Monitoring Stock Market Returns: A Stochastic Approach

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Abstract. Financial analysis plays a major role in investing the disposable income of various economic agents. Stock markets are predominantly made up of small investors with limited information and low capabilities for a suitable analysis. Researchers, as well as practitioners, are divided over the findings on the adequacy of technical analysis in investing. This paper examines the Markov chain process in the stock market to discover the essential links and probabilities for the stocks' transition through three states of stagnation, growth, and decline (i.e., stagnant, bull, and bear markets). The subject of analysis is a randomly selected portfolio of 20 shares traded on the New York Stock Exchange. The data suggest that the portfolio relatively quickly, in four trading days, achieves equilibrium probabilities that allow a certain amount of predictability of future movements. At the same time, when analyzing the expected time intervals for the first transition, we found that the portfolio returns to a state of growth much faster than a decline. In addition, the results negate the basic habits of frequent trading, herding, and taking a short position in events of negative price fluctuations. Our research contributes towards observing regularities and stock market efficiency with a clear goal of improving expectations and technical analysis for small individual investors.

Keywords: Markov processes, operational research, stock market, portfolio theory.

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1. Introduction

Operational research often becomes a structural part of corporate management and investment decision- making on both microeconomic and macroeconomic levels. Special applications in the microeconomic processes of firms and households are found to be usually related to financial problems and their complex solving systems. The quantitative analysis is a special mixture of statistical, econometric, and optimization methods, all emerging from broader mathematical modelling. Applications are related but not limited to project planning, resource allocation, investment decision-making in the real and financial sectors, process automation, etc. Mathematical models are used to approximate reality. They have never been nor will ever be perfect in depicting the deterministic (or stochastic) reality, but they are a crucial tool for modern economists. With their help we come to a general understanding of the underlying processes and factors which contribute to theory formulation and challenging. Recently, special attention is devoted to models used in formulating and making decisions. Depending on their precision, they can be a clear algorithm in decision making for various levels of management or even a breakthrough analysis for inclusion of other determinants which were previously unobserved. In

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addition, including qualitative factors is necessary, since reality is not completely quantitative, after all.

In this paper, we will study the stochastic nature of the stock market and the formulation of an investment strategy for a small and average investor. According to general theory, we assume that such investors make their decisions predominantly based on technical rather than fundamental analysis. By doing so, we aim to formulate a guide for investment decisions in a stochastic set-up of a fluctuating market. To account for market dynamics, we observe stocks traded at the New York Stock Exchange. Through a Markov chain process, our goal is to explain the transition dynamics of a random portfolio through three states of return. Formulating such fictional investment should simplify the market dynamics in the states of stagnation (objectively defined as portfolio return in the interval between -0.1% and 0.1%), growth, and fall, which are related to the concepts of bull and bear markets. The analysis is based on high-frequency (daily) data for the period between January 2, 2018, and November 30, 2021. This timeline includes the COVID-19 pandemic and its influence on price fluctuations. The random portfolio consists of 20 stocks of companies in basic industries - the energy, financial, health, and technology sectors, which have critical economic importance. Our findings support the main hypothesis that the market quickly reaches equilibrium, especially after initial plummeting in valuation. The time needed for transitioning to a state of growth from an initial state of fall, though minor, is shorter in the opposite case. We show that such results do not support panic selling after a day of bad results, opposing the concept of herding in financial investing. The random portfolio has a greater probability of growth in the following days compared to the probability for value decreasing. Due to the specific period, the stocks are observed to be rarely stagnant in their returns.

Formulating new rules and expectations for trading at the stock market is necessary. Through the incorporation of mathematical models in finance as an emerging research field, we contribute to the academic literature of operational research. Our study focuses on the financial markets through observation of transition probabilities and expected first passage times (EFPT) through three states of portfolio returns. By doing so, small investors can observe regularities and market efficiency, which assist in executing decisions and avoiding irrational behavior such as the phenomenon of herding.

The paper is structured as follows. Section 2 overviews the theoretical background of portfolio theory, stock investment, and Markov processes and their importance in modern academic work. Section 3 discusses the methodological approaches in operational research (OR hereafter), developing the research model, and data acquiring and analysis. Upon this, the paper relates the observed results with adequate elaboration, finishing with a systematic conclusion of the research.

2. Theoretical background

Operational research dates formally back to the 17th century with the first complex decision analysis, but the modern format of quantitative analysis was introduced in the 20th century. Most commonly today, the quantitative methods are applied in the financial and industrial systems alongside their usage in the sector of non-financial services. Process optimization and generating a unique insight into their nature is the main idea behind operational research, independent of whether we talk about stochastic or deterministic processes [7, 5]. Andrey Andreyevich Markov, who the stochastic process was named after, introduced the idea in the early 20th century through studying the interdependence between certain events and the law of large numbers in probability theory [15]. The practical usage of the Markov chain in economics is immense. Changes in the market structure of companies may follow the Markov process [18, 13], implying that firms transit between sizes in their lifetime. Similar concepts are found in studies of the agricultural sector [11], income distribution and disparities [14] as well as predicting human potential in ICT firms [10]. The literature dominantly deals with concepts such as service optimization, income maximization, or cost minimization.

With the rapid development of financial markets, portfolio optimization becomes a key component in formulating investment decisions. Markowitz [16] emphasizes the fundamental concept of rational financial investors who act according to their own 'subjective perception of event probabilities' in times of rising uncertainty. The relationship between such a concept with the expected returns and the risk of a stock portfolio is the starting point in our study. The dominant proportion of stock market investors is small and possesses significantly less information compared to the large institutional investors, limiting their ability for deep market analysis [3]. Some studies imply that portfolios of average individual investors lag behind the market return by an average of 1.5% annually, with increased market activity (evident in times of greater market distortions) further decreasing it [2]. Due to low aggregate levels of information in possession, small investors usually incorporate technical or partial fundamental analysis of market tendencies. The concept of technical analysis is often criticized. Brown and Jennings [4] suggest that it is impossible for the spot price to be completely determined by information contained in the previous periods. Following the results of Barber and Odean [2], Hoffman and Chefrin [9] note that the investors who are highly dependent on technical analysis usually achieve unsatisfactory results and trade speculatively. One of the main motivations behind our research is the paper of Zhu and Zhou [25], which suggests that the technical analysis (mainly the moving average analysis of prices) is especially useful in creating a sufficient portfolio return, on the condition that stock returns are predictable. Park and Irwin [19] conducted a general review on earlier studies related to the effectiveness of technical analysis, with 56 out of 95 modern studies indicating that such methods consistently generate economic profits for individual investors. For additional research revisiting the positive aspects of technical analysis, we strongly suggest the work of Shynkevich [22].

So far, the usage of Markov chains in explaining stock market movements is stationed around trend and volatility studies, with little attention placed on expected intervals of transition and their connection to technical analysis. Using a Markov process, Hamilton and Lin [8] analyze the monthly volatility of stocks in recession environments, combined with the respective ARCH and GARCH models. The authors conclude that recessions are the main source of fluctuation, coinciding with the specific timeline chosen for our study which includes the COVID-19 pandemic. The ability to form precise expectations is strengthened with the usage of Markov chain models and neural networks in the computer modeling works of Dai et al. [6]. Financial portfolios, such as the Tunisian public debt portfolio, can be studied through hidden Markov models (HMM) for the expected Value-at-Risk (VaR) changes [21]. Such approach was found to be robust and accurate enough for formulating predictions. Two-state and eight-state models were used in studying the Prague Stock Exchange Index, systematizing the states of growth and fall depending on their intensity [23]. However, the authors do not include the possibility of the market being in a stagnant state, which can be regarded as a fundamental shortcoming. Even though stocks are rarely stagnant, the inclusion of such a state may significantly change the state transition hypotheses. The COVID-19 pandemic and its relationship with the US stock market, volatility, correlation, and liquidity was also analyzed. Based on a Markov chain model, the results imply a connection between stock returns, volatility, and correlation, but not with the liquidity component during the turbulent period [12]. Changes in the traditional co-movement between variables usually appear a day prior to structural changes in returns, which may imply certain predictability of market risks. Recent research incorporating the Markov chain models study the Nigerian, Chinese, and London stock exchanges, [1, 24, 20] respectively.

Our study builds upon the preexisting knowledge and research with a clear goal of contributing towards stock market decision making of small individual investors. Through operational research incorporating the Markov chain models, we fill the gap in the literature left due to omittance of stagnant returns, technical analysis, and critical sector portfolios, as well as inclusion of the pre- and post-pandemic periods for the case of the New York Stock Exchange.

3. Methodological approach

The methodological approach in operational research (OR) is usually based on typical steps and procedures that allow for better creation, specification, and adequacy of the implemented mathematical models. In the following sections, we relate the standard OR procedures to the employed Markov chain model in the context of stock markets.

3.1. Defining the problem

Capital markets are fundamentally created by the agents, i.e., investors, rather than the publicly traded companies. In practice, the pool of investors is dominated by the small 'uninformed' investors, contrary to the large 'well-informed' ones. Classical portfolio theory suggests that, predominantly, market investors do not achieve above-average returns but rather a return proportional to that of the market in the long run. They do not possess significant expertise for deep quantitative analysis, so they focus on technical analysis quite often instead. Their decisions are based on historical price fluctuations with the single goal of finding regularities in their movements, which in turn can help predict the future. Starting from the behavioral concept of herding, a large number of investors base their decisions on general market tendencies rather than fundamental principles.

Since financial time series are often characterized with volatility clustering, with the Markov chain model we estimate the probability of persistent devaluation of stocks. Our main hypotheses are aimed at checking the presence of quick return stabilization of our respective random portfolio and faster transition from initial fall to a state of growth. Supporting such assumptions contributes towards developing a broader range of technical analysis and formulating new trading rules and expectations.

3.2. Developing the mathematical model

The main modeling approach in this study is the Markov process. We base the probability of a random portfolio being in a given state and transiting through predefined states on an examination of the return as a controlled variable in the model.

The Markov process is a model of real events which defines their randomness *a priori* It is suitable for studying processes which are not based on natural laws and do not follow a deterministic trajectory (e.g., the rotation of the Earth around its axis). Traditionally, it examines subsequent events that depend solely on the event that precedes them. The model excludes the conditionality of the present event on all past events. However, this does not imply that the information is not incorporated in the price of the asset. Even if today's status depends on that of yesterday, it is implicitly conditioned by the event that precedes yesterday's - and so forth in a chain. We employ a discrete-time Markov chain which is useful for monitoring random walks at the stock market. Having this in mind, the capability of trend analysis is of key importance irrespective of the type of the asset, e.g. securities, commodities, etc. [7].

Let X_t be a random variable in time t where $t = 0, 1, 2, 3, ..., n$. Subsequently, the state of the random variable at a given point in time is $X_t = \{X_0, X_1, X_2, ..., X_n\}$. For simplification, we define three 'market states' - stagnation, growth (i.e., bull market), and fall (i.e., bear market), making a state space $S = \{1, 2, 3\}$. Such set-up simplifies the subsequent matrix operations and is a fair representation of reality. The portfolio theory signals that the market efficiency depends on the level of informational integration in security prices, and so, semi-strong efficiency implies that all publicly available information and historical prices are already incorporated in asset prices [16]. This is a fair representation of the statement that the present stock price (not all previous) impacts the future price, which in turn is the main idea behind the Markov process. This conditional relationship can be stated as

$$P(X_{t+1} = s|X_t = s_t) \tag{1}$$

where P is the probability for reaching a certain state for each time period t throughout all possible states $s_0, s_1, s_2, ..., s$. The main output is the transition matrix which depicts the probabilities of transition through various states. In our case it is a square matrix with a (3×3) dimension which can be written as

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \tag{2}$$

such that the traditional definition of transition probability is $0 \leq p_{ij} \leq 1$ and where $p_{ij} = P(X_{t+1} = j|X_t = i)$ signals the probability that our process will transit from state i to state j in one trading day. The matrix rows indicate the state at time t while the columns show one step ahead at $(t+1)$. In our model, jumps between states are possible in any given time period, not implying any 0 or 1 restrictions in the transition matrix. Such structure follows the stochastic component of the stock market where interchangeable and reoccurring states are completely possible. Consequently, the proposed Markov chain of the study is irreducible i.e., ergodic. The stochastic model is said to be ergodic if it is possible to transition from any initial state to another state in a certain amount of steps, implied by

$$P_{ij}(t) > 0 \tag{3}$$

where P is the probability of transitioning from initial state i to state j in t steps, which does not necessarily one. Moreover, the employed model in this study restricts the possibility of absorbing states as there is no reason for such assumption when dealing with stock market series.

3.3. Data acquiring and analysis

For the purpose of our research, we used secondary data for the stocks traded at the New York Stock Exchange for the time period between 2018 and 2021. The data were collected from the NASDAQ - Stock Screener database [17]. Daily adjusted closing prices are taken for the stocks of companies coming from the basic industries - the energy, financial, health, and technology sectors. Only common stocks are used. The data are further transformed to obtain price returns through nominal prices

$$R_t = \frac{\Delta P}{P_{t-1}} \tag{4}$$

where ΔP is the one-period difference in prices and R is the return. is the one-period difference in prices and R is the return. We created a portfolio with 20 stocks based on a stratified sample of sectors. For the hypothetical portfolio an equal weighting system is employed, defining a portfolio return as

$$R_t{}^p = \sum_{i=1}^{n} \omega_i R_{it}, \forall\omega = \frac{1}{n} \tag{5}$$

where R^p is the portfolio return, R is the return of individual stock i , and ω denotes the wights given to each stock in the portfolio. General summary of the stocks traded at the NYSE classified by sectors and sizes used in the analysis are provided in Table 1.

Market Capitalization	***Nano-Micro (0-$300M)***		***Small-Medium ($300M-$10B***		***Large-Mega ($10B+)***	
Sector	*No. of companies*	*Share*	*No. of companies*	*Share*	*No. of companies*	*Share*
Basic Industries	5	3.85%	89	68.46%	36	27.69%
Energy	26	18.71%	84	60.43%	29	20.86%
Financial	187	27.3%	409	59.71%	89	12.99%
Health	8	6.67%	55	45.83%	57	47.5%
Technology	15	6.67%	140	62.22%	70	31.11%
Total	241	18.55%	777	59.82%	281	21.63%

Table 1: *Number and share of companies traded at the NYSE, classified by sector and market capitalization, December 2021.*

The random stratified sample portfolio consists of four stocks of companies per sector, getting a total of 20 stocks. To preserve anonymity, we deliberately decided not to provide the names of the companies used. In our fictional investment, 10% of the share are stocks of nano-micro companies, small-medium companies account for 75% of the stocks used, and the rest are large-mega corporations. All companies are American and their main activities include: oil and gas production, production of medical and dental instruments, medical specialization, management of hospitals and geriatric institutions, investment management, financial corporations with a wider range of operations, marketing, secondary car markets, non-life insurance, product packaging, and electronic data processing. The data cover the time period from January 2, 2018 to November 30, 2021, providing 986 observations for further analysis. The following figure depicts the nominal and relative price changes of our portfolio. Structural disruption is evident during the onset of the COVID-19 pandemic, after which the trend changes significantly from fairly stagnant to growing.

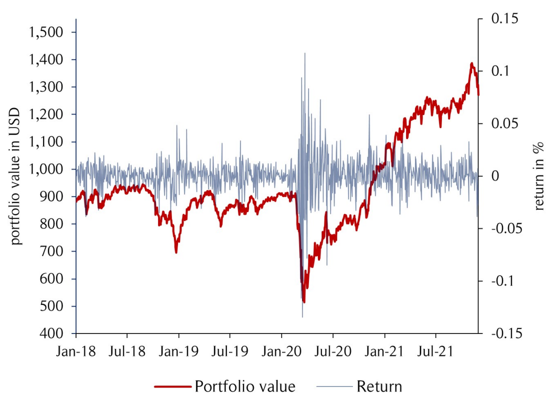

Figure 1: *Portfolio value and return, January 02,2018 - November 30, 2021.*

The main analysis is concentrated on measuring the probabilities for the portfolio to reach one of the three predefined states. We calculate them based on relative frequencies, depending on the number of days that the portfolio was at state s_i

$$P(X = s_i) = \frac{f_{s_i}}{\sum_{i=1}^{n} f_{s_i}} \tag{6}$$

where $i = 1, 2, 3$ are states of stagnation, growth and fall, respectively, and f is the frequency i.e., the number of daily occurrences for each state. The biggest problem is defining the stagnation state interval which should formally be fixed exclusively at 0% return, but for objective reasons such as too few observations, the mathematical model would be operable with only two states - something that does not represent reality. Arbitrarily, we define stagnation as $-0.1\% \leq s_1 \leq$

0.1%, with growth returns higher than the upper bound $s_2 > 0.1\%$ and decreasing returns below the lower bound of stagnation $s_3 < -0.1\%$. The absolute and relative frequencies (i.e., probabilities) of each state are presented in the following table 2.

State	Numerical notation	Interval	Frequency	Probability
Stagnation	1	$-0.1\% \leq s_1 \leq 0.1\%$	81	0.08223
Growth	2	$s_2 > 0.1\%$	488	0.49543
Fall	3	$s_3 < -0.1\%$	416	0.42234

Table 2: *Absolute frequencies and state probabilities of the Markov process.*

We checked for different intervals of return for the stagnation state, such as -0.5% and 0.5%, as lower and upper bound. The probability distribution changed significantly into 0.3472, 0.3492, and 0.3036 for each of the three states, respectively. However, defining the stagnation state in a narrower interval is favorable from both theoretical and practical aspects. Defining a too narrow stagnant state may result in it being practically non-existent. For example, if we further reduce the stagnation interval by half (between -0.05% and 0.05%), the frequency of stagnant states approximately halves to 44 observations and to a 0.0447 probability. This will logically impose changes in the probabilities of growth and fall states as well. A three-state Markov chain for monitoring stock returns is quite a simplification of reality. At this point, we ought to mention that the incorporation of more than three states or significant change in percentage intervals, as previously noted, may substantially change subsequent matrix operations. Consequently, caution is advised when interpreting the results.

4. Implementation, results, and discussion of the Markov process

The central idea of Markov chain processes is the formulation of the transition matrix between predefined states in which the random variable can be found. With it we define all nine outcomes in the specific case of a discrete stochastic process. Self-fulfilling outcomes with 0 and 1 restrictions are not possible in this study, as there is no reason why a portfolio could not transit in any state at any given time or remain 'trapped' in a specific one. The transition matrix can also be represented by the following diagram.

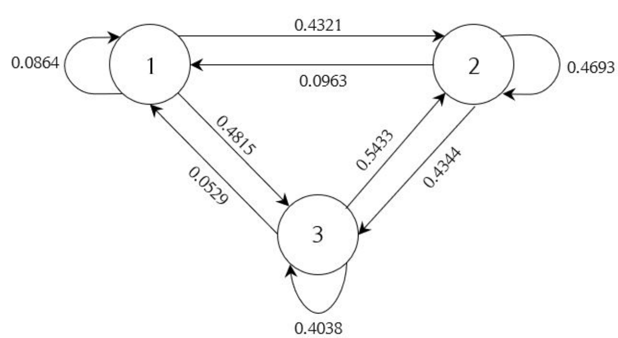

Figure 2: *Transition probabilities diagram.*

The obtained results are summarized within the transition matrix. Consequently, it can be observed that the portfolio is most likely to transition from a downtrend to a growth position with 54.33%, followed by the probabilities of a transition from stagnation to decline by 48.15%, and from growth to decline by 43.44%. If the portfolio is in a state of decline in the initial period, it is the least probable that in the next one it will 'jump' into a state of stagnation (5.29%). The same can be recorded in matrix form:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} 0.0864 & 0.4321 & 0.4815 \\ 0.0963 & 0.4693 & 0.4344 \\ 0.0529 & 0.5433 & 0.4038 \end{bmatrix} \tag{7}$$

However, the transition matrix indicates the probability of passing through states within one step (i.e., one time interval). We broaden the analysis considering a four-day trading interval after the initial period. With that we complete the five-day trading week on the stock exchange. The calculations of transition probabilities through n steps, which in our case is $n = 2, 3, 4$, are

$$P^2 = \begin{bmatrix} 0.0745 & 0.5017 & 0.4237 \\ 0.0765 & 0.4979 & 0.4256 \\ 0.0783 & 0.4972 & 0.4245 \end{bmatrix} \tag{8}$$

$$P^3 = \begin{bmatrix} 0.0772 & 0.4979 & 0.4249 \\ 0.0771 & 0.4980 & 0.4250 \\ 0.0771 & 0.4978 & 0.4251 \end{bmatrix} \tag{9}$$

$$P^4 = \begin{bmatrix} 0.0771 & 0.4979 & 0.4250 \\ 0.0771 & 0.4979 & 0.4250 \\ 0.0771 & 0.4979 & 0.4250 \end{bmatrix} \tag{10}$$

Transition matrices can be calculated even for the next 15, 100, or 500 periods, but after a certain number of steps (in our case, for the first time approximately 4 days from the initial moment) the system achieves a stable (long-term) state of transition probabilities. Essentially, this does not mean that they never change, but that it determines a certain level of predictability of the future state of the system [5]. After conducting additional power operations, we found that the stable state of the system has the following distribution

$$X \sim (\Pi_1 \; \Pi_2 \; \Pi_3) \sim (0.07709 \; 0.49788 \; 0.42503) \tag{11}$$

which can be explained as the portfolio having a 7.709% chance of being in a stable state, 49.788% chance of being in a state of growth, and 42.503% chance of being in a state of fall, irrespective of the initial state. The system distribution can be analyzed from two aspects - if we observe what happens on a particular day or if we draw conclusions without observing based on the previously determined probabilities. Whether we *a priori* determine a certain probability of achieving a particular state or study the system though observations, the distribution stabilizes at a predetermined level over a certain period.

In technical analysis, it is useful to know the probability of the portfolio transiting between states for the first time after certain number of trading days. We often encounter statements that stocks face difficulties returning 'on track' and that such phenomenon is evident in cases of multiple-day devaluation. In that manner, we define the probability that the portfolio will transit from state i to state j after exactly n periods. *Is it more likely that the portfolio will reach a growth position for the first time in 4 trading days after initial period decline or maybe the opposite is more probable?* The results of this analysis can be related to the initially set hypothesis. Expected first passage times (EFPT) are obtained through first passage probabilities calculated as

$$f_{ij}^{(n)} = p_{ij}^{(n)} - \sum_{k=1}^{n-1} f_{ij}^{(k)} p_{ij}^{(n-k)} \tag{12}$$

where $f_{ij}^{(n)}$ is a first passage probability and $p_{ij}^{(n)}$ is a transition probability, both from state i to state j in n trading days. Aggregated, these probabilities of first transition may be observed in matrix form $F^{(n)}$ with the resulting output presented in table 3.

	State in period 0	State in period n 1	2	3
$F^{(1)}$	1	0.0864	0.4321	0.4815
	2	0.0963	0.4693	0.4344
	3	0.0529	0.5433	0.4038
$F^{(2)}$	1	0.0671	0.2989	0.2293
	2	0.0682	0.2777	0.2502
	3	0.0737	0.2423	0.2614
$F^{(3)}$	1	0.0649	0.1425	0.1279
	2	0.0641	0.1340	0.1395
	3	0.0668	0.1136	0.1481
$F^{(4)}$	1	0.0598	0.0670	0.0632
	2	0.0590	0.0631	0.0778
	3	0.0617	0.0534	0.0826

Table 3: *First transition probabilities.*

We found that after two trading days, first-time transiting from stagnant to growth state has the highest probability. After three and four trading days, the highest probability of first transition is noted for the self-returning state of fall (value decreasing). Consequently, our indication leads to a higher portfolio probability of devaluation for the first time after a longer period rather than returning to growth. This means that the investment regains positive momentum fairly quickly, even after initial fall. Such results oppose the statement that a longer period is needed to regain value after achieving bad results due to various factors.

Apart from using first-passage probabilities, we can calculate the expected first passage time (EFPT) based on the initial state. According to Carter et al. [5], expected values can be obtained through solving several systems of simultaneous equations based on transition probabilities

$$E(T_{ij}) = m_{ij} = 1 + \sum_{k=1}^{N} p_{ik} m_{kj}, \ \textit{such that } k \neq j \tag{13}$$

where m is expected number of trading days until first passage from state i to state j. In cases where $i = j$ we discuss special cases of expected reoccurrence times. Those probabilities do not need to be calculated through simultaneous systems since we only need stable state probabilities - something that we already calculated. Consequently, we obtain the following set of equations:

$$m_{11} = \frac{1}{\Pi_1} \tag{14}$$

$$m_{12} = 1 + p_{11}m_{12} + p_{13}m_{32} \tag{15}$$

$$m_{13} = 1 + p_{11}m_{13} + p_{12}m_{23} \tag{16}$$

$$m_{21} = 1 + p_{22}m_{21} + p_{23}m_{31} \tag{17}$$

$$m_{22} = \frac{1}{\Pi_2} \tag{18}$$

$$m_{23} = 1 + p_{21}m_{13} + p_{22}m_{23} \tag{19}$$

$$m_{31} = 1 + p_{32}m_{21} + p_{33}m_{31} \tag{20}$$

$$m_{32} = 1 + p_{33}m_{32} + p_{31}m_{12} \tag{21}$$

$$m_{33} = \frac{1}{\Pi_3} \tag{22}$$

Simultaneously, we can solve equations 15 and 21, obtaining an expected 2.08 trading days for the first transition from initial stagnant state to a state of growth and expected 1.86 trading days for the first transition from initial fall to growth.
Proof. The simultaneous nature of the m_{12} and m_{32} variables defines a two-equation system as

$$\begin{cases} m_{12} = 1 + p_{11}m_{12} + p_{13}m_{32} \\ m_{32} = 1 + p_{33}m_{32} + p_{31}m_{12} \end{cases}$$

which can be further transformed into

$$\begin{cases} (1 - p_{11})m_{12} = 1 + p_{13}m_{32} \\ (1 - p_{33})m_{32} = 1 + p_{31}m_{12} \end{cases}$$

After incorporating transition matrix probabilities into our system, we obtain

$$\begin{cases} 0.9136m_{12} = 1 + 0.4815m_{32} \\ 0.5962m_{32} = 1 + 0.0529m_{12} \end{cases}$$

$$\begin{cases} m_{12} = \dfrac{1 + 0.4815m_{32}}{0.9136} \\ 0.5962m_{32} = 1 + 0.0529m_{12}\dfrac{1 + 0.4815m_{32}}{0.9136} \end{cases}$$

$$\therefore 0.56832m_{32} = 1.0579$$

leading us to the solutions

$$\begin{cases} m_{32} = 1.8615 \\ m_{12} = 2.076 \end{cases}$$

Solving equations 16 and 19, we found that the expected first passage time is 2.17 trading days from stagnation to fall and 2.28 trading days from initial growth to fall. In a general context, the portfolio is expected to recover quickly from initial downfall. Combining equations 17 and 20 yields expected first transition intervals - from growth and fall to stagnation - of 12.82 and 13.36 trading days, respectively. This may be accounted to the especially rare state of stagnation for the analyzed portfolio. Studying the results for the reoccurring states, we found that the shortest interval is noted for growth (2.01 trading days), followed by fall (2.35 trading days), and approximately 13 days for reoccurring stagnation. Based on these results, small investors can further develop their market expectations and capabilities for technical analysis. Instead of herding and focusing on extremely short-term trading strategies, it would be opportunistic to form a longer-term approach in investing, having in mind the frequent readjustment of the portfolio value. Moreover, these results signal market efficiency, which was initially expected for highly developed capital markets in the USA. To avoid repetition and over-extensiveness in stating solutions, we present a compact proof of the obtained results.
Proof. The simultaneous nature of the m_{21} and m_{31} and the m_{13} and m_{23} variables define two systems presented as

$$\begin{cases} m_{21} = 1 + p_{22}m_{21} + p_{23}m_{31} \\ m_{31} = 1 + p_{32}m_{21} + p_{33}m_{31} \end{cases} \quad and \quad \begin{cases} m_{13} = 1 + p_{11}m_{13} + p_{12}m_{23} \\ m_{23} = 1 + p_{21}m_{13} + p_{22}m_{23} \end{cases}$$

which can be further transformed into

$$\begin{cases} (1 - p_{22})m_{21} = 1 + p_{23}m_{31} \\ (1 - p_{33})m_{31} = 1 + p_{32}m_{21} \end{cases} \quad and \quad \begin{cases} (1 - p_{11})m_{13} = 1 + p_{12}m_{23} \\ (1 - p_{22})m_{23} = 1 + p_{21}m_{13} \end{cases}$$

obtaining the solutions after incorporating transition probabilities from equation 7

$$\begin{cases} m_{31} = 13.357 \\ m_{21} = 12.818 \end{cases} \quad and \quad \begin{cases} m_{23} = 2.2785 \\ m_{13} = 2.172 \end{cases}$$

Even though our results support the initially set hypotheses, there are some limitations to our research. First, to simplify matrix operations we used a three-state system. Even though a perfect approach doesn't exist, we believe that defining a wider range of accessible states is favorable. Second, we note that the main restriction in our study is the assumption of return intervals. There is no clear-cut border between different levels of return, which may be perceived as a subjective approach. Even though the results portray a quick shift from negative to positive portfolio returns, this may not mean that returning to a growth state is sufficient to offset the initial devaluation. An important limitation to point out is the chosen data frequencies in this study. Since the research employs daily returns, the usage of intraday returns, for example, may significantly change the transition and subsequent steady-state probabilities. We ought to explore such a case in a separate study since obtaining intraday data at this point was out of reach. Being completely aware of the noted research imperfections, we deliberately leave these questions and limitations open for our future work and for inspiring other scholars' research.

5. Conclusion

Through the implementation of Markov chain models in financial time series, we monitored the predictability of NYSE portfolio returns throughout the 2018-2021 period. We managed to formulate a rule of thumb in the technical analysis for small individual investors, confirming that frequent trading, immediate short positions in the initial downfall of portfolio value, and herding are unnecessary behavioral reactions. The resulting matrix of transition indicates a greater probability for transiting into a state of growth after initial fall for the analyzed period, with a possibility of reaching long-term steady-state probabilities in approximately four trading days. Moreover, the expected first passage times (EFPT) suggest that the hypothetical portfolio needs the least time (1.86 trading days) to jump from a state of fall to growth, unlike the reverse situation which needs approximately 2.28 trading days to occur. However, the three-state system defined in our study is not a clear definition of reality but rather a necessary simplification. A clearer image could be obtained through a larger set of multi-level states of return, discussing the sufficiency of achieved returns in offsetting initial fluctuations. Moreover, our initial analysis can be further broadened with a larger sample of observations, preferably at least a decade-long window. Additionally, we leave the study of a non-US stock market with larger volatility open for future work. Finally, the capacity for structural changes in portfolio returns may vary and is dependent on exogenous shocks, policy changes, as well as the investors' risk perceptions. The impact of such variables should be studied in a different environment and model setting rather than strictly through Markov chains.

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