Union of Mathematicians of Macedonia - ARMAGANKA

IX SEMINAR OF DIFFERENTIAL EQUATIONS AND ANALYSIS

and

1st CONGRESS OF DIFFERENTIAL EQUATIONS, MATHEMATICAL ANALYSIS AND APPLICATIONS

CODEMA 2020

Proceedings of the CODEMA 2020
Зборник на трудови од CODEMA 2020

Skopje, 2021

WORKING WITH MATHEMATICALLY GIFTED STUDENTS IN PRIMARY EDUCATION – PART TWO

ISBN978-608-4904-09-0 UDC: 373.3.011.3-056.317:51

Risto Malcheski[1)], Aleksa Malcheski[2)], KaterinaAnevska[3)], Metodi Glavche[4)]

Abstract. Most commonly, the differentiation of mathematics instruction is realized by working with mathematically gifted students. Traditionally this is as a part of mathematical sections, by preparing the students for mathematical competitions. However, we consider that it is going to be much more efficient if it is implemented continuously and following a specially prepared program for these students. For this purpose in this paper, we present the integral teaching program for working with gifted students in Grade VII (Grade VI) in the nine-year long (eight-year long) primary education. This teaching program is an upgrade of the corresponding teaching programs for the given grades, developed by the same authors. Alongside, almost the complete teaching program is supplemented by a system of tasks for particular topics (solved and unsolved), given in [10] – [19].

1. INTRODUCTION

Most often, studying creativity, giftedness and talent comes down to defining them and creating instruments for recognizing and identifying talented and gifted children. In this paper, we are not going to analyze these questions, because they are already analyzed in the existing literature, such as in [1], [2], and [20]. Additionally, the gifted students and their education, i.e. the methods and forms of work with the gifted students are another important aspect in the work with these students. The existing literature also analyzes these questions, and integral programs for work with gifted students in mathematics, aged 7 -11, are presented in [3] –[5].

Systems of tasks from different areas are also present in the previously mentioned works and they can serve for work with the mathematically gifted students aged 7 to 11. These systems of tasks are compiled in the frames of the European MATHEU project, in which they are overambitiously called didactic pillars. We can say that the use of particular systems of tasks when working with the mathematically gifted students is used by several authors many years prior to the realization of the MATHEU project. For example, such a system of tasks is presented in [6], which is smaller in volume and it is intended for acquisition of the scientific methods by the mathematically gifted students.

2010 *Mathematics Subject Classification.* Primary: 97-XX
Key words and phrases. mathematically gifted students, primary education

In the following analysis, we are going to present an integral teaching program for work with mathematically gifted students aged 12-13, and for the topic Theory of numbers, we are going to present a system of tasks, which we believe will serve when working with the mathematically gifted students.

2. PROGRAM FOR WORKING WITH MATHEMATICALLY GIFTED STUDENTS AGED 12-13

Integral teaching programs when working with mathematically gifted students in the initial education, i.e. ages 7 -11 are presented in [3] – [5]. This part will also cover it in terms of work with students aged 12-13, i.e. with students from seventh (sixth) grade in the nine-year(eight) long primary education. In the preparation of this program, as well as in the preparation of the previous programs, we implemented experiences from the work with mathematically gifted students from several countries, the ones coming from the Russian Federation, Bulgaria, Romania, Serbia, Croatia, Bosnia and Herzegovina and Macedonia being the most prominent.

The goals of the teaching program for students aged 12-13 are the following:

- The students understand the term fraction, do the operations with fractions and use them when solving tasks,
- The students represent values in terms of percentages and use percent operations,
- The students understand the need for the introduction of negative numbers and generating the set of integers, as well as to acquire the term absolute value and learn the arithmetic operations in the set of integers,
- The students understand the construction of the set of rational numbers and to learn the arithmetic operations and their properties in the set of rational numbers,
- The students solve equations with one variable and use them when solving a word problem tasks,
- The students understand the term mapping, and use axial and central symmetry to map figures,
- The students distinct axial and central symmetric figures as well as to determine the axis of symmetry and center of symmetry of figures
- The students understand the characteristics of the triangle and quadrangle, their properties and classification,
- The students calculate the perimeter of triangles and quadrangles,
- The students understand the relation of congruent triangles and use the attribute of congruence in simple tasks,
- The students understand the need for proving a theorem and are able to prove some theorems,
- The students are able to solve logical tasks,

- The students learn the basic combinatory principles and combinatory configurations in an informal way,
- The students are able to solve tasks with coloring, covering and cutting figures into simpler figures, and creating a new figure from the cut pieces,
- The students use the method of invariants (in an informal way),
- Develop the qualities of thinking, such as: elasticity, pattern-making, depth, rationalization, and critical thinking,
- The students learn in an informal way the scientific methods: observation, comparison, experiment, analysis, synthesis, and the axiomatic method
- The students learn in an informal way the types of conclusion-making: induction, deduction, and analogy, while presenting suitable examples from which the students will learn that analogy based conclusion is not always correct.

The following content needs to be learned to achieve the previously mentioned goals:

Topic I. ***Integers and rational numbers*****:** direction, positive and negative numbers, additive inverses, set of integers, absolute value and comparing integers, fractions, types of fractions, extending and reducing fractions, creating common denominators for the fractions and comparing fractions, arithmetic operations with fractions (additions, subtraction, multiplication and division), complex fractions, order of the arithmetic operations and calculating the value of a numerical expression, percentages: concept for percent and percent value, transforming a decimal number into percent, as well as a percent in the form of a fraction and a decimal number, the set of rational numbers, absolute value of a rational number and comparing rational numbers, arithmetic operations with rational numbers (addition, subtraction, multiplication and division) and their properties, calculating the value of a numerical expression with rational numbers, solving linear equations containing rational numbers, exponentiation with a natural number as an exponent, operations with exponents.

Topic II. ***Theory of numbers*****:** general and specific indicators for divisiveness (divisiveness with 7, 8, and 11), numeric systems, greatest common divisor, Euclid's algorithm and lowest common multiplier, prime and composite numbers, Eratosthenes sieve, infinity of the set of prime numbers, basic arithmetic theory, elementary Diophantine equations.

Topic III. ***Word problem tasks*****:** tasks with numbers and numerals, tasks with measure numbers, tasks with percentages, and tasks involving money.

Topic IV. ***Geometry*****:** mapping, definition and basic properties, axial symmetry, definition and basic properties, mapping figures by axial symmetry, axisymmetric figures, line segment bisection, angle bisection and its properties, perpendicular lines, distance from point to plane, central symmetrical figures, triangle: elements of a triangle and types of a triangle, altitude of a triangle and orthocenter of a triangle, median of a triangle and a triangle centre, bisection of a side of a triangle and a center of a excircle of a triangle, bisection of an angle of a triangle and a center of an incircle of a triangle, circumscribed circles of a

triangle, tangent of a circle, congruent figures, congruent triangles, indicators for congruent triangles: the indicator side-angle-side (SAS), the indicator angle-side-angle (ASA) and the indicator side-side-side(SSS), properties of an isosceles triangle, parallel lines, parallel postulate, transversal of parallel lines, angles of a transversal, angles with parallel sides and angles with normal sides, sum of interior angles of a triangles and sum of exterior angles of a triangle, median of a triangle, relation between sides and angles of a triangle, basic constructive tasks, construction of a tangent of a circle and construction of a triangle with given elements, elements of a quadrangular, sum of angles in a quadrangular, quadrangular types, parallelogram, parallelogram properties, rhombus and square, trapezoid, elements of a trapezoid and its properties, isosceles trapezoid and kite, construction of a quadrangular with given elements, perimeter of a quadrangular: parallelogram, trapezoid and kite, concept of area, area of a triangle and a quadrangular, concept for volume, volume of a square and a cuboid.

Topic V. ***Sets, logic and combinatorics*****:** sets, number of elements of a set, operations with a set, Venn diagram, determining a set in given conditions, logical tasks, games and strategies, Dirichlet's principle (intuitive use), counting and recounting by using the principles of sum, difference and product (intuitive use), coloring, covering and cutting, elementary games and strategies.

3. AN EXAMPLE OF A SYSTEM OF TASKS FOR WORKING WITH MATHEMATICALLY GIFTED STUDENTS AGED 12-13

Similarly, as the programs presented in [3], [4] and [5], for the realization of the suggested program for working with mathematically gifted students aged 12-13, we must prepare adequate teaching aids, i.e. textbooks that will be mandatorily supplemented by adequate collections of tasks. Further on, we are going to give an example of a system of tasks for this age group on the topic of the Theory of numbers.

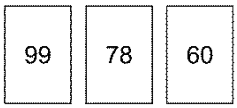
Task 1. What is the sum of all five-digit numbers created with the digits 1,2,3,4 and 5, each digit occurring exactly once?

Task 2. Between the digits of any two-digit number written with the same digits insert two zeroes. The new number is exactly 91 times greater than the initial number. Prove!

Task 3. Two numbers act like 19 : 8. If the sum of these numbers is divided by their difference, we get a quotient of 2 and a remainder of 20. What are these numbers?

Task 4. Let a,b,c,d be different digits and each of them be a prime number. Write all $\overline{ab10cd}$ divisible by 264.

Task 5. On 3 cards, Marko wrote 6 different numbers, one on each side of the cards. The sum of the two



numbers written on each of the cards is equal. Marko set the cards in such a way that only three numbers can be seen, as on the image on the right. We know that the hidden numbers are prime numbers. Find the hidden numbers.

Task 6. Determine a two-digit number such that the sum of the number and the number written with the same digits in a reverse order is a square of a natural number.

Task 7. Let T be a three-digit number. A new three-digit number is formed by switching the position of the last two digits. Their sum is a four-digit number which begins with 195. Determine all such three-digit numbers.

Task 8. The first row of a table with three columns contains the numbers a, b and c. The second row below contains the numbers $a_1 = a - b$, $b_1 = b - c$ and $c_1 = c - a$. Analogously, the third row contains $a_2 = a_1 - b_1, b_2 = b_1 - c_1$ and $c_2 = c_1 - a_1$, etc. Find the greatest possible number in the order in which the number 2013 can occur in an adequate choice of the starting numbers.

Task 9. Determine the four-digit number $\overline{xyzt}$ for which the following is satisfied:

$$\overline{xyzt} + 4 \cdot \overline{yzt} + 2 \cdot \overline{zt} = 2018.$$

Task 10. Instead of a and b use such digits so that the number $\overline{4a1b}$ is divisible by 12.

Task 11. When dividing the number $n+125$ with the number 19 there is a remainder of 7. Calculate the remainder when dividing the number n by 19.

Task 12. Find the numbers x, y so that the number $\overline{x74y}$ is divisible by 15.

Task 13. Determine the digits a and b so that the number $\overline{a783b}$ is divisible by 56.

Task 14. Let $S(n)$ be sum of the digits of the natural number n. Is there exist a natural number for which the following applies?

a) $S(k) + S(k^2) = 2008$,

b) $S(k) + S(k^2) = 2009$.

Task 15. The product of two natural numbers is 384, and their lowest common multiplier is 48. Determine the numbers.

Task 16. Find the greatest four-digit number for which when divided by 3, 4, 5, 6, and 7, the remainder is 2.

Task 17. Determine all three-digits natural numbers for which when divided by 7 the remainder is 2, when divided by 9 the remainder is 4, and when divided by 12 the remainder is 7.

Task 18. The number 333 express as a product of two numbers so that each of the multipliers is smaller than 10.

Task 19. The difference between a two-digit number and the number written with the same digits but in reverse order is 45. The sum of these numbers is an

exact square of a natural number. List all two-digit numbers that have these properties.

Task 20. Determine the integers a,b and the prime number p so that $|ab|\,p = 4022$.

Task 21. The product of two three-digit numbers is written with only several threes. What are the numbers?

Task 22. If p is a prime number, prove that

a) $p^3 + 1987$ is a composite number,

b) $p^{1987} + 1987$ is a composite number.

Task 23. Calculate all natural numbers n so that the numbers $3n-4$, $4n-5$ and $5n-3$ are prime numbers.

Task 24. Calculate all prime numbers p for which the following inequations are true: $\frac{3}{16} < \frac{5}{p} < \frac{2}{7}$.

Task 25. Calculate all prime numbers p so that

$$\frac{665}{1993} < \frac{5}{p} < \frac{997}{1994}.$$

Task 26. Calculate all prime numbers p,q and r so that

$$2p + 3q + 4r = 2022.$$

Task 27. A natural number n, when divided by 3, has a remainder a, when divided by 6, it has a remainder b, and when divided by 9, it has a remainder c. We know that $a+b+c=15$. Calculate the remainder, when the number n is divided by 18.

Task 28. What are the natural numbers a and b for which the following applies

$$\text{NZD}(a,b) = 8 \text{ and } \text{NZS}(a,b) = 168.$$

Task 29. Are there exist natural numbers x and y so that

$$\text{NZD}(x,y) + \text{NZD}(x+1, y+1) = x - y\ ?$$

Task 30. The houses from the left side of the street are numerated with odd numbers, and the houses on the right side of the street are numerated with even numbers. The sum of all house numbers on one side is 1309, and on the other is 2162. How many houses are there on this street?

Task 31. Calculate all integers n for which $\frac{n+4}{3n-2}$ is an integer.

Task 32. Calculate all pairs of integers (a,b) for which the following applies $a = \frac{4b-5}{b-2}$.

Task 33. Solve the equation in the set of integers.

$$10xy + 16x + 5y = 2006.$$

Task 34. Solve the equation in the set of natural numbers

$$\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = 1.$$

Task 35. Determine all natural numbers a,b,c,d,e so that $2<a<b<c<d<e$ and

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}=1.$$

Task 36. Calculate the three-digit number that is divisible by 9, whose digit for tens is for 4 smaller than the digit of the ones, and the product of its digits is equal to 0.

Task 37. Let n be a natural number greater than 1. Prove that the value of the fraction $\frac{10^n+8}{36}$ is an integer.

Task 38. Determine the smallest natural number which when multiplied by 2 becomes the square of a natural number, and when multiplied by 3 becomes the cub of another natural number.

Task 39. Write the number 31322_4 in base 10 system.

Task 40. Write the number $5a19_{11}$ in base 10 system.

Task 41. Find a natural number b so that in a b base system, the number 792 is divisible by 297.

Task 42. Write the tables for addition and subtraction of nonnegative integers smaller than the base of the base 8 numeral system.

4. CONCLUSIONS

In the previous analysis, we discussed only the work with gifted students in math in seventh (sixth) grade in the nine-year long (eight-year long) education. We need to emphasize that the work with the gifted students in math should be part of the differentiation of the instruction, and:

- The work with the gifted students, especially the students gifted in math, should not only be a declarative effort of the responsible institutions, which according to the practice thus far consider that for its realization it is sufficient to give accreditations for organizing competitions and say that it should be done by the teacher in the programs,
- A special teaching program should be developed for each age group, such as the program that is contained in this paper, and it needs to be carried out in the course of the whole school year, not right before the math competitions, which is the case in the current practice.
- Adequate didactic materials will be prepared for the realization of the program for work with gifted students, supported by the respon-

sible institutions, which as far as we know is not the case in any country of our immediate surroundings.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

AUTHOR'S CONTRIBUTIONS

All authors contributed equally and significantly to writing this paper. All authors read and approved the final manuscript.

References

[1] Đorđević, B.: *Školski oblici podrške u razvoju nadarenih učenika*, Nastava i vaspitanje, Beograd, 1990
[2] Gallagher, J. J.: *Teaching the gifted children*, Boston, Allyn & Bacon, 1985
[3] Glavche, M., Anevska, K., Malcheski, R.: Working with mathematically gifted students in the initial education – part one, Teacher, Faculty of education, Bitola
[4] Glavche, M., Anevska, K., Malcheski, R.: Working with mathematically gifted students in the initial education – part two, Conference Procededings of the Second International Scientific Conference: Gifted and talented creators of the progress, Faculty of Education – Bitola
[5] Glavche, M., Anevska, K., Malcheski, R.: Working with mathematically gifted students in the initial education – part three, Conference Procededings of the Second International Scientific Conference: Gifted and talented creators of the progress, Faculty of Education – Bitola
[6] Malčeski, R., Gogovska, V.: *The Role of Educational method in teaching of gifted and talented students*, The 10th International Congress on Mathematical Education, July 4-11, 2004, Copenhagen, Denmark
[7] Гроздев, С., Кендеров, П.: *Инструментариум за откриване и подкрепа на изявени ученици по математика*, Математика и метаматическо образование, 34 пролетна конференция на СМБ, Боровец, 6-9 април 2005, София, 53-64, 2005
[8] Кендеров, П., Гроздев, С.: *Диференцираното обучение по математика в европейските училища*, Математика и метаматическо образование, 35 пролетна конференция на СМБ, Боровец, 5-9 април 2006, София, 39-49, 2006

[9] Кендеров, П., Гроздев, С.: *Ев ропейския проект MATHEU: откриване, мотивиране и подкрепа на математическите таланти в европейските училища*, Математика и метаматическо образование, 33 пролетна конференция на СМБ, Боровец, 1-4 април 2004, София, 39-49, 2004
[10] Малчески, А., Малчески, Р., Главче, М., Малчески, С., (2019). Математички талент 12 (нерешени задачи за натпревари по математика – трет дел), Армаганка, Скопје
[11] Малчески, Р. (2019). Математички талент 10 (нерешени задачи за натпревари по математика – втор дел), Армаганка, Скопје
[12] Малчески, Р. (2020). Математички талент 16 (збирка задачи за VII одделение – втор дел), Армаганка, Скопје
[13] Малчески, Р., Малчески, А., Аневска, К. (2019). Решавање на текстуални задачи (четврто издание), Армаганка, Скопје
[14] Малчески, Р., Малчески, А., Аневска, К. (2020). По патеките на шампионите за математика (второ издание), Армаганка, Скопје
[15] Малчески, Р., Малчески, А., Малчески, С. (2020). Математички талент 20 (збирка задачи за VI и VII одделение), Армаганка, Скопје Скопје
[16] Малчески, Р., Малчески, А., Мисајлески, З., Костадинова, С., Димовски, Т. (2020). Математички талент 23 (збирка задачи за VI и VII одделение), Армаганка, Скопје
[17] Малчески, Р., Малчески, С., Аневска, К. (2018). Математички талент 4 (збирка задачи за VII одделение), Просветно дело, Скопје Скопје
[18] Малчески, Р.: *Методика на наставата по математика* (трето *издание*), Армаганка, Скопје, 2019

[1)] International Slavic University G. R. Derzhavin, Sv. Nikole, Macedonia
E-mail address: risto.malceski@gmail.com
[2)] SS. Cyril and Methodius University in Skopje, Faculty of Mechanical Engineering, Skopje, Macedonia
E-mail address: aleksa.malceski@gmail.com
[3)] FON University, Skopje, Macedonia
E-mail address: anevskak@gmail.com
[4)] SS. Cyril and Methodius, Faculty of Pedagogy, Skopje
E-mail address: mglavche@gmail.com

Publisher
Union od Mathematicians of Macedonia – ARMAGANKA

Editor in chief
Prof. d-r Aleksa Malcheski

CIP - Каталогизација во публикација
Национална и универзитетска библиотека "Св. Климент Охридски", Скопје

51(082)

PROCEEDINGS of CODEMA 2020 = Зборник на трудови од CODEMA 2020. -
Skopje : Armaganka, 2021. - 375 стр. : илустр. ; 25 см

Текст на мак. и англ. јазик. - Фусноти кон текстот. - Библиографија кон трудовите

ISBN 978-608-4904-09-0

а) Математика -- Зборници

COBISS.MK-ID 53570309