Robust Localization Algorithm for Large Scale 3D Wireless Sensor Networks

Article *in* International Journal of Ad Hoc and Ubiquitous Computing · August 2016

DOI: 10.1504/IJAHUC.2016.10000216

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Abstract: Nodes positioning has recently been of great interest in wireless networks owing to its crucial role in many applications. In wireless sensor networks (WSNs), the task of localising sensor nodes with unknown position is important for efficient network configuration and operation. This challenge has stimulated research of various localisation algorithms. In this paper we propose robust localisation algorithm for large scale three-dimensional (3D) WSNs based on multidimensional scaling (MDS). Our approach has two main improvements over classical MDS algorithm. Firstly, it uses heuristic approach for distance matrix calculation, and secondly, it applies Levenberg-Marquardt (LM) method for absolute map refinement using received signal strength (RSS) measurements. Furthermore, the performance of the proposed approach is compared to other 3D WSN localisation techniques and it is shown that the proposed approach outperforms other techniques for 3D localisation.

Keywords: WSN; wireless sensor network; 3D localisation; node positioning; MDS; multidimensional scaling; LM; Levenberg-Marquardt.

Reference to this paper should be made as follows: Saeed, N. and Stojkoska, B.R. (xxxx) 'Robust localisation algorithm for large scale 3D wireless sensor networks', *Int. J. Ad Hoc and Ubiquitous Computing*, Vol. x, No. x, pp.xxx–xxx.

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The authors have contributed equally to this article.

1 Introduction

The advancement of wireless communication technologies and micro-electro mechanical systems (MEMS) has fostered the development of multi-functional and low-power wireless sensor nodes, that are capable of data collection, information processing and wireless communication. Wireless sensor network (WSN) is a collection of these low-cost randomly placed heterogeneous wireless sensor nodes that are programmed to perform a specific task (Akyildiz et al., 2002).

WSN are generally self organising networks, where large number of wireless sensor nodes are deployed. The purpose of WSN is to collect and process the data from the sensor nodes to obtain useful information.

WSNs bridge up the gap between the physical world and the digital world, as they are widely used in many applications. Together with the wireless technology expansion, new and interesting problems are arising for WSNs. The typical examples are 'internet of things' (IoT) and e-health with wireless body area networks IEEE 802.15.6 (Bari et al., 2013; Shabana et al., 2015). Some other common application scenarios of WSNs include industrial, medical, household, marine, military and environmental monitoring.

The fundamental problem for all of the above mentioned WSN applications is that the collected information is useless without knowing the accurate nodes locations. Therefore, localisation of the nodes is one of the main concerns in WSNs. Location in WSNs generally refers to determine the geographical coordinates of every sensor node in the network (Pal, 2010). One way to solve localisation problem is to equip every sensor node with global positioning system (GPS), but this is a costly solution. Although, GPS works well for outdoor environment, it is useless for indoor localisation since GPS signals are very weak (Bulusu et al., 2000). Therefore, efficient and optimised localisation techniques needs to be developed for WSNs for harsh and indoor environments.

Localisation of an unknown node is achieved using distance or angle calculations between the nodes. The most exploited ranging techniques are: received signal strength (RSS), time of arrival (ToA), time difference of arrival (TDoA) and angle of arrival (AOA). In RSS measurements, the distance between two nodes is estimated from the received power at receiving node (Youssef and Youssef, 2007). Generally, propagation loss is computed and is converted into distance. As the distance between two nodes increases, the received power is getting weaker and vice versa. RSS technique is a very cheap solution for ranging because it does not require any extra hardware. However, its performance is not good comparing to other ranging techniques owing to multipath, shadowing and fading (Wang and Yang, 2011). In Elnahrawy et al. (2004), the authors propose extra hardware to overcome these limitations in RSS measurements. ToA measurement considers speed, wavelength and time of radio signal traveling between the unknown node and anchor node. ToA measurements are much more accurate compared to RSS, but need extra hardware to calculate the ToA of the radio signal. TDoA technique considers the time difference between two different kind of signals arriving at the received node. For TDoA approach, the nodes need to be equipped with two kinds of extra devices (Hara et al., 2013) which can detect both kind of signals. Unknown node calculates the time difference between the two different signals and computes the distance information from it. Finally, the AoA ranging measurements are based on the angle of reception at receiver node. Generally, AoA technique provides very accurate localisation (Wang et al., 2014) but its cost is much more higher than RSS. In this paper we consider low-cost RSS based ranging measurements because the sensor nodes are very low powered devices and equipping the nodes with extra hardware reduces the lifetime of the sensor nodes.

In this paper, we propose a novel 3D WSN localisation algorithm based on multidimensional scaling (MDS). Our algorithm uses classical MDS approach with two main improvements. First, for distance calculation between the non-neighbouring nodes, instead of using Dijkstra algorithm, we used the heuristic approach presented in Stojkoska (2014), as it was shown that this technique better approximates the actual distances (Stojkoska, 2014). Second, for local map construction, we use a refinement phase based on Levenberg-Marquardt (LM) optimisation technique, which produces more accurate positioning (Saeed and Nam, 2014). Our approach actually combines two previous algorithms (Stojkoska, 2014; Saeed and Nam, 2014) and the achieved accuracy outperforms both. Henceforth, the acronym MHL-M will be used for our MDS algorithm with Heuristic approach and LM refinement. The proposed MHL-M algorithm is more robust in presence of independent and identically distributed (i.i.d) zero mean Gaussian noise.

The rest of the paper is organised as follows. Section 2 reviews literature related to 3D WSN localisation techniques. In Section 3, MDS is briefly discussed with its theoretical background. Section 4 explains in details our novel algorithm. Results from the performed simulations are presented in Section 5. Finally, Section 6 concludes the proposed work.

2 Related work

Indoor and outdoor localisation are becoming increasingly important for many applications, especially within the last few years. Although this topic has been interesting for the research community more than a decade, it is becoming even more attractive. The main reason for this is that the concepts of WSN are becoming universally accepted concepts in mobile computing. In the beginning, WSNs were designed for a specific applications requirement. Today, when almost everyone has a smartphone, the mobile networks and IoT can be considered as a natural extension of WSN. As the range of mobile applications is becoming endless, the challenges associated with WSN are being reinvented with additional requirements. In the case of nodes localisation, the accuracy of the estimated position is still in the focus.

The most straightforward method for localisation is trilateration (or triangulation), which is used by the GPS satellites. The first algorithms for WSN localisation were based on these techniques (Niculescu and Nath, 2001). However, in cases where the density of the anchor nodes is limited to only a small portion of all nodes, the indirect lateration is necessity. Namely, each node, to obtain its own position, uses its already localised neighbouring nodes as anchors. After this, the node itself would become a new anchor. This process produces huge cumulative error, thus, these algorithms provide very poor accuracy. Although these methods are almost being abandoned, the principle of lateration is still being used even in the newer approaches, usually for the refinement phase.

The era of MDS algorithms stared after 2003 and 2004 (Shang et al., 2003, 2004). The first algorithm was named MDS-MAP (Shang et al., 2003). This is a centralised approach, where all distance measurements between the nodes are collected at the central station (sink node) and using MDS are being converted into two-dimensional array. Today, there are dozens of algorithms in the literature based on MDS that tend to increase the accuracy of MDS-MAP. Most of them are distributed versions, where part of the job is performed inside the network by more powerful nodes. One of the first cluster based approaches was presented in Stojkoska et al. (2008). In this cluster-based MDS algorithm (CB-MDS), the network is initially divided into clusters, where one member of the cluster is usually with unlimited resources. For example, it can be a node equipped with more powerful microcontroller and connected to steady power supply. This node is known as cluster-head and is responsible to collect the measurements from the members of its own cluster. After performing the localisation, cluster-heads distribute this information to the sink node. Since cluster-heads provide only local maps, an iterative process of map merging is performed at the sink node to obtain global, or absolute map of the WSN. Approaches based on hierarchical networks are especially suitable for irregular topologies, where centralised MDS approaches produce very large localisation error. The authors of Stojkoska et al. (2008) evaluated the performances of CB-MDS for irregular topologies (C-shape, L-shape, etc.) and have shown that this approach outperforms MDS-MAP in terms of accuracy. Other approaches focus to extend MDS-MAP with one more step, which performs refinement of the process of coordinate alignment. In Saeed and Nam (2014), the authors use iterative refinement algorithm based on LM method and prove fast convergence of the algorithm.

Optimisation techniques are also used for WSN localisation (simulated annealling (Kannan et al., 2006), particle swarm optimisation (Monica, and Ferrari, 2013), semidefinite programming (Stojkoska et al., 2010), etc.), as well as machine learning approaches (support vector machines (Yong et al., 2012), neural networks (Rahman et al., 2009), etc).

What is common for the above-mentioned algorithms is that most of them were implemented, simulated and investigated exclusively for two-dimensional networks. In the last few years, the new applications impose a demand for three-dimensional localisation. Thus, there was an evident need for development of new algorithms dedicated for three-dimensional environments. Many of the researchers propose modifications of the existing well known algorithms for 2D WSN. However, simple extension of 2D algorithms is not always feasible owing to some specific characteristics of the 3D environments. Generally, the 3D WSN algorithms can be divided into two main categories:

1. Node self-localisation approach, where each node performs set of measurements to obtain its own position. Nodes usually use at least 4 anchors that have stronger radio signal. Anchor nodes can be either stationary or mobile nodes (e.g., airplane). These approaches fit the distributed paradigm.
2. Source (Sink) localisation approach, where all measurements between the nodes are collected at a central point where further computation is done. These approaches can be either centralised or distributed (e.g., in hierarchical networks).

Variations of MDS based algorithms for 3D WSN exist. In Stojkoska (2014) and Chaurasiya et al. (2014), the authors use novel approach for distance calculation between the non-neighbouring nodes, instead of using Dijkstra algorithm. In both approaches, the distance is achieved through more complex geometrical relationships. In Stojkoska (2014), the non-neighbouring distance represents an average of possible distances, while in Chaurasiya et al. (2014) it is obtained on more accurate iterative way, which, on the other side, imposes greater computational overhead compared with Stojkoska (2014). The results of both algorithms prove improvement over traditional MDS-MAP algorithm. These are centralised, source localisation algorithms. Mobile beacon-based localisation using classical MDS is proposed in Kim et al. (2010). This is a self-localisation approach. The mobile beacon flying over the 3D terrain broadcasts messages that are further used by each node in the network to obtain range measurements. These measurements are used to construct distance matrix for MDS algorithm. The CB-MDS algorithm (Stojkoska et al., 2008) has been modified for 3D networks and named D3D-MDS (Fan et al., 2015). The simulation analysis of the D3D-MDS algorithm shows increased localisation accuracy compared with 3D-DV-HOP and 38.6% compared with 3D-MDS-MAP. The authors propose their own clustering algorithm which strongly depends on the position of the anchors. The main assumption in Fan et al. (2015) is that the anchors are placed on the network edges, which is hardly true in real WSNs.

3 Description of 3D MDS

Multidimensional scaling (MDS) represents a set of analytical techniques (Cox and Cox, 1994). For a given set of multidimensional objects, MDS reduces the dimensionality of each object. MDS takes as input the distances between each pair of objects in the set (usually calculated as Euclidean distances). The aim of MDS is to present data in a visual (two or three dimensional) form that is more explicable. MDS has been used for many years in different disciplines, like scientific visualisation and data mining in fields such as cognitive science, information science, statistics, psycho-physics, psychometry, marketing and ecology.

The analogy between object distances and node distances in a network is used for the purpose of WSN localisation. MDS algorithm can use inter node distances to produce two or three dimensional representation, which corresponds to the real nodes deployment. Since nodes are capable to measure the inter node distances with respect to their neighbouring nodes, the only problem remains obtaining the non-neighbouring inter node distances. In MDS-MAP, these distances are approximated with the distances calculated by Dijkstra algorithm.

Distance measurements between each pair of neighbouring nodes are being collected at the central station (sink). The remaining (non-neighbouring) distances would be calculated by the sink. Thus, MDS can be classified as centralised, range-based localisation algorithm. The well known MDS-MAP for 3D WSN consists of three steps:

1 Calculate shortest distances between every pair of nodes (using either Dijkstra or Floyd all pairs shortest path algorithm). This is the distance matrix that serves as an input to the MDS in step 2.

2 Apply classical MDS to the distance matrix. Use the first 3 largest eigenvalues and eigenvectors to construct a relative map with relative location for each node.

3 Transform the relative map into absolute map using sufficient number of anchor nodes (at least 4). This process usually includes translation, rotation and reflection. This transformation is also known as Euclidean or Rigid transformation, because it preserves the shape and the size. There are many algorithms proposed in the literature that compute a rigid 3D transformation. Among them, the singular value decomposition (SVD) is the most stable (Lorusso et al., 1995). Detailed mathematical description of each step can be found in Stojkoska (2014).

3.1 Time complexity of MDS-MAP for 3D-WSN

In step 1, distance matrix construction using Dijkstra's or Floyd's algorithm requires $O(N)^3$, where N is the number of nodes in the network. In step 2, applying MDS to the distance matrix has complexity of $O(N)^3$ owing to SVD. In step 3, computing the rigid transformation takes $O(w)$ time, while applying the transformation (rotation and translation) to the whole relative map takes $O(N - w)$ time, where w is the number of anchors ($w << N$).

4 MHL-M algorithm

Multidimensional scaling algorithm as a mathematical tool is an exact method to locate the nodes. It assumes knowing the distances between each pair of nodes in the network, which in practice is not fusible, owing to two main reasons:

- limited radio range of the sensor nodes
- presence of obstacles in the sensor field.

Under these circumstances, MDS is prone to error, which results in uncertain node locations. To overcome this drawback of MDS, original MDS pipeline for WSN localisation can be extended with two more phases:

i Pre-processing phase:

Preprocessing phase is applied immediately after step 1, i.e., after the distance measurements. In this phase, different geometric calculations can be used to correct distances between the nodes. These corrected distances are used to fill the distance matrix, which serves as an input to step 2. This approach is actually presented in Stojkoska (2014) and Chaurasiya et al. (2014), where authors use geometric characteristics of the network.

ii Post-processing (or refinement) phase:

After obtaining the relative map of the network in step 3 of MDS pipeline, different refinement techniques can be used to correct the estimated positions. The distances between the anchor nodes and their neighbouring nodes are used as a reference measurements. The approach presented in Chaurasiya et al. (2014) implements a refinement based on LM. To the best of our knowledge, all approaches in the literature based on MDS adopted only one of these phases. In this paper, we proposed a new algorithm named MHL-M. Our approach, at the same time implements both phases (pre-processing based on the algorithm in Stojkoska (2014) and post-processing based on the algorithm in and Chaurasiya et al. (2014).

In this section, we will explain in details the background of the algorithms Stojkoska (2014) and Chaurasiya et al. (2014), which are combined in our approach.

4.1 MDS with improved distance estimation (IMDS)

The main problem with MDS based localisation is the shortest path distances between every user in the network. Generally, the similarity/dissimilarity matrix used in MDS is obtained using Dijkstra algorithm that calculates the shortest path routes between every node in the network. Dijkstra algorithm calculates the longest possible theoretical distance between two non-neighbouring nodes. This approximation produces an error in MDS. IMDS (Stojkoska, 2014) algorithm uses heuristic approach (HA) for distance matrix calculation, where lightweight computation is performed to average the difference between the shortest and the longest possible distance. This approach was firstly implemented and evaluated for two-dimensional networks (Stojkoska and Kirandziska, 2013), but later it was extended for the 3D networks (Stojkoska, 2014). The evaluation of this algorithm shows improvement over classical MDS for both two-dimensional and three-dimensional networks.

The basic idea behind this averaging can be seen from the Figure 1. Here, the distance between the neighbouring nodes A and B is known (d_1). Also, the distance between node B and it neighbouring node C is known (d_2), although the exact positions of all nodes are not known. The radio range is the same for all nodes in the network, i.e., R. The distance between the non-neighbouring nodes A and C has to be calculated. Dijkstra algorithm will calculate the distance as $a = d_1 + d_2$, assuming that node C lays on node C_2. On the other hand, there is also a real possibility that node C will lay very close to node C_1, thus the distance would be just a little greater than R. HA assumes that node C lays in the middle of the curve C_1C_2. Using geometrical relationships, the distance a is calculated as

$$a^2 = d_1^2 + d_2^2 - 2 \cdot d_1 \cdot d_2 \cdot \cos(\theta), \quad (1)$$

where $\theta = \angle ABC$. To find θ

$$\theta = \phi + \zeta, \tag{2}$$

where $\phi = \angle ABC_1$ and $\zeta = \angle C_1BC$. From Figure 1, ϕ is calculated as

$$\phi = \arccos\left(\frac{d_1^2 + d_2^2 - R^2}{2 \cdot d_1 \cdot d_2}\right), \tag{3}$$

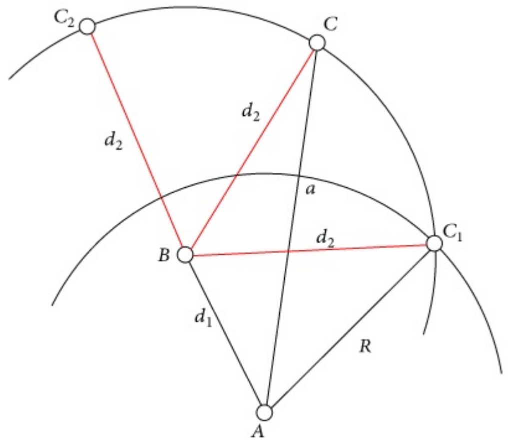
and $\zeta = \frac{\pi}{2} - \frac{\phi}{2}$. Evaluating equation (1) in terms of ϕ and ζ we get

$$a^2 = d_1^2 + d_2^2 - 2 \cdot d_1 \cdot d_2 \cdot \sin(\frac{1}{2}(\phi)), \tag{4}$$

which can be rewritten as

$$a = \sqrt{d_1^2 + d_2^2 + 2 \cdot d_1 \cdot d_2 \cdot \sin\left(\frac{1}{2}\arccos\left(\frac{d_1^2 + d_2^2 - R^2}{2 \cdot d_1 \cdot d_2}\right)\right)}. \tag{5}$$

Figure 1 Distance approximation (see online version for colours)



4.2 *Multidimensional and Levenberg-Marquardt with heuristic approach (MHL-M) based localisation*

In Saeed and Nam (2014) the authors proposed WSN network localisation scheme based on MDS technique that uses LM method (MDS-LM). This is an efficient subsequent iterative algorithm for 2D WSN. The problem with this algorithm is that it assumes the shortest path distances based on Dijkstra algorithm which accumulates large error when the network is not uniformly distributed. The error is large when the nodes are in multi-hop scenario, which is very common for WSNs.

In this paper, we have developed new algorithm that combines the advantages of both above mentioned algorithms (Stojkoska, 2014; Saeed and Nam, 2014). Here, we are going to give a detail description of our algorithm. A network of $N = v + w$ sensor nodes in a three-dimensional space is considered where v is the number of unknown nodes and w is the number of anchor nodes respectively. Let $\boldsymbol{X} = \{\boldsymbol{\Theta}_i\}_{i=1}^{N}$, where $\theta_i = [x_i; y_i; z_i]^T$ are the actual coordinates of the ith node. Based on classical MDS, the proposed MHL-M algorithm consists of the following three steps:

1 First of all, the neighbourhood information between all the nodes in the network are computed. Based on the neighbourhood information shortest path distances between every node in the network is obtained using HA. Distance matrix is denoted by D and is written as

$$\mathbf{D} = \begin{bmatrix} 0 & \cdots & \zeta_{1N}^2 \\ \vdots & \ddots & \vdots \\ \zeta_{N1}^2 & \cdots & 0 \end{bmatrix} \tag{6}$$

where ζ_{ij}^2 are the HA based shortest path distances between every node in the network. Matrix D in equation (6) is a square symmetric matrix with $\zeta_{ii} = 0$ and $\zeta_{ij} = \zeta_{ji}$

2 Apply classical MDS to the distance matrix to get the relative configuration of the sensor nodes. Kruskal (1956) defined the stress function for the minimisation of loss function as

$$\mathfrak{S} = \sqrt{\sum_{i=1}^{N} \frac{\left(\hat{\zeta}_{ij} - \zeta_{ij}\right)^2}{\hat{\zeta}_{ij}}} \tag{7}$$

when the MDS perfectly embeds the distance information data $\hat{\zeta}_{ij} = \zeta_{ij}$ and $\mathfrak{S} = 0$. The minimisation of stress function $\mathfrak{S}$ is computed by gradient descent approach. Then the relative configuration is achieved from the 3 largest singular values $\mathfrak{h}$ and the corresponding Eigen vectors $\mathfrak{k}$ as

$$\hat{\boldsymbol{X}} = \mathfrak{k}\sqrt{\mathfrak{h}}, \tag{8}$$

where $\hat{\boldsymbol{X}}$ shows the relative estimated position of each node in the network. Using linear transformations (scaling, translation and rotation) the absolute positions of the sensor nodes are obtained. The transformation to absolute positions is achieved using iterative LM refinement approach.

3 The estimated positions $\hat{\boldsymbol{X}} = \{\hat{\boldsymbol{\Theta}}_i\}_{i=1}^{N}$ are the initially estimated positions by MDS, while $\theta_l = [x_l; y_l; z_l]^T$ are the known coordinates of the l-th anchor node, where $l = 1, 2, \ldots, w$ and $w \geq 4$ for 3D positioning. Then from the RSS measurements the log normal path loss model can be expressed as

$$\ln(P_{r,l}) = \ln(C) + \ln(P_t) - \beta \ln(d_l) + \eta_{rss}, \tag{9}$$

where $P_{r,l}$ is received power, C accounts for all other factors that affect the received power, P_t is the transmitted power, η_{rss} is the Gaussian distributed noise with variance $\sigma_{\mathrm{rss},l}^2$. β is the path loss exponent and d_l is the Euclidean distance between the unknown node and the l-th anchor. Let

$$r_{\mathrm{rss}} = \ln(P_{r,l}) - \ln(C) - \ln(P_t). \tag{10}$$

The RSS is simplified to

$$r_{\mathrm{rss}} = -\beta \ln(d_l) + \eta_{rss}, \tag{11}$$

and in vector form equation (11) is written as

$$\boldsymbol{r}_{rss} = \boldsymbol{G}(\boldsymbol{x}) + \boldsymbol{\eta}_{rss}, \tag{12}$$

where $\boldsymbol{r}_{\text{rss}} = \{r_{\text{rss1}}, r_{\text{rss2}}, ...r_{\text{rssw}}\}$,
$\boldsymbol{\eta}_{\text{rss}} = \{\eta_{\text{rss1}}, \eta_{\text{rss2}}, ...\eta_{\text{rssw}}\}$ and

$$\boldsymbol{G}(\boldsymbol{x}) = -\beta \begin{bmatrix} \ln(d_1) \\ \ln(d_2) \\ \vdots \\ \ln(d_w) \end{bmatrix}, \tag{13}$$

where

$$d_l = \sqrt{(x_i - x_l)^2 + (y_i - y_l)^2 + (z_i - z_l)^2} \tag{14}$$

is the distance between the ith and the lth node. The probability density function s(pdf) for the $\boldsymbol{r}_{rss}$ in equation (11) is determined as

$$f(\boldsymbol{r}_{\text{rss}}) = \frac{1}{(2\pi)^{\frac{w}{2}} \sqrt{\boldsymbol{\sigma}_{\mathbf{rss},l}}} \exp\left(-\frac{1}{2}\left(\boldsymbol{r}_{rss} - \boldsymbol{G}(\boldsymbol{x})\right)^T \boldsymbol{\sigma}_{\text{rss},l}^{-1} \left(\boldsymbol{r}_{\text{rss}} - \boldsymbol{G}(\boldsymbol{x})\right)\right), \tag{15}$$

where $\sigma^2_{\text{rss},l}$ is the noise variance. The cost function is computed based on the noisy RSS measurements and initial estimated positions $\hat{\boldsymbol{X}}$ of the sensor node position as

$$\Psi(\hat{\boldsymbol{X}}) = \sum_{l=1}^{w} \left(\boldsymbol{r}_{rss} - \sqrt{(x_i - x_l)^2 + (y_i - y_l)^2 + (z_i - z_l)^2}\right)^2. \tag{16}$$

The cost function tries to minimise the error between the true Euclidean distance and RSS based shortest path estimated distance. To find the minimum of cost function LM efficient iterative algorithm is proposed to solve the non linear optimisation problem. The iterative LM procedure to estimate the 3D location of all the nodes in the network is

$$\hat{\boldsymbol{X}}^{p+1} = \hat{\boldsymbol{X}}^p - \left(\boldsymbol{U}_p^T \boldsymbol{U}_p + \lambda(\boldsymbol{I})\right)^{-1} \times \boldsymbol{U}_p \left(\boldsymbol{r}_{rss} - \Psi(\hat{\boldsymbol{X}}^p)\right), \tag{17}$$

where p is the number of iterations, λ is the step size, $\boldsymbol{I}$ is the corresponding identity matrix and $\boldsymbol{U}$ is the Jacobin matrix for the cost function given as

$$\boldsymbol{U}_p = \begin{bmatrix} \frac{\hat{x}_i - x_1}{d_{i1}} & \frac{\hat{y}_i - y_1}{d_{i1}} & \frac{\hat{z}_i - z_1}{d_{i1}} \\ \frac{\hat{x}_i - x_2}{d_{i2}} & \frac{\hat{y}_i - y_2}{d_{i2}} & \frac{\hat{z}_i - z_2}{d_{i2}} \\ \vdots & \vdots & \vdots \\ \frac{\hat{x}_i - x_w}{d_{iw}} & \frac{\hat{y}_i - y_w}{d_{iw}} & \frac{\hat{z}_i - z_w}{d_{iw}} \end{bmatrix}, \tag{18}$$

where w are the total number of anchor nodes and

$$d_{iw} = \sqrt{(x_i - x_w)^2 + (y_i - y_w)^2 + (z_i - z_w)^2}. \tag{19}$$

Final locations of all the sensor nodes in the network are computed using equation (17).

4.3 *Time complexity of MHL-M*

The MHL-M algorithm consists of basic three steps. Firstly, distance matrix construction using HA costs $O(N^3)$, where N is the total number of nodes in the network. The second step consists of applying the MDS to the distance matrix which also have time complexity of $O(N^3)$, while the refinement process using LM method takes $O(N - w)$ where w is the number of anchors. The overall complexity of MHL-M is

$$\mathbf{T} = 2 \times \left(O(N^3)\right) + O(N - w). \tag{20}$$

Note that the proposed approach has the same time complexity as Stojkoska (2014).

5 Performance evaluations

In this section the performance of the proposed MHL-M is evaluated and compared to already existing 3D WSNs localisation algorithms.

5.1 *Simulation setup*

To study the behaviour of the proposed MHL-M algorithm, we performed extensive simulations in MATLAB. Wireless sensor nodes are uniformly deployed in 3D cubic area. We assume that every node has the same communication range R. Two nodes succeed to communicate only if $d \leq R$, where d is the distance between the nodes. We assume Gaussian noise in distance measurements, that is, the measured distance $\hat{d}_{ij}$ between nodes i and j is defined in terms of the true distance d_{ij} as $\hat{d}_{ij} = N(d_{ij}, \sigma^2_{ij})$. All the nodes in the network are considered to be static and the distance between adjacent nodes is measured through RSS. We evaluated the performance of the proposed MHL-M in $(100 \times 100 \times 100)$ cubic area and compared the results to with IMDS and MDS-LM algorithms. The impact of various parameters, like error variance, transmission range and network density on the performance of the proposed algorithm is evaluated. We considered root mean square error (RMSE) as the performance metric for all the simulation results, where the RMSE is given as

$$\textbf{RMSE} = \left(\sqrt{\frac{\sum_{i=1}^{v}(\hat{x}_i - x_i)^2 + (\hat{y}_i - y_i)^2 + (\hat{z}_i - z_i)^2}{v}}\right) \cdot 100\%. \tag{21}$$

We evaluated our MHL-M algorithm in terms of localisation accuracy and we compared it with four other algorithms. Namely, we compare MHL-M with two centralised algorithms, i.e., its predecessor IMDS (Stojkoska, 2014) and MDS-LM (Saeed and Nam, 2014). We also implemented two centroid-based approaches from the literature, which are a good example of distributed algorithms (Chen et al., 2008; Blumenthal et al., 2007). We did not perform time complexity analysis, because we have previously shown that MHL-M, IMDS and MDS-LM have the same computational complexity. On the other hand, in case of the distributed approaches (Chen et al., 2008; Blumenthal et al., 2007), each node is performing local computation to obtain its own location based on simple calculation.

5.2 Simulation results

Before we present the results from the simulation setup, we visualise the network to show the localisation error more intuitively. Figures 2 and 3 represent localisation results obtained with our MHL-M algorithm and with MDS-LM (Saeed and Nam, 2014) respectively. This network consists of 100 nodes randomly deployed in the monitored area with radio range of 35 m and average connectivity of 11.6. The squares (□) show the actual location of the sensor nodes while the red lines show the localisation error. Absolute map is achieved using only 4 anchors. As can be seen from the figures, the proposed MHL-M performs much better than MDS-LM (Saeed and Nam, 2014).

Figure 2 Localisation error for the proposed MHL-M algorithm (see online version for colours)

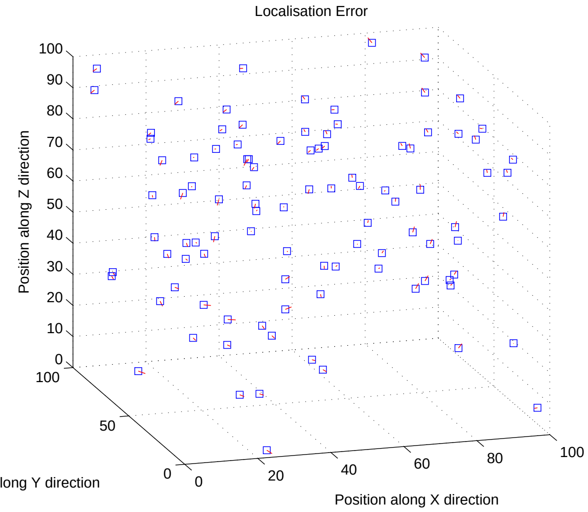


Figure 3 Localisation error for MDS-LM algorithm (see online version for colours)

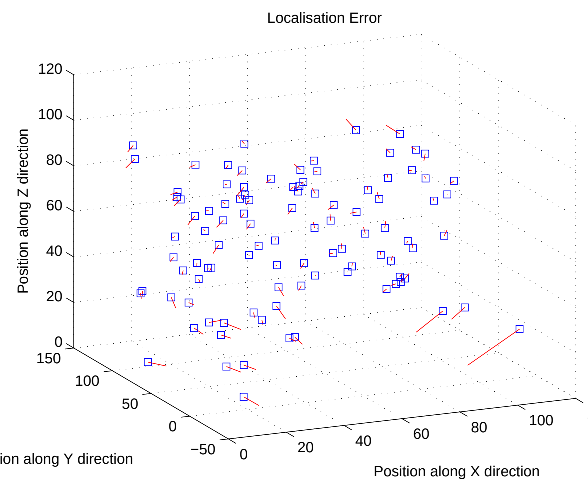


Table 1 shows the results outcomes of the proposed MHL-M algorithm with respect to the other localisation algorithms used for comparison. For this simulation setting, the localisation error in

Table 1 is presented in metres.

Table 1 Localisation Error of MHL-M, MDS-LM (Saeed and Nam, 2014), IMDS (Stojkoska, 2014), Centriod (Chen et al., 2008) and Weighted Centriod (Blumenthal et al., 2007) localisation algorithms

Scheme	*Localisation error* (m)
MHL-M	4.8
MDS-LM (Saeed and Nam, 2014)	6
IMDS (Stojkoska, 2014)	17.54
Centriod (Chen et al., 2008)	20.22
Weighted Centriod (Blumenthal et al., 2007)	19.5

Figure 4 shows the impact of network connectivity to the RMSE. It can be noticed that when the connectivity in the network increases the RMSE decreases as there are more direct connections between the nodes in the network. MHL-M gives much better estimation compared to IMDS (Stojkoska, 2014) and MDS-LM (Saeed and Nam, 2014. The performance of the proposed algorithm is also compared with centriod localisation (Chen et al., 2008) and weighted centriod localisation (Blumenthal et al., 2007). MHL-M gives much better estimation compared to centriod and weighted centriod localisation algorithms because both of them strongly depend on the number of anchors and their corresponding locations. Further it can be seen from the figure that at certain connectivity level, the proposed approach achieves the theoretical bound, that is the Cramer Rao lower bound (CRLB). For this simulation setting, the absolute map is achieved using only 4 anchors.

Figure 4 RMSE vs. connectivity (see online version for colours)

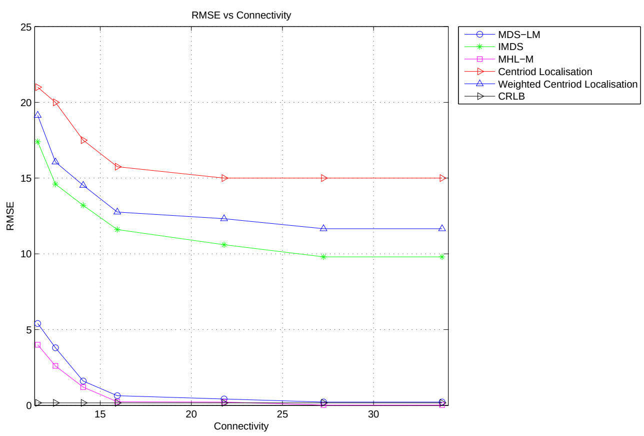


Figure 5 shows the impact of range error variance on RMSE, i.e., the proposed approach is more robust to noise comparing to other 3D WSN localisation techniques. The connectivity for this network is 11.6, while the number of anchors is 4 again. It is important to mention here that all three algorithms based on MDS are very resilient to range error variance. The algorithms are very stable especially for range error greater than 30%. This is very important characteristic, as range measurements are prone to error. In the case of RSSI measurements, this error is at least 20%, but more often is greater (Jianwu and Lu, 2009).

Figure 5 RMSE vs. range error variance (see online version for colours)

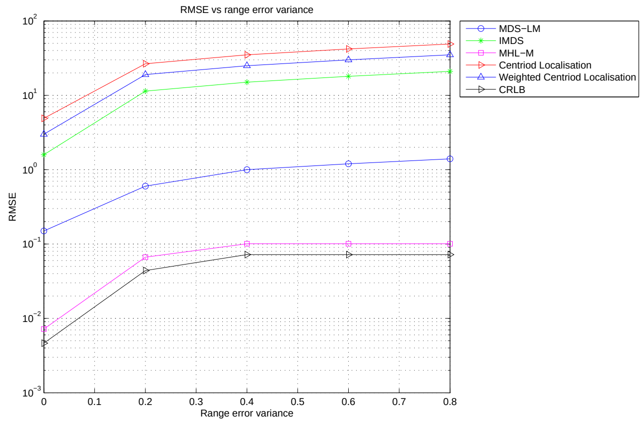
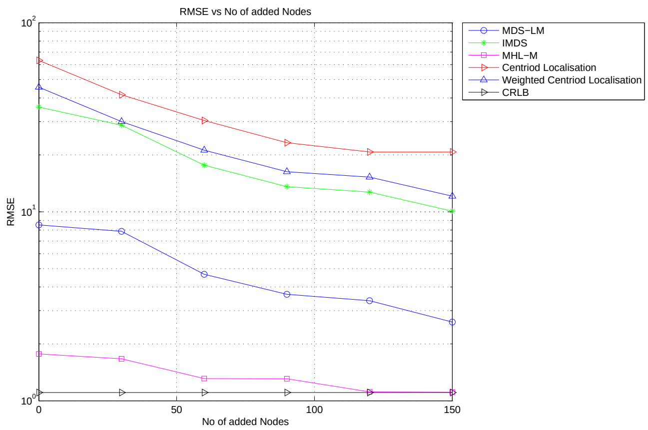


Figure 6 shows how adding more number of nodes improves the accuracy of the proposed algorithm. It is expected, since number of nodes affects the density of the network. When the density of the WSN increases, the connectivity level also increases, which improves the localisation accuracy. The connectivity level for this network varies from 4.45 to 11.8.

Figure 6 RMSE vs. no of nodes added (see online version for colours)



Figures 7 and 8 show the impact of the number of anchors on the localisation error for different connectivity levels and range errors of 10% and 50%, respectively. It can be seen, that increasing the number of anchors does not have a crucial influence on the localisation error, localisation error decreases slightly, improving the performance of the proposed MHL-M algorithm. This is especially notable for large connectivity levels, where MHL-M achieves the CRLB regardless of the number of anchors.

Figure 7 RMSE vs. no of anchors with 10% range error (see online version for colours)

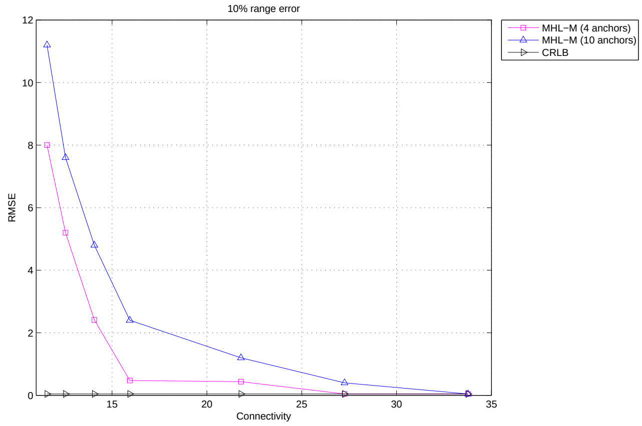
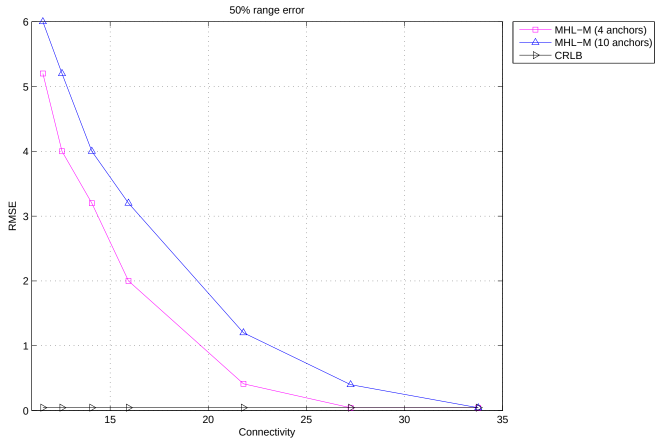


Figure 8 RMSE vs. no of anchors with 50% range error (see online version for colours)



6 Conclusion

Multidimensional scaling techniques have been extensively used for localisation in WSNs. In this paper, we have proposed a novel robust MHL-M localisation algorithm based on MDS for 3D WSNs, which is build upon previous research and it combines a heuristic approach for shortest path error reduction between non-neighbouring nodes and a LM based refinement of the absolute network map.

Statistical performances of the proposed algorithm were analysed through extensive simulations, regarding different network parameters, like network density, number of anchor nodes and range error measurements. Considering network density, our algorithm outperforms other 3D localisation algorithms owing to the fact that it is not dependent on the configuration of the anchors. Furthermore, at certain connectivity level, MHL-M achieves the theoretical bound, i.e., the Cramer Rao lower bound (CRLB). It was shown that the algorithm is very resilient to range error variance, achieving stability for range errors greater than 30%. The algorithm performance and the number of anchors are not strongly coupled and the increase in anchor number brings only a small improvement.

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