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WORKING WITH MATHEMATICALLY GIFTED STUDENTS IN PRIMARY EDUCATION – PART ONE

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Abstract. The mathematics content enables differentiation of instruction as early as initial education. For this purpose, it is essential to discover and identify mathematically gifted students. After discovering and identifying the mathematically gifted students, we need to organize instruction that will result in faster advancement. For this goal, in this paper we have provided an integral teaching program for working with mathematically gifted students in Grade VI (Grade V) in the nine-year (eight-year) long primary education. This teaching program is an upgrade of the corresponding teaching programs presented in [3] – [5] intended for students in the initial education, i.e. students in Grades II – V in the nine-year long education, i.e. students in Grades I – IV in the nine-year long education. Alongside, almost the complete teaching program is supplemented by a system of tasks for particular topics (solved and unsolved), given in [10] – [17].

1. INTRODUCTION

Work with mathematically gifted students is usually carried out as a part of mathematical sections which help the students prepare for the upcoming mathematics competitions. Practice shows that the educational process does not give as much attention as needed to identify the mathematically gifted students or to work with them. Our analysis will not focus as much on identifying these students, because this matter is thoroughly treated in the existing literature, such as [1], [2], and [18].

During the last several years we have seen efforts to organize systematic and continuous work with mathematically gifted students. Nevertheless, these efforts were accompanied by numerous contradictions, such as the absence of adequate programs, as well as lack of necessary literature for realizing these programs. Having this into consideration, we made an effort to develop integral programs for work with mathematically gifted students in the initial education, i.e. students in grades III-V (I-IV) in the nine-year long (eight-year) primary education in the papers [3], [4] and [5]. These papers also recommend adequate literature that covers the suggested programs in a significant part. The literature has been enriched with new collections of tasks in recent years.

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This paper is a continuation of the papers [3], [4] and [5], and its goal is to offer adequate integral programs for work with mathematically gifted students aged 11-12. Taking into consideration the experiences in the Republic of Macedonia and some other countries, we are going to make an effort to offer a system of tasks for one of the topics in the program. This system of tasks can be used to discover and identify mathematically gifted students as well as work with them (done in [3], [4], and [5] for the students in the initial education).

2. PROGRAM FOR WORKING WITH MATHEMATICALLY GIFTED STUDENTS AGED 11-12

In this part we are going to present an integral program for work with mathematically gifted students aged 11-12, i.e. students in grade VI (grade V) in the nine-year long (eight) primary education. This teaching program should be realized continuously, not only when the students are practicing for mathematics competitions.

Goals of the program for students aged 11-12:

- The student understands and adopts operations on sets and uses them when solving tasks,
- The student distinguishes finite and infinite sets,
- The student understands the decimal number system and completes arithmetic operations with natural numbers,
- The student understands the commutative and associative laws of addition and multiplication, and the distributive law and can apply these laws when calculating the numerical expressions,
- The student is able to solve word problems arithmetically,
- The student is able to solve equations with one variable and can apply them when solving word problems,
- The student adopts divisibility rule of 2, 3, 4, 5, 8, and 9 and can use this knowledge when solving appropriate tasks,
- The student adopts the terms greatest common divisor and least common multiple and can use them when solving tasks,
- The student adopts the terms prime and composite number and can solve tasks with prime and composite numbers,
- The student can compare and reduce fractions and does operations with fractions,
- The student transforms fractions into decimal numbers and percentages and can do operations with decimal numbers,
- The student distinguishes the basic terms (point, line, plane, distance) from the derivative terms (semi-line, line segment, angle, polygon, circle, etc.),
- The student adds line segments graphically and arithmetically,
- The student adds angles graphically and arithmetically,

- The student adopts the measurement units for mass, length, time, temperature, area and volume,
- The student operates with named numbers and transforms single-named into multi-named numbers,
- The student calculates the perimeter and area of a rectangle and a square, as well as the area of more complex shapes composed of rectangles and squares,
- The student calculates the area of a cube and a cuboid, as well as more complex shapes composed of cubes and cuboids,
- The student solves basic logical tasks,
- The student understands the basic combining principles and combining configurations,
- The student solves elementary tasks with coloring, covering and dissecting figures into simpler figures,
- The student uses the invariant method at an elementary level,
- The student develops qualities of thinking, such as elasticity, pattern-making, depth, rationalization, and critical thinking,
- Efforts will be made for the student to adopt the scientific methods informally: observation, comparison, experiment, analysis, and synthesis,
- Efforts will be made for the student to adopt the types of conclusion-making informally: induction, deduction, and analogy, while presenting suitable examples from which the students will learn that analogously based conclusion is not always correct.

The following content needs to be learned in order to achieve the previously mentioned goals:

Topic I. *Sets*: definition and notion for a set, ways of presenting sets, a subset, equal sets, set operations (union, intersection, difference, and Cartesian product), and tasks with Venn diagrams.

Topic II. *Natural numbers*: the set of natural numbers, extended set of natural numbers, notion for a finite and an infinite set, decimal number system, operations with natural numbers (addition, multiplication, subtraction, and division) and their properties, numerical expression, order of arithmetical operations, order of natural numbers, inequalities, degrees, extended form of a natural number in a decimal entry, solving equations with one variable, solving numerical rebus puzzles with addition, subtraction, multiplication and division, and sequences of numbers which satisfy a certain property and magical figures.

Topic III. *Number theory*: divisibility in the set of natural numbers, division with remainders, general divisibility properties, divisibility properties of 2, 3, 4, 5, 9 and 11, prime and composite numbers, decomposing a composite number into prime factors, common divisors and common multiples of two or more natural numbers, greatest common divisor and least common multiple, elementary Diophantine equations.

Topic IV. *Fractions*: Decimal numbers: fractions, reading and writing fractions, representing fractions on a number line, expanding and reducing

fractions, comparing fractions, arithmetic mean of natural numbers, addition and subtraction of fractions, decimal fraction and a decimal number, comparing decimal numbers, operations with decimal numbers (addition, subtraction, multiplication and division), transforming fractions into decimal numbers and vice versa, finite and infinite decimal number, periodic decimal number and decimal rounding, and solving equations with one variable with decimal numbers.

Topic V. ***Solving word problems*****:** tasks with numbers and numerals, tasks with measure numbers, percentage tasks, and money tasks.

Topic VI. Geometry: point and line, mutual position of two lines, the distance between two points, semi-line, and line segment, length of a line segment, basic and derivative terms, graphic and arithmetic line segments addition, length of a broken line and perimeter of a polygon, and a circle, common position of a circle and a point, circle and a line, and of two circles, the notion of an angle, adjacent, linear and vertical angles, measuring angles, types of angles according to size, graphic and arithmetic angles addition, complementary and supplementary angles, bisection of a line segment and bisection of an angle, distance from a point to a line, convex and concave polygons, types of polygons, triangle (axial symmetry of a triangle, triangle inequality), elementary constructive tasks, area and perimeter of a square and a rectangle and more complex shapes consist of squares and rectangles, area of a cube, a cuboid and more complex shapes built of cubes and cuboids.

Topic VII. ***Logic and combinatorics*****:** elementary logical tasks, classical logical tasks (finding the culprit, liar, etc.), Dirichlet's principle (intuitive use), method of invariants (elementary level), counting and recounting by using the principles of sum, difference, and product (intuitive use), coloring, covering and dissection, comparison of weight and pouring liquids, elementary games and strategies.

3. AN EXAMPLE OF A SYSTEM OF TASKS FOR WORKING WITH MATHEMATICALLY GIFTED STUDENTS AGED 11-12

Adequate instructional materials, i.e. textbooks, accompanied by collections of tasks have to be developed to realize the suggested program for work with mathematically gifted students aged 11-12. The textbook and the collection of tasks must contain adequate tasks that will be used to discover and identify mathematically gifted students. We are going to present an example of a system of tasks for this age group. The tasks cover the topic Sets, part of the collections of tasks [10] – [17].

Task 1. Let

$A=\{a \mid 2 \text{ is devisor of } a, a \in \mathbb{N}\}$ and $B=\{b \mid 3 \text{ is devisor of } b, b \in \mathbb{N}\}$. and. Determine the set $A \cap B$.

Task 2. Let

$$A=\{a \mid a\in\mathbb{N}, a\le 7\} \text{ and } B=\{b \mid b\in\mathbb{N}, 4\le b<9\}.$$

Determine the elements of the set C if

$$C=\{c \mid c\in\mathbb{N}, c=a-b, a\in A, b\in B\}.$$

Task 3. Let

$A=\{a \mid a\in\mathbb{N}, a<7\}$, $B=\{b \mid b\in\mathbb{N}, 5<b<6\}$ and $C=\{c \mid c\in\mathbb{N}, 5<c<8\}$.

Determine the set $(B\cap C)\setminus A$.

Task 4. Determine the elements of the set B if

$$A\cup B\cup C=\{a,b,c,d,e,m,n,n,p,q\},\ A\setminus B=\{e,m\},$$
$$A\cap C=\varnothing,\ C\setminus B=\{p,c\}.$$

Task 5. Let $A=\{1,2,3\}$ and $B=\{3,4,5\}$. Determine the sets X if $A\cup X=A$ and $B\cap X=A\setminus(A\setminus B)$. How many solutions are there?

Task 6. Let $S=\{1,2,3,4,5,6,7,8,9,10\}$. Are there exist sets A and B such that $A\cup B=S$, $A\cap B=\varnothing$ and the sum of the elements of the set A equals the sum of the elements of the set B?

Task 7. Let $A=\{1,2,3,4,5,6,8\}$ and $B=\{2,4,5,6,8\}$. Determine the set S for which the following equations are true

$$A\cap S=\{3,4\} \text{ and } B\cup S=\{2,3,4,5,6,7,8,9\}.$$

Task 8. Let

$$A=\{a,b,c,d,e\}, B=\{a,d,f\}, C=\{b,e,f,g\}, D=\{a,f,g,h\}.$$

Determine the set S given that

$$S\subseteq A,\ S\cap(B\cup D)=\varnothing,\ (A\cap C)\setminus S=\varnothing,\ \{c\}\setminus S=\{c\}.$$

Task 9. Let $A=\{5,2x+2\}$ and $B=\{2x+1,y-3\}$, for x and y natural numbers.Determine all the values of x and y such that B is the subset of A.

Task 10. Let $A=\{x \mid x=2k, k\in\mathbb{N}\}$ and $B=\{x \mid x=2k-1, k\in\mathbb{N}\}$. Using symbols determine the relationship between the sets

a) A and $\mathbb{N}$, b) B and $\mathbb{N}$,

c) A and B, d) $A\cup B$ and $\mathbb{N}$.

Task 11. The elements 1, 2, 3, 4, 5, 6 7, 8 are the only elements that form the sets A,B and C. Determine the sets A,B and C given that the following conditions are fulfilled:

a) none of the elements belong to all three sets,

b) the elements 3 and 8 belong only to set B,

c) each of the elements 1, 4, and 7 belong to only one of the sets A,B and C,

d) the set A contains only two elements,

e) the element 2 belongs exactly to two of the sets A,B and C,

f) $1\notin B, 2\notin A, 4\in A, 6\in A, 6\in B, 7\notin B$.

Task 12. Determine the sets A and B which fulfill the conditions: $A\cup B=\{2,3,4,5,6,7\}$, $A\cap B=\{3,4,5,6\}$, $7\notin A\setminus B$ and $2\notin B\setminus A$.

Task 13. Determine the sets A, B and C which fulfill the conditions:

$$A \cup B \cup C = \{1,2,3,4,5,6,7,8,9\},$$
$$A \cup B = \{1,2,3,6,7,8\},$$
$$A \cup C = \{1,2,3,4,5,7,9\},$$
$$A \cap B = \{1,2\}, \quad A \cap C = \{3,7\}.$$

Task 14. Let $S = \{x \mid x = 3a + 5, x < 20\}$.

a) For which values of the number a from the set $\mathbb{N}_0$ the set S will not be empty?

b) For which values of the number a from the set $\mathbb{N}$ the set S will be empty?

Task 15. Let

$A = \{0,2,3,5,9\}$, $B = \{1,2,7,8,9\}$, $C = \{2,4,5,6,7\}$ and $D = \{2,4,5,6,7,9\}$.

Represent the set D in terms of the sets A, B, C and the set operations.

Task 16. Let:

$$A = \{x \mid x \in \mathbb{N}, x \le 10\}, \ B = \{x \mid x \in \mathbb{N}, 5 \le x < 15\} \text{ and}$$
$$C = \{x \mid x \in \mathbb{N}, x \le 12 \text{ and } x \text{ is an even number}\}.$$

Prove that for the given sets, the following is true

$$A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C).$$

Task 17. Determine the sets A, B, C such that the following hold true:

$$A \cup B = \{2,3,4,5,6,7,8\}, \qquad A \cap B = \{2\},$$
$$B \cup C = \{1,2,4,6,8\}, \qquad B \cap C = \{2,4,8\},$$
$$C \cup A = \{1,2,3,4,5,7,8\}, \qquad C \cap A = \{2\}.$$

Task 18. Let $A = \{x \mid x \in \mathbb{N} \text{ and } x < 6\}$ and $B = \{4,5,6,7,8\}$. Determine the set X if $X \subset (A \cup B)$, $X \cap A = A \setminus B$, $X \cap B = B \setminus A$.

Task 19. Determine the sets A, B and C, given that

$$A \cup B \cup C = \{1,2,3,4,5,6,7,8\},\ A \cap B \cap C = \{3\},\ C \setminus (A \cup B) = \{5,8\},$$
$$A \cap B = \{1,3\},\ B \setminus C = \{1,2\},\ A \setminus C = \{1,4\},\ B \cap C = \{3,6\}.$$

Task 20. Determine the sets A, B and C, if

$$A \cup B \cup C = \{n \mid n \in \mathbb{N} \text{ and } n \text{ is a one digit number}\},$$
$$A \cap B \cap C = \{2\},\ C \setminus (A \cup B) = \{1,3,5\},\ (B \cap C) \setminus A = \{6,7\},$$
$$(A \cap C) \setminus B = \varnothing,\ A \cap (B \cup C) = \{2,4,5\}.$$

Task 21. Determine the sets A and B if

$$A \cup B = \{1,2,3,4,5,6,7,8,9,10,11,12\},\ A \cap B = \{8,9,10,11,12\}$$

and the sum of all the elements of A equals the sum of all the elements of B.

Task 22. Let $A = \{1,2,3,4\}, B = \{0,1,2\}, C = \{5,6\}$. Check if the following equations are correct:

$$(A \cap B) \times C = (A \times C) \cap (B \times C) \text{ and } (A \setminus B) \times C = (A \times C) \setminus (B \times C).$$

Task 23. Each of the members of one team plays football or tennis. How many members are there in the team if 18 play football and tennis, 23 play football and 21 tennis?

Task 24. An international convention was the place where a group of people from Spain, Finland, Mongolia and Vietnam, a total of 21 people met. Five of them speak Spanish, 14 Finnish, 14 Mongolian and 10 Vietnamese. One Spaniard speaks only his native tongue. Two people from Finland speak Mongolian and Vietnamese, but not Spanish. Of the remaining people, none speak more than two languages, and eight people speak Finnish and Mongolian. How many people from Spain speak Vietnamese?

Task 25. A sports club has 60 members, of which 39 play football, 28 handball and 16 play both football and handball. Are there members of the club who neither play football nor handball?

Task 26. A class has 31 students. Each of the students can either swim or drive a bicycle, and 11 students can both swim and ride a bicycle. How many students can swim and how many can ride a bicycle if there are twice more students who can swim than students who can ride a bicycle?

Task 27. A class has 33 students. If 21 play basketball, 18 football and 6 neither basketball nor football, how many students play both football and basketball?

Task 28. In Gorjan's school in sixth grade, the students learn three foreign languages: 2 French, English and Russian, 9 only French and English, 13 French and English, 12 Russian and English, 29 English, 6 learn only French and 7 learn only Russian.

a) How many students in total are there in sixth grade?

b) How many students learn the Russian language?

c) How many students learn the French language?

Task 29. Let A be the set of natural numbers less than 2018, divisible by 4, B is a set of natural numbers less than 2018 which can be divided by 6 and C is a set of natural numbers less than 2018 which can be divided by 15. Determine the number of elements of the set $A \setminus (A \cap B \cap C)$.

Task 30. Let A be a set of natural numbers less than 500 and divisible by 2, B be a set of natural numbers less than 500 divisible by 3, C be a set of natural numbers less than 500 divisible by 4 and D be a set of natural numbers less than 500 divisible by 5. Determine the set $A \cap B \cap C \cap D$.

4. CONCLUSIONS

One of the forms for differentiation of instruction in primary education is the work with mathematically gifted students. This work is usually carried out within the preparation of the students for participation in many mathematical competitions. This practice, in our opinion, is neither a successful differentiation of instruction nor well-organized work with mathematically gifted students. The

work with the mathematically gifted students should be organized in the following way:

- The acknowledgement and identification of the mathematically gifted students will be carried out in the upper grades as well, because when this is done only during the initial years of elementary education, most often than not, the gifted students characterized with width, critical thinking and depth, remain off the radar of the teacher,
- A special program will be developed for every age group, such as the program in this paper and it will be realized during the entire school year. Content which is not included in the regular program will be learned during extracurricular instruction, where material from the regular program will be upgraded, and
- For realizing the program with work with the gifted students adequate didactic material will be developed (a complete textbook or mathematical manuals for the content not included in the regular program and collections of tasks that cover the entire program for work with mathematically gifted students).

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

AUTHOR'S CONTRIBUTIONS

All authors contributed equally and significantly to writing this paper. All authors read and approved the final manuscript.

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