aligns the two sets in terms of least square errors, i.e., we seek a rotation matrix  $\mathbf{R}$  and a translation vector  $\mathbf{t}$  such that

$$(R, t) = \arg \min_{R, t} \sum_{i=1}^N \|(Rp_i + t) - q_i\|^2. \quad (1)$$

This transformation is also known as Euclidean or Rigid transformation, because it preserves the shape and the size.

There are many algorithms purposed in the literature that compute a rigid 3D transformation [14]. The most explored are based on Singular Value Decomposition (SVD), as it is known to be the most stable [15]. Finding the optimal rigid transformation with SVD can be broken down into the following steps:

- Compute the weighted centroids of both point sets

$$\bar{p} = \frac{1}{N} \sum_{i=1}^N p_i, \bar{q} = \frac{1}{N} \sum_{i=1}^N q_i, \quad (2)$$

- Compute the centered vectors

$$p_i' := p_i - \bar{p}, q_i' := q_i - \bar{q}, i=1, \dots, N \quad (3)$$

- Compute the 3x3 covariance matrix

$$H = P'Q'^T, \quad (4)$$

where  $P'$  and  $Q'$  are the 3xN matrices that have  $p_i'$  and  $q_i'$  as their columns, respectively.

- Compute the singular value decomposition

$$H = U\Sigma V^T, \quad (5)$$

The rotation we are looking for is then

$$R = VU^T, \quad (6)$$

- Compute the optimal translation as

$$t = \bar{q} - R\bar{p}. \quad (7)$$

#### C. Time complexity of MDS-MAP for 3D-WSN

In step 1, distance matrix construction using Dijkstra's or Floyd's algorithm requires  $O(n^3)$ , where  $n$  is the number of nodes in the network. In step 2, applying MDS to the distance matrix has complexity of  $O(n^3)$  due to singular value decomposition. In step 3, the relative map is transformed through linear transformations. Computing the rigid transformation takes  $O(N)$  time for computing  $P$  and  $Q$ , while computing SVD takes only  $O(3^3)$  time (since the dimension of covariance matrix  $H$  is 3x3). Applying the transformation (rotation and translation) to the whole relative map takes  $O(n \cdot N)$  time, where  $N$  is the number of anchors ( $N \ll n$ ).

### III. IMPROVED MDS-BASED APPROACH FOR WSN POSITIONING

In this section, we will explain in details our improved multidimensional scaling algorithm (IMDS) for nodes localization in WSN.

MDS is very accurate technique for dimensionality reduction. If the correct distance matrix is given as input, MDS algorithm will reconstruct the map of the network without error. But, calculating distance matrix for networks

where only distances between neighboring nodes are known is not a trivial task. This problem in MDS-MAP is solved by applying Dijkstra's (or Floyd's) all pairs shortest path algorithm. Dijkstra's algorithm is a graph search algorithm that solves the single-source shortest path problem. In WSN localization problem, the sensor network is represented as a graph with non-negative edge path costs, while the real, Euclidian distance between two non-neighboring nodes is replaced with the distance calculated using Dijkstra algorithm. But the assumption that Dijkstra distance between two nodes correlates with their Euclidean distance is hardly true. This approximation produces an error, i.e., the positions obtained as MDS output usually differ from the correct positions. The difference between the real and the predicted positions is known as estimation error. The error is bigger when the nodes are in multi-hop communication range, which is a common case in obstructed environments. It is usually caused by the presence of obstacles or terrain irregularities that can obstruct the line-of-sight between nodes or cause signal reflections. Fig. 1 shows two examples when Dijkstra algorithm will calculate much larger distance between non-neighboring nodes. Left side of the picture shows an example of two nodes A and B that are far from each other. The distance between A and B will be calculated as  $AB=a+b+c+d$ , which is much longer than the real Euclidean distance. This scenario is present when the network is deployed on vast regions where the radio range of the nodes is short compared with the length of the region. On the right side of Fig. 1, there is an example where two nodes can't communicate directly although they are very close to each other. The reason for this is the presence of an obstacle that obstructs the line-of-sight. In this scenario, Dijkstra algorithm is completely inapplicable as it calculates a few times longer distance.

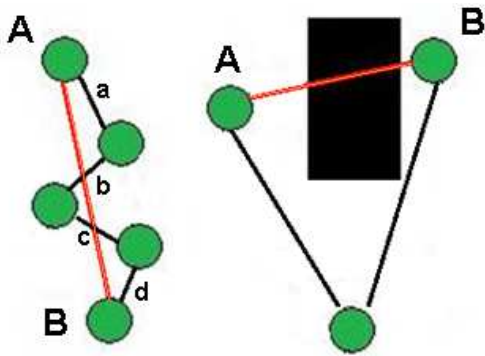


Figure 1. Distance approximation

As it can be seen from the two examples presented in Fig. 1, the distance calculated using Dijkstra algorithm always increase the real distance. In order to reduce this distance, in this paper, we propose an alternative heuristic approach. By reducing the distance matrix error, we intend to reduce the overall estimation error.

A. Distance matrix calculation

Consider there are three nodes in a network: A, B and C (Fig. 2), with known distances between nodes A and B ( $d_1=AB$ ), and between nodes B and C ( $d_2=BC$ ). Since distance matrix requires the distances between every pair of nodes in the network, the distance between nodes A and C has to be obtained. We will refer to this distance as  $a$ .

If maximum radio range of the nodes in the network is  $R$ , then, we know for sure that node C can lay anywhere on the curve  $C_1C_2$ . If Dijkstra's algorithm is used for this purpose, it will calculate the distance  $a$  as  $a=AB+BC$ , which is the longest possible theoretical distance between nodes A and C. More precisely, C will lay exactly on  $C_2$ . On the other hand, if we calculate the shortest possible theoretical distance between nodes A and C, it will be very close to  $R$ . We can conclude that:

$$R < a \leq d_1 + d_2. \tag{8}$$

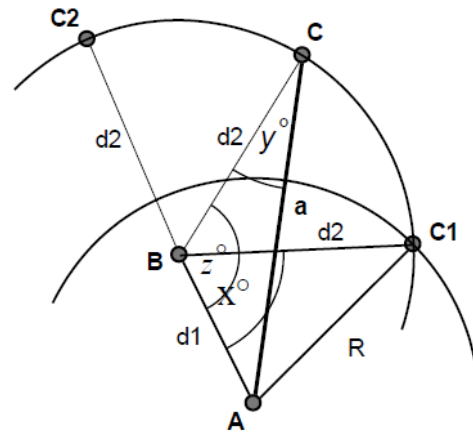


Figure 2. Distance approximation

To minimize the possible error, we propose a heuristic solution that assumes that the node C lies exactly in the middle of the curve  $C_1C_2$ . Hence, the distance  $a=AC$  can be calculated using cosine formula as:

$$a^2 = d_1^2 + d_2^2 - 2 \cdot d_1 \cdot d_2 \cdot \cos(\sphericalangle ABC). \tag{9}$$

In order to calculate the distance  $a$ , first, we need to find the angle using cosine formula:

$$\sphericalangle ABC = \sphericalangle ABC_1 + \sphericalangle C_1BC \tag{10}$$

The angle  $\sphericalangle ABC_1$  can be calculated again with the cosine formula:

$$\sphericalangle ABC_1 = \arccos\left(\frac{d_1^2 + d_2^2 - R^2}{2 \cdot d_1 \cdot d_2}\right) \tag{11}$$

Since

$$\sphericalangle C_1BC = \sphericalangle C_1BC_2, \tag{12}$$

$$\sphericalangle C_1BC = \frac{1}{2} \sphericalangle C_1BC_2, \tag{13}$$

$$\sphericalangle C_1BC = \frac{1}{2} (\pi - \sphericalangle ABC_1), \tag{14}$$

$$\begin{aligned} \sphericalangle ABC &= \sphericalangle ABC_1 + \frac{1}{2} (\pi - \sphericalangle ABC_1), \\ \sphericalangle ABC &= \frac{\pi}{2} + \frac{1}{2} \sphericalangle ABC_1 \end{aligned} \quad (15)$$

Finally,

$$\begin{aligned} a^2 &= d_1^2 + d_2^2 - 2 \cdot d_1 \cdot d_2 \cdot \cos(\sphericalangle ABC) = \\ &= d_1^2 + d_2^2 - 2 \cdot d_1 \cdot d_2 \cdot \cos\left(\frac{\pi}{2} + \frac{1}{2} \sphericalangle ABC_1\right) = \\ &= d_1^2 + d_2^2 + 2 \cdot d_1 \cdot d_2 \cdot \sin\left(\frac{1}{2} \sphericalangle ABC_1\right), \end{aligned} \quad (16)$$

where

$$ABC_1 = \arccos\left(\frac{d_1^2 + d_2^2 - R^2}{2 \cdot d_1 \cdot d_2}\right) \quad (17)$$

We note here that our algorithm preserves the time complexity of MDS-MAP algorithm.

#### IV. PERFORMANCE EVALUATION

The performance of the algorithms for WSN localization depends on different network parameters, such as the network topology, the number of anchors (i.e., the anchor-to-node ratio), the radio range, the density of nodes, etc. Hence, the location estimation error is going to be evaluated as a function of different parameters.

##### A. Network model

We assume a typical sensor network composed of hundreds (or thousands) of sensor nodes deployed uniformly across three dimensional monitored area (valley or mountain). Each sensor is equipped with an omni-directional antenna and has limited resources (CPU, battery, memory, etc.). Since radio signals are omni-directional, only nodes within certain radio range R can communicate with each other. If two nodes are within each others transmission range they are called neighbors. Further, we made following assumptions:

- Nodes are static and unaware of their location.
- There is a path between every pair of nodes.
- Nodes deployed in close proximity to each other exchange messages.
- Each node uses RSSI (or any other) method for distance estimation.
- RSSI provide accurate neighboring sensor distance estimation.

We simulated both techniques (MDS-MAP and IMDS) on different surface WSNs with Matlab.

We considered:

- Different network topologies:
  - 100 nodes randomly deployed on valley terrain (topology I)
  - 100 nodes randomly deployed on mountain terrain (topology II)

- 4, 6, 10 and 15 anchors for absolute map construction (for 3D rigid transformation SVD method was used)
- Different radio ranges (R) that lead to different average connectivity (average number of neighbors).
- Radio range error *er* (from *er*= 0%R to *er*=30% R with step 5% of R)

Thus 280 different networks were simulated (2 x 4 x 5 x 7) and each node location was discovered with both MDS-MAP and IMDS technique. The connectivity parameter and the estimation error for each scenario represent average over 30 trials for both algorithms. The average estimation error is normalized by the radio range R:

$$Error = \frac{\sum_{i=1}^n \text{distance}(pos_i^{(estimated)} - pos_i^{(true)})}{(n - N) \cdot R} \cdot 100\%, \quad (18)$$

where n is the number of nodes in the network, N is the number of anchor nodes,  $pos_i^{(estimated)}$  is the estimated location and  $pos_i^{(true)}$  is the true location of the i-th node.

##### B. Comparison of MDS-MAP and IMDS for 3D surface WSN

It is expected that MDS-based algorithms for WSN localization will not work well for such scenarios, basically because of multi-hop distance between each pair of nodes. Our improved heuristic approach presented in this paper is expected to achieve more acceptable accuracy.

Fig. 3 shows an example of two typical 3D surfaces. On the upper picture there is a surface, which represents a valley, while the lower surface represents a mountain. In our simulations, two scenarios are constructed to emulate a terrain with a valley and a terrain with a mountain. 100 nodes are deployed randomly with a uniform distribution over these two surfaces.

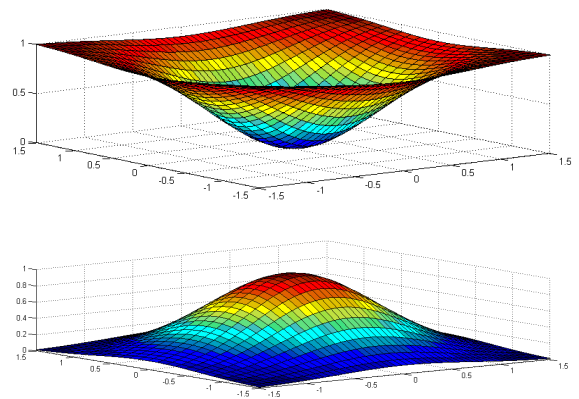


Figure 3. Typical 3D surface, valley (upper) and mountain (lower)

Fig. 4 and Fig. 5 compare the results of MDS-MAP and IMDS for topology I and topology II respectively.

In the case of topology I (Fig. 4), when  $er$  is small, both IMDS and MDS-MAP produce very similar estimation error. This error is much more affected by the number of anchors. As  $er$  increases, IMDS performs much better than MDS-MAP for all connectivity levels, regardless of the number of anchors.

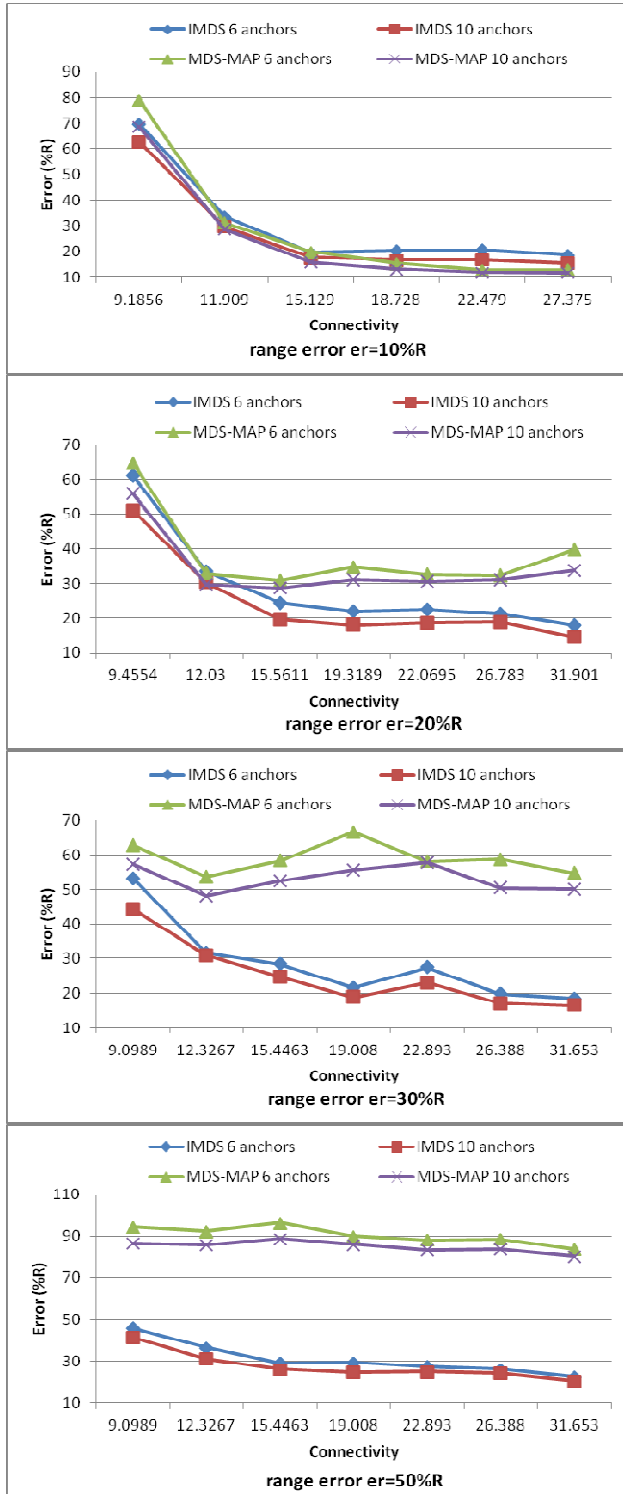


Figure 4. Comparison of MDS-MAP and IMDS for topology I

In case of topology II, for small  $er$  MDS-MAP has smaller estimation error than IMDS (Fig. 5). For large values of range error  $er$ , IMDS is better than MDS-MAP in terms of accuracy.

The characteristics of IMDS to produce smaller estimation error than MDS-MAP for large range error  $er$  is very important, as range measurement in the real applications is prone to error. When adopting distance measurement based on RSSI, the range error measurement is at least 10%R. The results presented in [10] show average range error measurement between 5%R and 30%R for longer radio range R. Similar research that investigates RSSI is conducted in [16] and [17], reporting average error around 20%R.

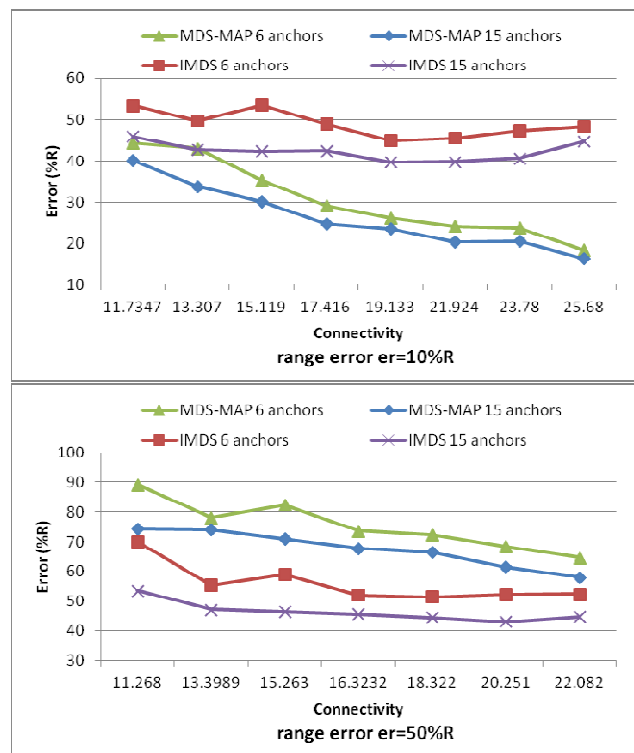


Figure 5. Comparison of MDS-MAP and IMDS for topology II

The average performance of IMDS as a function of connectivity for valley WSN is given on Fig. 6. IMDS is very stable and predictive. Estimation error decreases as connectivity increases. The radio range error  $er$  affects the estimation error in a way that larger  $er$  deteriorates the performance of IMDS.

As expected, the number of anchors affects the results, i.e., having more anchors slightly improves performance for all connectivity levels (Fig. 7).

If we compare the results for topology I and topology II, we can notice that both MDS-MAP and IMDS show better performance for topology I (valley terrain). The main reason for this is the characteristic of the terrain. Valley terrain is

very regular because all nodes that are within radio range R can communicate with each other. Mountain terrain should be considered as an irregular topology. The mountain presents an obstacle that obstructs the radio propagation between the nodes, which means that sometimes nodes that are very close to each other cannot communicate, i.e., cannot measure the distance between each other. For terrains with obstacles, nodes localization problem should be solved differently. IMDS algorithm should manage hierarchical network organization based on cluster formation. This cluster-based approach, which is already developed and implemented for 2D networks in [18], should be considered for 3D surface networks.

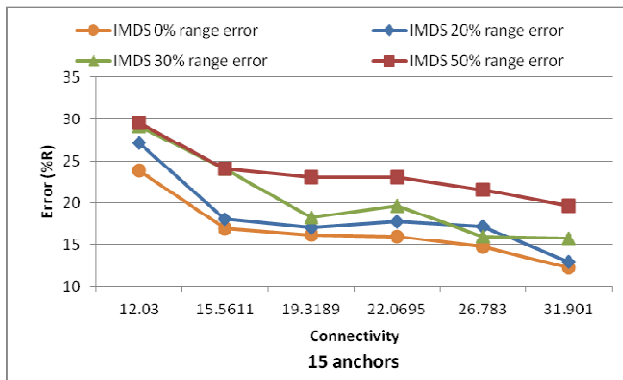


Figure 6. The effect of range error on the estimation error for topology I

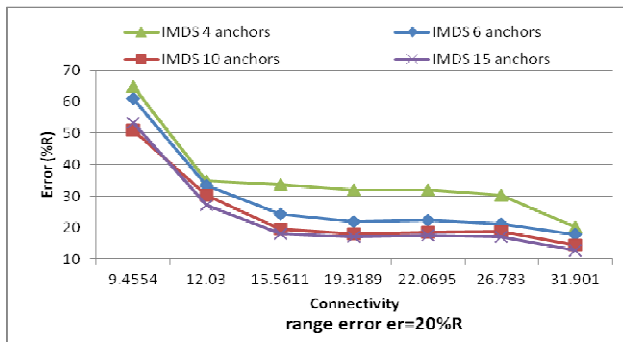


Figure 7. The effect of number of anchors on the estimation error for topology I

V. CONCLUSION AND FUTURE WORK

In this paper, we implemented improved MDS-based algorithm (IMDS) for nodes localization in 3D surface WSN. In IMDS, a novel technique for distance matrix refinement was introduced in order to reduce the estimation error. We investigated two surface network models (valley and mountain) and we showed that our approach outperforms MDS-MAP in terms of accuracy. IMDS performs much better than MDS-MAP especially when radio range error *er* is large.

For future work, we intend to investigate our algorithm on network where nodes are deployed on more complex 3D

terrains. It is also a challenge to simulate radio propagation model in such complex 3D terrains, which is not a trivial task due to the presence of obstacles.

This way, we believe this work will contribute for future development of smart network technologies in different domains, especially for context-aware applications.

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