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A METHOD FOR CALCULATING THE PROBABILITY OF RUIN OF AN INSURANCE COMPANY

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Abstract Insurance companies frequently ask their actuaries to do calculations of the probability of the company being ruined. These calculations are based on complex data analysis of the last insurance period, which requires building mathematical models and using the data as an input parameter in these models. For this purpose, an algorithm, which presents a simplified mathematical model and provides approximate results of the outcomes (later used for managerial decisions), was prepared. Modeling of different outcomes based on various different inputs shows that the probability of the company to become ruined is inversely proportional to the written premium.

The algorithm developed in this paper is illustrated with tables. The model is presented to the reader in a way that the reader can reproduce the calculations and build a custom data model.

Keywords: probability of ruin, insurance, premium, claim, gross premium, risk premium, pure risk premium, safety loading, expences, profit.
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1. INTRODUCTION

The prediction of the outcome in which company would be ruined ([1]) is a problem with 3 unknowns: the number of claims k, the amount of the claims z and the time t when claim happened. This process can be depicted as a pool which has the received premium as input, and the paid claims as output (Figure 1). If we analyze the period $T = [T_0, T_1]$, for example exactly one year, then we would not be interested in the exact date of the claim, but just the number of claims that have been claimed K_T and the sum of their amounts

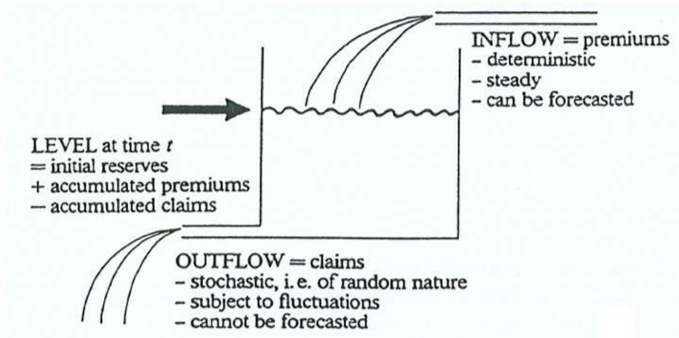

Fig. 1. Ruin of an insurance company (from [1], p. 8).

z_i, $i = 1, 2, \ldots, K_T$. The total value of claims for period T is Z_T, where

$$Z_T = \sum_{i=1}^{K_T} z_i$$

Each of these two parameters Z_T and K_T is stochastic and the overall probability for ruining of the company is dependent of their values. Accumulated balance during period T is calculated by the formula:

$$U_T = U_0 + B - Z_T \tag{1}$$

where U_0 is initial reserve, B is premium income and Z_T is total of claims cost in period T (Figure 2). A ruin of the company is a situation in which the accumulated balance of the insurance company during the period T is less than zero. Consequently, a ruin probability [1] of the company in a period T is the probability that the accumulated balance of the insurance company during the period T is less than zero, i.e., $P\{U_T < 0\}$.

This paper aims to offer an algorithm based on mathematical method for calculation of the probability of insurance company ruining. Initially the model is simplified with using only one variable - average claim amount. This simplified model can be implemented in several insurance types (for example in a third party liability or any other obligatory liability).

2. THE MODEL

We will consider a model which can be applied to insurance classes where the range of a limit of coverage is limited, known and narrow. In this case we can approximate the limit of coverage (also called a sum insured) with the average value SI. For example, one such class in our country is third party liability (TPL), which have same limit of coverage depending of the type of the vehicle.

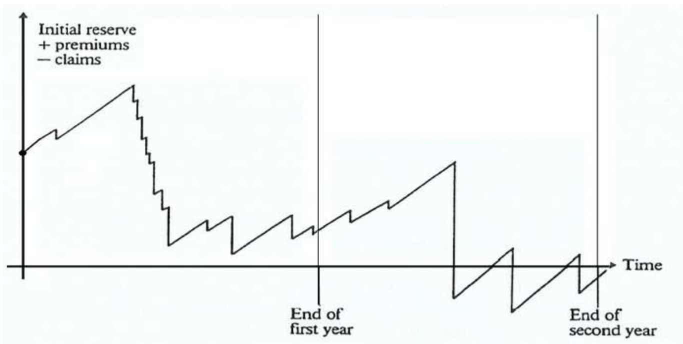

Fig. 2. Ruin of an insurance company (from [1], p. 8).

Let us assume that an insurance company sell policies of only one insurance class, and in all of them we can approximate the limit of coverage with a same value SI and gross premium p_g.

Gross premium of an insurance company may be represented as shown on Figure 3.

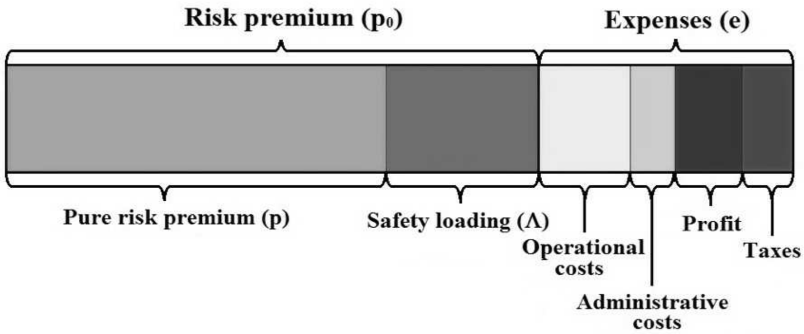

Fig. 3. Gross premium split (from [1], p. 8).

Pure risk premium coefficient or loss of the sum insured coefficient p is ratio of the amount of claims in the last X years to the sum insured. Safety loading coefficient Λ defines the part of the risk premium intended for compensation of the claims and provides protection against any uncertainty ([1], [2]). If safety loading is set to 0, the anticipated claims may occur in the beginning of the insurance period and the premium collected is not sufficient to cover these claims, although when considering a longer period of time ideally pure risk premium would be in balance with the claims. The sum of the pure risk premium coefficient (p) and the safety loading coefficient (Λ) determine the net premium coefficient $p_0 = p + \Lambda$, as shown in Figure 3. The ratio of the safety loading coefficient (Λ) to the pure risk premium coefficient (p) is denoted by $\lambda = \Lambda/p$ and is called safety loading ratio.

Net insurance or risk premium for one insurance is $P = SI \cdot p_0$, pure risk premium is $P_p = SI \cdot p$ while the safety loading is $\Lambda \cdot SI$.

Example 1. Let suppose that in one company the pure risk premium coefficient and the safety loading coefficient are $p = 0,0300\%$ and $\Lambda = 0,0075\%$. In Table 1 are illustrated the terms defined above.

Sum insured	Pure risk premium coefficient	Safety loading coefficient	Risk premium coefficient	Safety loading to risk premium ratio	Safety loading ratio	Risk premium
SI	p	Λ	$p_0 = p + \Lambda$	Λ/p_0	$\lambda = \Lambda/p$	$P = SI \cdot p_0$
1	2	3	4	5	6	7
			(2+3)	(3/4)	(3/2)	(1· 4)
100000	0,03%	0,0075%	0,0375%	0,2	0,25	37,5

Table 1 Illustration of the parameters of an insurance company

The expenses (e) are umbrella term to cover all the different kinds of administrative and operational costs, taxes and profit, i.e., a part of the premium that is not intended for direct compensation of claims. The expenses, for the time being, will not be split into parts, but will be taken as a whole. Gross premium coefficient is $p_g = p_0 + e$. The ratio of the expenses coefficient (e) to the gross premium coefficient (p_g) is expenses margin and is denoted by e_m.

The expenses are expressed as $E = SI \cdot e$ and Gross premium is $P_g = p_g \cdot SI$. From the above follows that $P_g = p_g \cdot SI = (p_0 + e) \cdot SI = p_0 \cdot SI + e \cdot SI = P + E$, i.e., $P_g = P + E$.

We will consider that the whole premium income is collected and earned.

Example 2. If we let the expenses coefficient e in the company from the *Example 1* to be 0,0050% then we obtain the results given in Table 2.

We will consider that insurance companies sold number of policies (i) during the period of calculation (T) and they have the same probability for claim occurrence. Consequently, they have the same size and structure of the gross premium (in practice, only large companies have enough date to create structure of gross premium split (tariff). Smaller companies take and apply tariffs of large companies). Accordingly, every company will have a total net premium $B = P \cdot i$, total insured sum $TSI = SI \cdot i$ and the expected value of

Pure risk premium	Expenses coefficient	Gross premium coefficient	Expenses margin	Expenses loading	Gross premium
$P_p = SI \cdot p$	e	$p_g = p_0 + e$	$e_m = e/p_g$	$E = SI \cdot e$	$P_g = p_g \cdot SI = P + E$
7a	8	9	10	11	12
$(1 \cdot 2)$		(4+8)	(8/9)	$(1 \cdot 8)$	$(1 \cdot 9)$=(7+11)
30,00	0,005%	0,0425%	11,765	5,0	42,50

Table 2 Illustration of the parameters of an insurance company

claim amount is calculated as the product of the pure risk premium coefficient and the total sum insured ($z = p \cdot TSI$).

Now we will simplify the classical model. Classical actuarial approach applies a function of the distribution of the amount of claims. Instead of using the stochastic value of the amount of the claims occurred, we are going to use the previously known statistical value of the average claim amount (m). This substitution can be done only if we have a stabile statistical evaluation of the average claims for several years and the value of the dispersion of the claims is not greater than the average value of the claims. With the introduction of this simplification we can get a model of an insurance company with only one stochastic value - the number of occurred claims. The expected average number of claims will be the ratio between the expected total claim amount and the mean claim amount ($k = z/m$). Also, we will assume that all claims to a given portfolio will occur and be solved during the observed period T.

Let us consider the cases where the number of claims occurred (J_T) is greater than the expected number k in the period T. These cases are the subject of particular attention because they may ruin the insurance company. It is clear that buffer for the increased number of claims is the total safety loading amount ($L = \Lambda \cdot SI \cdot i$), which is proportional to the number of sold policies (i). The total amount of safety loading amount is the difference between total net premium and the expected total amount of claims ($L = B - z$).

The number of claims that can be buffered by safety loading amount and the initial reserve may be calculated by the following formula $J_\Lambda = [(L + U_0)/m]$.

Now, let us consider the expenses. The only part of expenses that may be used to buffer the consequences of the excessive number of claims is the profit.

If we take that the planned ratio of profit is P_D percentages of the gross premium, the planned profit from policy is $D = P_D \cdot P_g$. The total planned profit of the company D_T is the product of the planned profit of policy D and the number of insured risks i. Let us consider the case when the number of occurred claims is larger than the planned number and the source of their compensation is the sum of the total safety loading amount (L), the part of the expenses intended for profit (D_T) and the initial reserve (capital) of the companies.

The number of additional claims before a ruin of the company may be calculated by the following formula $J_{\Lambda_D} = [(L + U_0 + D_T)/m]$. It means that the number of claims that company can survive is

$$S = k + J_{\Lambda_D} = \left[\frac{z + L + U_0 + D_T}{m}\right] \tag{2}$$

In the case when the number of occurred claims is greater than S it leads to ruining the company because $(S + 1) \cdot m > B + D_T + U_0$.

Lets consider the probability of ruining the company.

We will build a model for the probability of ruining the company by the number of claims incurred (J_T).

Let $p_i(J_T)$ be the probability of occurring exactly J_T claims while i is the number of risks insured, and p_k is the probability of claim occurrence in a period T.

$$p_i(J_T) = C_i^{J_T} \cdot p_k^{J_T} \cdot (1 - p_k)^{i - J_T} \tag{3}$$

The probability p_k in this model is a relationship between the expected number of claims k and the number of risks insured i and is calculated as $p_k = k/i$.

The equation (3) is practically unusable because for large i and J_T we get huge numbers that are hard to calculate even on a computer. For example, if we have a company that sold 8000 polices and the expected number of claims is 48, then

$$p_k = \frac{k}{i} = \frac{48}{8000} = 0,006 \tag{4}$$

Now, if we want to calculate the probability that there will be 60 claims, we have

$$p_{8000}(60) = C_{8000}^{60} \cdot p_k^{60} \cdot (1 - p_k)^{8000-60} = C_{8000}^{60} \cdot 0,006^{60} \cdot 0,994^{7940} \tag{5}$$

Therefore, in order to calculate the probability that a given number of claims appeared, we will apply Poisson formula which gives a good approximation at low probabilities. According to Poisson formula

$$p_i(J_T) \approx \frac{a^{J_T}}{J_T!} e^{-a} \tag{6}$$

where a is the mathematical expectation of the number of claims incurred. In our model, $a = ip_k = k$. Therefore,

$$p_i(J_T) = C_i^{J_T} \cdot p_k^{J_T} \cdot (1 - p_k)^{i - J_T} \approx \frac{k^{J_T}}{J_T!} e^{-k} \tag{7}$$

Let μ be the random variable - the number of occurred claims. The probability $P(\mu \leq S)$ to occur at most S claims, where $S = k + J_{\Lambda_D}$, which is the probability for the company to survive, is a sum of the probabilities to occure exactly 0, 1, 2, ..., S claims, and is calculated with the following formula:

$$P(\mu \leq S) = \sum_{j=0}^{S} C_i^j \cdot p_k^j \cdot (1 - p_k)^{i-j} = 1 - \sum_{j=S+1}^{\infty} C_i^j \cdot p_k^j \cdot (1 - p_k)^{i-j} \tag{8}$$

Applying the Poisson formula, we get:

$$P(\mu \leq S) = \sum_{j=0}^{S} \frac{k^j}{j!} e^{-k} = 1 - \sum_{j=S+1}^{\infty} \frac{k^j}{j!} e^{-k} \tag{9}$$

The probability of ruining the company is:

$$P(\mu > S) = 1 - P(\mu \leq S) = \sum_{j=S+1}^{\infty} \frac{k^j}{j!} e^{-k} \tag{10}$$

where S is defined with (2).

In what follows, we will illustrate the model on three insurance companies: Company 1 that sold 1.000 policies, Company 2 - 8.000 and Company 3 - 16.000 policies, as presented in Table 3 and Table 4. Let the mean claim amount for all three companies is the same and set to 5.000. Let suppose that the sum insured for all three companies is $SI = 100000$, the pure risk premium coefficient is $p = 0.03\%$ and safety loading coefficient is $\Lambda = 0.0075\%$. Therefore, we can use the calculations from Table 1 and Table 2 in *Example 1* and *Example 2*. We will assume that initial reserve (capital) of the companies (U_0) is equal to zero.

In Table 3 and Table 4 are given the calculations for these three companies. In the first column of Table 3 and Table 4 (the column denoted with 13) is given the number of risks insured i for each of the three companies.

In the columns denoted with 14 and 15 are given the risk premium income B and the total sum insured TSI, respectively. In a column 16 is calculated the total expected size of claims z. Using these values and the assumption that the mean claim size for all three companies is $m = 5000$ (column 17), in column 18 is calculated the expected number of claims k for each company. In column 19 is given the number of claims occurred J_T, when J_T runs from

Number of risks insured	Risk premium income	Total Sum insured	Total expected size of claims	Mean claim size	Expected number of claims	Number of claims occurred in period T	Total of claims cost in period T
i	$B = P \cdot i$	$TSI = SI \cdot i$	$z = p \cdot TSI$	m	$k = z/m$	J_T	$Z = m \cdot J_T$
13	14	15	16	17	18	19	20
	$(7 \cdot 13)$	$(1 \cdot 13)$	$(2 \cdot 15)$		$(16/17)$		$(17 \cdot 19)$
1000	37500	100000000	30000	5000	6	6	30000
						7	35000
						8	40000
8000	300000	800000000	240000	5000	48	48	240000
						49	245000
						50	250000
						51	255000
						52	260000
						53	265000
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
						62	310000
						63	315000
16000	600000	1600000000	480000	5000	96	96	480000
						97	485000
						98	490000
						99	495000
						100	500000
						101	505000
						102	510000
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
						125	625000
						126	630000

Table 3 Illustration of the method applied to three insurance companies

Number of risks insured	Number of claims occurred in period T	Initial reserve	Profit coefficient planned	Profit planned per insurance	Total expected profit	Gross accumulated balance during period T	Number of claims compensated by safety loading amount plus initial reserve plus profit
i	J_T	U_0	P_D	$D = P_D \cdot P_g$	$D_T = D \cdot i$	$U_D(T) = U_0 + B + D_T - Z$	$J_{\Lambda_D} = [U_D/m]$
13	19	21	22	23	24	25	26
				(22 · 12)	(23 · 13)	(21+14+24-20)	(25/17)
1000	6	0	4%	1,7	1700	9200	**1**
	7					4200	
	8					-800	
8000	48	0	4%	1,7	13600	73600	**14**
	49					68600	
	50					63600	
	51					58600	
	52					53600	
	53					48600	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	62					3600	
	63					-1400	
16000	96	0	4%	1,7	27200	147200	**29**
	97					142200	
	98					137200	
	99					132200	
	100					127200	
	101					122200	
	102					117200	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	125					2200	
	126					-2800	

Table 4 Illustration of the method applied to three insurance companies

the expected number of claims to the number of claims that will ruin the company. Knowing the mean claim size m, in the column 20 is given the total of claim costs Z for each possible value of J_T. In the columns 21 and 22 are given the assumptions that the initial reserve U_0 of each company is 0 and the profit P_D of each company is 4% of the gross premium. The planed profit per insurance D and the total expected profit D_T are given in a column 23 and 24, respectively. The sum of the initial reserve U_0, the total risk premium income B and the total expected profit D_T decreased by the total claims cost Z give the gross accumulated balance during a given period (column 25). From this column we can see how the gross accumulated balance changes when the number of the occurred claims increases. Clearly, the balance of the company decreases when the number of occurred claims increases. This number divided by the mean claim size gives the number of claims above the expected number of claims that can be served using the safety loading amount $L = B - z$, the initial reserve U_0 and the expected profit D_T (column 26).

Number of risks insured	Risk premium income	Expected number of claims	Aggregate claim amount expected	Safety loading amount	Initial reserve	Total expected profit
i	$B = P \cdot i$	$k = z/m$	$z = m \cdot k$	$L = B - z$	U_0	$D_T = D \cdot i$
13	14	18	16	27	21	24
	$(7 \cdot 13)$	(16/17)	$(17 \cdot 18)$	(14-16)		$(23 \cdot 13)$
1000	37500	6	30000	7500	0	1700
2000	75000	12	60000	15000	0	3400
4000	150000	24	120000	30000	0	6800
6000	225000	36	180000	45000	0	10200
8000	300000	48	240000	60000	0	13600
9000	337500	54	270000	67500	0	15300
10000	375000	60	300000	75000	0	17000
12000	450000	72	360000	90000	0	20400
14000	525500	84	420000	105000	0	23800
16000	600000	96	480000	120000	0	27200

Table 5 Calculating the probability of ruin of the company - part 1

If we are interested only for the number of additional claims that can be served using the safety loading amount, the initial reserve and the expected profit, there is no need to make the whole table. In that case, it is enough to make the calculations in the row in which the number of occurred claims J_T is equal to the expected number of claims, i.e., the first row for each company in the table.

From Table 3 we can see that the probability of claim occurrence in a period T is same for all the three companies, and is calculated as:

$$p_k = \frac{k}{i} = \frac{6}{1000} = \frac{48}{8000} = \frac{96}{16000} = 0,006 \tag{11}$$

Number of risks insured	Number of claims amortised by safety loading amount plus initial reserve	Number of claims amortised by safety loading amount, initial reserve plus profit	Number of claims that company can survive	Probability of survive	Probability of ruin
i	$J_\Lambda = [(L+U_0)/m]$	$J_{\Lambda D} = [(L+U_0+D_T)/m]$	$S = k + J_{\Lambda D}$	$P(\mu \leq S)$	$P(\mu > S)$
13	26a	26	28	29	30
	(27+21)/17	(27+21+24)/17	(18+26)		(1-29)
1000	1	1	7	0,74398	0,25602
2000	3	3	15	0,84442	0,15558
4000	6	7	31	0,93224	0,06776
6000	9	11	47	0,96804	0,03196
8000	12	14	62	0,97841	0,02159
9000	13	16	70	0,98481	0,01519
10000	15	18	78	0,98927	0,01073
12000	18	22	94	0,99458	0,00542
14000	21	25	109	0,99625	0,00375
16000	24	29	125	0,99807	0,00193

Table 6 Calculating the probability of ruin of the company - part 2

In Table 5 and Table 6 is obtained the probability of ruin of the company for different values of the number of insured risks i. In a column 26a is given the number of claims J_Λ that can be served by the safety loading and the initial reserve, while in the column 26 is given the number of claims J_{Λ_D} that can be amortized if the profit is also used for this purpose. The number of claims S that the company can survive is given in a column 28. The company will survive if the number of occurred claims μ is smaller than S. Using equation (9) we obtain the probability that the company will survive (a column 29). In a column 30 is given the probability that the company will not survive, i.e., the probability of a ruin of the company.

Obviosly, the probability of ruin is a function that depends on the number of risks insured, which is shown in Table 6. It is apparent from the Table 6 that the probability of ruining the company declines with the number of risks insured, and the function is not linear.

Based on results given in Table 6, we built the chart shown on Figure 4.

The red line on the graph marks the probability of ruin of 0,01 (the probability of ruin of 0.01 is often the mandatory limit that the regulators or companies themselves set to protect the company from ruin). At the conditions assumed within this model, the function of ruin intersects the line of 0,01 probability at the number of risks insured close to 10.000. This means that when the company exceeds the number of 10.000 policies sold, then the probability of ruin is less than 0,01.

The graph of the function $P(\mu > S)$ - probability of ruin as a function of the number of risks insured, is very steep at the beginning, which means that the insurance company with less then 6.000 risks insured is unstable. For example, if the company operates at chance of ruin at 5%, it should be ruined during 20 years of its existence, or if there are 20 companies like this, one of them will be ruined, according to the mathematical expectation.

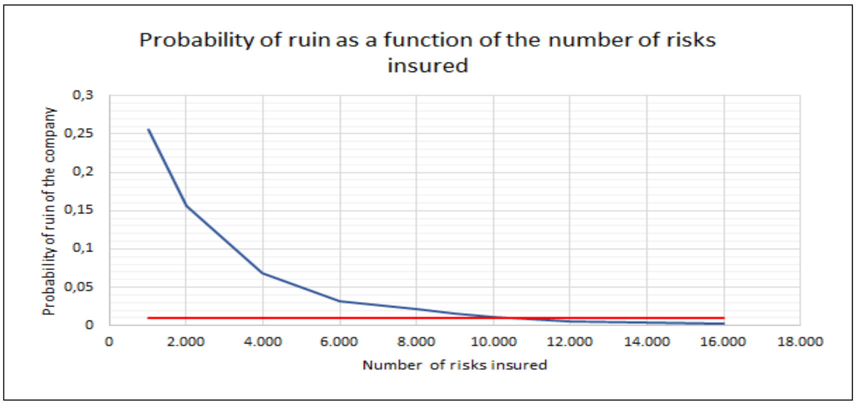

Fig. 4. The probability of ruin as a function of the number of risks insured.

In the example, for a simplification and better overview U_0 is equal to zero, but the upper stated formulas are applicable with any stated amount of the initial reserve U_0.

3. CONCLUSION

With this algorithm we can simulate the performance of the Insurance Company, we can model different outcomes and we can project certain aspects of the results of new insurance products market introduction. Software based to the model offered in this paper may be built in excel very easy. In the paper we have proved that as the number of policies sold grows, the probability of ruining the company decreases, i.e., the probability of the company to become ruined is inversely proportional to the written premium.

The insurance company can project the next business actions based on the model explained in this paper and can manage the risk and compliance with the standing regulation limits in the country.

The consequence of the stated in the paper are the the following:

- Smaller insurance companies have to increase sales;
- Smaller insurance companies have to reinsure its portfolio to lower the ruin probability (at additional costs);
- Increased sales of policies decrease the insurance risk, and contributes to the stability of the insurance company;
- Insurance risks lead to commercial risks, and they are strongly interconnected.

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