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Contribution of generator-load pairs in distribution networks power losses

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ABSTRACT

The focus of the research presented here is developing a method to calculate contribution of each generator-load pair in total distribution network power losses. To do this we calculate current component from any generator to any load and then multiply each of these conjugate current components by difference between corresponding generator node voltage and load node voltage. This way there are no quadratic expressions and problems with non-separability of losses. Also, the procedure breaks down the losses in such a way that one may investigate how each power transaction contributes to the losses. The idea to create such kind of method is to assist network users (producers and consumers) obtaining detailed information about distribution of power losses among network branches, and base on this information to consider corresponding transparent and non-discriminatory actions. Developing the method, no problematic assumptions or simplifications were used. Hence, the method is exact and do not consider privilege to any network user. In addition, it handles PV nodes without additional inconveniencies. Accordingly comparing this method with methods allocating losses to nodes it is understandable to identify dissimilarities. Moreover, this version of the method is applicable to radial distribution network with neglected influence of line shunt susceptances. The whole procedure is illustrated with a simple numerical example and it is also applied to a bigger system. The MATLAB code is given as an open-source for further research.

1. Introduction

The deregulation of electrical power industry has posed many new challenges to electrical engineers. Transmission and distribution networks, which are crucial segments for fair competition in generation and supply, should be accessible on a fair basis for all users. This means that an adequate price for network services should be calculated. Having in mind that power losses are considerable portion of network operating costs, the problem of electricity tracing gains importance as it offers a solution to the loss allocation (LA) problem [1].

Proactive government support in promoting renewable energy sources results in an increased number of distributed generators (DGs) in the distribution networks, which alters the power flows and affects network losses. Nowadays, a lot of distribution networks are with multiple sources with different distances from sources to loads. Therefore, the LA methods should be able to identify the contribution of all participants in total network losses [2].

A relevant review with comparison of various LA methods, specifically in presence of DGs, is given in [2,3].

In one of the first papers two-step approach is used [4]: in the first step losses are allocated to loads without considering DGs and in the second step the network is solved once again including DGs and the change in losses are allocated to DGs only. Any mismatch in total and allocated losses is distributed to DGs in proportion to their apparent power.

There is a group of methods using decomposition of branch quantities into nodal injections dealing with branch flows expressed as current [5], power [6], and energy [7]. These methods contain a degree of arbitrariness since they inherently operate with squares of sums of branch currents, powers or energy components, which makes it impossible to divide losses into sum of terms uniquely attributable to generation or load due to crossed terms when quadratic expressions are expanded. In [5] squares of real and imaginary part of branch currents are represented as product of the respective current components and real and imaginary components of a “virtual” voltage which is calculated using a copy of the original network but with the branch series impedances replaced by their resistive terms. In [6,7] an approximation is introduced by using quadratic allocation of crossed terms. The problem of crossed terms and their approximation is still present in the literature nowadays [8]. In addition, struggling to fit into such common

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Nomenclature

$'$ and $''$ as an upper index denotes real and imaginary part of the complex quantity, respectively
$\mathbf{E}_N$, $\mathbf{E}_{NG}$ vectors of ones with length N and NG, respectively
I'_i, I''_i sum of real and imaginary parts of currents coming into node i, respectively
IC'_i, IC''_i sum of real and imaginary part of load current components coming into node i, respectively
IB'_{ik}, IB''_{ik} total real and imaginary part of the current in branch connecting nodes i and k, respectively
IBC'_{ik} sum of real parts of load current component in branch connecting nodes i and k
IBC''_{ik} sum of imaginary parts of load current component in branch connecting nodes i and k
IG'_k, IG''_k total real and imaginary part of generator current at node k, respectively
IL'_k, IL''_k total real and imaginary part of load current at node k, respectively
J'_{ki} real part of load current component at node k supplied by generator at node i
J''_{mi} imaginary part of load current component at node m supplied by generator at node i
$\mathbf{J}'$, $\mathbf{J}''$ $N \times NG$ matrices where element in row k and column i is J'_{ki} and J''_{ki}, respectively
$\mathbf{JJ}'$, $\mathbf{JJ}''$ $N \times NG$ matrices only having nonzero elements at position kk, where JJ'_{kk} and JJ''_{kk} are equal to the portion of IG'_k and IG''_k that supplies load at node k, respectively
$\mathbf{LD}$ $N \times NG$ matrix where element LD_{ki} is equal to the portion of network power losses caused by current J_{ki}
N, NG number of nodes and generator nodes in the network, respectively
V_i, $\mathbf{VD}$ voltage at node i and $N \times NG$ matrix where element in row k and column i is difference between voltage at generator node i and voltage at node k, respectively
$\mathbf{V}^g$, $\mathbf{V}$ vector of generator node voltages and vector of all node voltages, respectively
α_i^{real} set of nodes directly connected to node i and sending real part of the current to node i (see Fig. 3)
α_i^{imag} set of nodes directly connected to node i and sending imaginary part of the current to node i
β_i^{real} set of nodes directly connected to node i and node i sends real part of the current to each of them (Fig. 3)
β_i^{imag} set of nodes directly connected to node i and node i sends imaginary part of the current to each of them
μ_{i-k}^{real} set of generators whose real parts of the current flow by way of branch i-k
μ_{i-k}^{imag} set of generators whose imaginary parts of the current flow by way of branch i-k

scheme DG are required to be modelled as PQ generators with constant power factor. In this paper we do not apply any of these assumptions/approximations and operate with the original network only, using the basic circuit equations.

In the method from [9], again, losses are allocated to loads and DGs in two separate steps. Allocated losses are not matched with the total losses, hence third step with normalization of allocated losses is required.

Aiming to fairness in loss allocation, the game theory-based solutions are also applied. The problem is modelled as a cooperative game using Shapley value in which all participants are responsible for the system losses [10,11]. The applications of such methods are limited to the systems with small number of players (loads and DGs) because the problem dimension increases with the factorial of the number of players. To overcome dimensionality constraint, an analytical expression for loss allocation using Shapley value is given in [2]. However, in [2] DGs are represented as negative loads and loss normalization is applied.

The problem with crossed terms is avoided with the "exact method" proposed in [12,13], which is applicable only to systems without DGs. In this paper we expand it to include DGs and we use different notations. Recent article [14] introduces a method comprised of two parts. The method is too complicated since it first requires a participation matrix to be defined in order to determine which buses mostly impact the loss of each branch and in the second step the loss share of participants in each branch is calculated. Furthermore, the method approximates the cross terms of branch currents, which is also seen in other methods, but here authors use contractual power instead of actual load demand meaning that the approximation becomes more complicated and harder to follow and justify.

Following the main idea from the "exact method" [13], we calculate branch losses as a product of the voltage drop in the branch and the branch conjugate current components. In such a way, the losses are expressed as linear functions of branch current components and there are no problems with crossed terms since we are not using any quadratic expansion at all. This subtle difference from other methods is the main contribution of this paper which overcomes difficulties encountered elsewhere. This solves the main problem identified and stated in the conclusions of [3] as "all loss allocation procedures provide some kind of arbitrariness due to non-linear relation between system losses and power flows in a network which prohibits an explicit breakdown of losses among end-users."

The notable features of the presented approach are:

- Authentic treatment of DGs, i.e. we handle them as power sources that supply load. We trace currents from all sources to all loads thus offering a possibility to obtain losses in branches being used by all participants and therefore allocate losses for each generator-load pair.
- Losses are expressed as a product of branch voltage drop and conjugate current components, i.e. losses are linearly coupled to currents. Consequently, there are no quadratic expressions and problems with non-separability of losses. They are uniquely attributable to each generator-load pair.
- There is no need for normalization of allocated losses. The sum of allocated losses exactly matches with the total losses in the network obtained by a power flow method.
- To apply the approach, it is necessary to have currents of every load and every generator but to obtain these currents it is enough to use any complete power flow method that can handle all types of load and generator representation, including PV nodes. At the beginning, the network has to be solved with a selected power flow method only once.
- Except for the initial power flow calculation, the method does not need to use any data for generator or load power and line impedances. Also, there is no need for any simplification or additional assumption.
- The proposed method allocates losses for each power transaction for each generator-load pair using exact network representation. The output is a matrix where the element in row k and column i is equal to the portion of the power losses caused a transaction between generator i and load k. The sum of all elements of this matrix is exactly equal to the total network losses calculated with the power flow method.

The method allocates line losses to all combinations of generator-load pairs, including the slack node, which is treated as any other source in the network. However, we do not allocate losses to any node and only give an evidence of authentic conditions in the network supplying all participants with useful information which may be used in the process of transparent and non-discriminatory loss allocation per nodes. Such unprejudiced results can be exploited as a good base in the process of making fair agreement for costs distribution between network users. The article contains nine sections: Section 2 explains the use of current components to obtain contribution of each generator-load pair in network power losses; Section 3 creates auxiliary matrix **JJ** and vectors **IIG** and **IIL**; Section 4 develops relations between components of node current flow and components of branch currents; Section 5 elaborates decomposition into components of node current flows and branch currents; Section 6 forms equations to obtain current components corresponding to generator-load pairs; Section 7 presents results obtained for two illustrative examples; Section 8 gives the information about the code for the method developed in this article; Section 9 summarizes the conclusions.

2. Loss calculation by current components

In this section we explain the essence of the proposed method using a simple distribution network (illustrative example) shown in Fig. 1 where there are loads and distributed generators at nodes 2–5. Index 1 is given to the slack (supply) node. Table 1 gives real and reactive power of loads and generators. The voltage at supply (slack) node is $V_1 = 10\angle 0^\circ$ kV and the impedance of each branch is Z = (2.05 + j1.8) Ω. As can be seen in Table 1 we have different power balances at nodes: (a) active and reactive generator power at node 2 are smaller than load; (b) active generator power at node 3 is less but reactive power is greater than load; (c) active and reactive generator power at node 4 are greater than load; (d) active generator power at node 5 is greater than load but it absorbs reactive power.

Before starting with the proposed approach, using corresponding power flow method (which can handle all types of generator and load models), we obtain node voltages, total load currents, total generator currents and total branch currents. These total currents in the network example are shown in Fig. 1. Node voltage magnitudes and angles are given in Table 1. The complex power of the generator at the slack node is $(817.779 + j275.611)$ kVA, so that the total network losses are $\Delta S = (17.779 + j15.611)$ kVA.

Since currents are complex quantities, which are linear combination of their real and imaginary parts, we may apply the principle of superposition and in the further procedure calculate load current components for real and imaginary part separately. The process of decomposition of total branch currents (obtained by power flow calculation) into corresponding load current component is explained in Section 5. For the five-node example network Fig. 2 shows real parts of load current components for each generator and branch, so one may trace the routes of currents from generators to each load in the network. Similar figure can be drawn for imaginary parts of load current components as well.

Let us assume for a moment that there is only one generator in the network at node 1, and branch *l-m* directly connects nodes *i* and *k*. Thus, current in branch *l-m* is equal to the sum of currents of all loads supplied through that branch

$$IB_{lm} = \sum_{k\in\eta_m} IL_k, \tag{1}$$

where η_m is the set of nodes supplied by means of branch *l-m*, including node *m*.

Following the proposals in [12], using the results of a converged power flow calculation and without additional assumptions, we calculate power losses in branch *l-m* as a product of the branch voltage drop $VD_{lm} = V_l - V_m$ and the conjugate branch current, i.e.

$$\Delta S_{l-m} = (V_l - V_m)\cdot IB_{lm}^* = VD_{lm}\cdot IB_{lm}^* \tag{2}$$

Substituting (1) into (2), we get

$$\Delta S_{l-m} = VD_{lm}\cdot \sum_{k\in\eta_m} IL_k^* \tag{3}$$

The total losses in the network, caused by the load at node *k*, can be calculated as a sum of portions of branch losses for all branches in set ξ_k, forming the path between the supply node and node *k*, i.e.

$$\Delta S_k = \left(\sum_{(l-m)\in\xi_k} VD_{lm}\right)\cdot IL_k^* \tag{4}$$

The sum on the right hand side of (4) is a sum of voltage differences for all branches on the path between the supply node (having index 1) and node *k*, and it is obvious that it is equal to $V_1 - V_k$. Therefore, we may write (4) as

$$\Delta S_k = (V_1 - V_k)\cdot IL_k^* \tag{5}$$

In addition, we use the term 'load current component'. It is the portion of the load current at a node related to a generator at another node or at the same node. In case of load at node *k* and generator at node *i* we assign corresponding load current component as J_{ki}. To get the contribution in total power losses of a pair of load *k* and generator *i* we may simply write

$$LD_{ki} = (V_i - V_k)\cdot J_{ki}^* \tag{6}$$

Having in mind that we represent every load by its current whose components are linear combination of generator currents; following the principle of superposition, we may conclude that in the case with multiple generators in the network the total losses due to the load at node *k* are

$$\sum_{i\in\gamma} LD_{ki} = \sum_{i\in\gamma} [(V_i - V_k)\cdot J_{ki}^*] \tag{7}$$

where γ is the set of generator nodes in the network.

Principally a current component from each generator can supply any load. We consider that all these components may be conveniently stored in $N \times NG$ matrix **J**. Therefore, the load current component at node *k* supplied by a generator at node *i* is stored in row *k* and column *i* of matrix **J**. In Section 5 it is explained how to obtain elements of matrix **J**.

After the power flow calculation, we know voltages at all nodes. Therefore, we can calculate voltage difference between voltage at each generator node and voltage at each other node. The difference between voltages at generator node *i* and voltage at node *k* are stored in row *k* and column *i* in $N \times NG$ matrix **VD**. We calculate all these voltage differences using

$$\mathbf{VD} = \mathbf{E}_N\cdot(\mathbf{V}^g)^T - \mathbf{V}\cdot(\mathbf{E}_{NG})^T \tag{8}$$

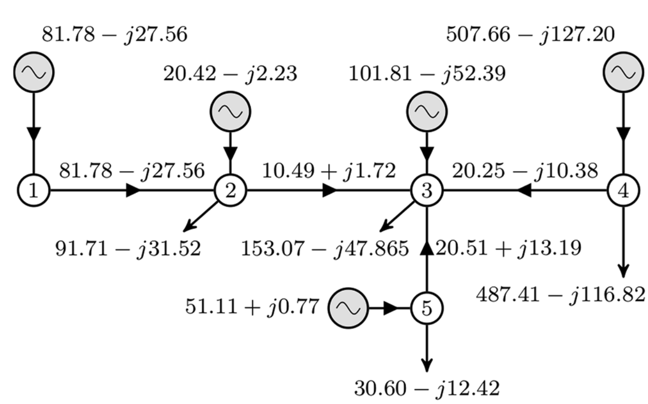

Fig. 1. Simple illustrative example – Complex currents (in amperes).

Table 1
Load and generator data for five-node system.

Node	Load power (kVA)	Generator power (kVA)	Voltage magnitude (kV)	Voltage angle (deg)
2	900 + j300	200 + j20	9.7832	−0.531
3	1500 + j450	1000 + j500	9.7650	−0.664
4	4800 + j1100	5000 + j1200	9.8250	−0.571
5	300 + j120	500 − j10	9.7828	−0.288

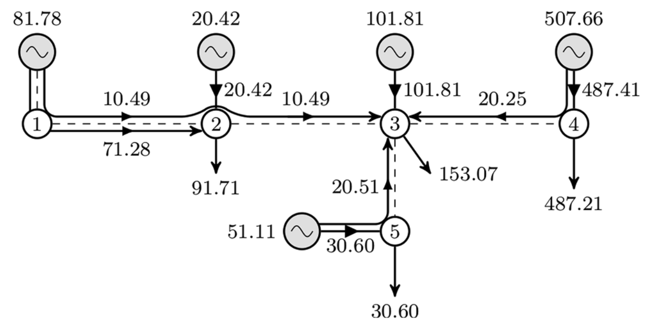

Fig. 2. Real parts of load current components (in amperes).

In case of generator at node i and load at node k their participation in network power losses is $(V_i - V_k)\cdot J_{ki}^*$. Therefore, we can calculate all these participations by matrix equation

$$\mathbf{LD} = \mathbf{VD} \odot \mathbf{J}^*, \tag{9}$$

where symbol $\odot$ denotes element-wise multiplication.

3. Definitions of vectors IIG, IIL and matrix JJ

In this Section we define vector of node net injected generator current $\mathbf{IIG} = \mathbf{IIG}' + \mathrm{j}\mathbf{IIG}''$, vector of node net injected load current $\mathbf{IIL} = \mathbf{IIL}' + \mathrm{j}\mathbf{IIL}''$, and $N \times NG$ matrix $\mathbf{JJ} = \mathbf{JJ}' + \mathrm{j}\mathbf{JJ}''$, which are later used in next Sections.

Please note that here all currents are calculated for real and imaginary parts separately. This should be done that way since direction of the real part of the branch current may be different from the direction of the imaginary part and the developed procedures are strongly dependent on the current direction. In addition, we suppose that in all cases $IG_k' \geqslant 0$, $IL_k' \geqslant 0$ and $IL_k'' \leqslant 0$, but IG_k'' can be positive, negative or zero.

Element IIG_k' is equal to the partial of real part of generator current at node k that flows in the direction of the network and element IIG_k'' is equal to the partial of imaginary part of generator current at node k that flows in the direction of the network, i.e. for any k, elements IIG_k' and IIG_k'' can be obtain using the following relation, respectively

$$IIG_k' = \begin{cases} IG_k' & \text{if } IL_k' = 0 \\ IG_k' - IL_k' & \text{if } IG_k' > IL_k' \\ 0 & \text{if } IG_k' = 0 \text{ or } IG_k' \leqslant IL_k' \end{cases},$$

$$IIG_k'' = \begin{cases} IG_k'' & \text{if } IL_k'' = 0 \\ IG_k'' - IL_k'' & \text{if } -IG_k'' > -IL_k'' \\ 0 & \text{if } IG_k'' = 0 \text{ or } -IG_k'' \leqslant -IL_k'' \end{cases}$$

Similarly, element IIL_k' is equal to the partial of real part of load current at node k flowing in the opposite direction of the network and element IIL_k'' is equal to the partial of imaginary part of load current at node k flowing in the opposite direction of the network, i.e. for any k, elements IIL_k' and IIL_k'' can be obtain using the next relation, respectively

$$IIL_k' = \begin{cases} IL_k' & \text{if } IG_k' = 0 \\ IL_k' - IG_k' & \text{if } IL_k' > IG_k' \\ 0 & \text{if } IL_k' = 0 \text{ or } IL_k' \leqslant IG_k' \end{cases},$$

$$IIL_k'' = \begin{cases} IL_k'' & \text{if } IG_k = 0 \\ IL_k'' - IG_k'' & \text{if } -IL_k'' > -IG_k'' \\ 0 & \text{if } IL_k'' = 0 \text{ or } -IL_k'' \leqslant -IG_k'' \end{cases}$$

Elements of matrix $\mathbf{JJ}'$ and $\mathbf{JJ}''$ in row k and column k, can be expressed as, respectively,

$$JJ_{kk}' = \begin{cases} IG_k' & \text{if } IL_k' \geqslant IG_k' \\ IL_k' & \text{if } IL_k' < IG_k', \\ 0 & \text{if } IG_k' = 0 \end{cases} \quad JJ_{kk}'' = \begin{cases} IG_k'' & \text{if } -IL_k'' \geqslant -IG_k'' \\ IL_k'' & \text{if } -IL_k'' < -IG_k'' \\ 0 & \text{if } IG_k'' \geqslant 0 \end{cases}$$

However, for all $i \neq k$ elements of $\mathbf{JJ}'$ and $\mathbf{JJ}''$ are equal to zero (i.e.$JJ_{ik}' = 0$and $JJ_{ik}'' = 0$).

4. Expressions regarding node current flow components and branch current components

In this Section we develop relations between components of node current flows and components of branch currents. In the next Sections these relations will be used in decomposing total currents into corresponding current components.

As in [1] we use the term ‘node current flow’. Nevertheless, we underline that for node i we discriminate the total node current flow I_i from the node current flow IC_i. As represented in (10) the total node current flow I_i is the sum of total currents coming into node i obtained by power flow calculation, i.e.

$$I_i = IIG_i' + \sum_{k \in \alpha_i^{\text{real}}} IB_{ki}' + j\left(IIG_i'' + \sum_{m \in \alpha_i^{\text{imag}}} IB_{mi}''\right) \tag{10}$$

Further, IC_i is the sum of all load current components from generators coming into node i, as shown in (11), i.e.

$$IC_i = IIG_i' + \sum_{k \in \alpha_i^{\text{real}}} IBC_{ki}' + j\left(IIG_i'' + \sum_{m \in \alpha_i^{\text{imag}}} IBC_{mi}''\right). \tag{11}$$

Fig. 3 shows a general case with real part of the currents associated to node i. The Figure helps in easier understanding of the explanations in this Section. Of course, similar Figure can be drawn for imaginary part of the currents.

In branch i-l we discriminate current and current IBC_{il}. Even though current and current IBC_{il} have equal total value they contain different components. By we assign the total current in branch i-l obtained by power flow calculation. On the other hand, by IBC_{il} we assign the sum of load current components passing through branch i-l. It is important to note that these load current components are stored separately. The analogous is valid for I_i and IC_i.

Now, supposing that for node i we know I_i', IC_i' (i.e. all load current components in IC_i') and IB_{il}', where $l \in \beta_i^{real}$, we are going to obtain all load current components in branch i-l, i.e. we are able to acquire IBC_{il}'.

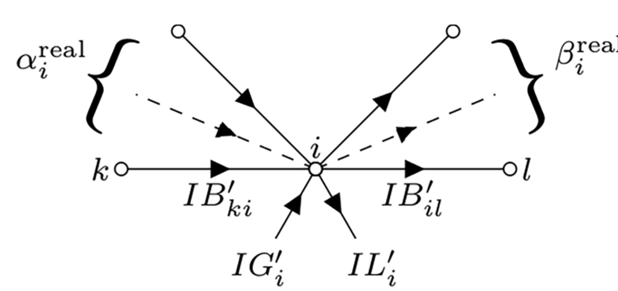

Fig. 3. Real part of currents at node i.

Similar is valid for IBC''_{il}. Because $IBC_{il} = IB_{il}$ and $IC_i = I_i$ these currents satisfy

$$\frac{IB'_{il}}{IBC'_{il}} = \frac{I'_i}{IC'_i} \text{ and } \frac{IB''_{il}}{IBC''_{il}} = \frac{I''_i}{IC''_i}$$

In view of (10) from the last equations follows

$$IBC'_{il} = \frac{IB'_{il}}{I'_i}\cdot IC'_i = \frac{IB'_{il}}{I'_i}\cdot\left(IIG'_i + \sum_{k\in\alpha_i^{\text{real}}} IBC'_{ki}\right);\ l \in \beta_i^{\text{real}} \tag{12}$$

$$IBC''_{il} = \frac{IB_{il}}{I''_i}\cdot IC''_i = \frac{IB''_{il}}{I''_i}\cdot\left(IIG''_i + \sum_{k\in\alpha_i^{\text{imag}}} IBC''_{ki}\right);\ l \in \beta_i^{\text{imag}}. \tag{13}$$

From (12) we understand that the real part of the current in line *i-l* contains the same components as current IC'_i but multiplied by IB'_{il}/I'_i and that is valid for each branch *i-l* where $l \in \beta_i^{\text{real}}$. Similarly, (13) shows that imaginary part of the current in line *i-l* contains the same components as current IC''_i, but multiplied by IB''_{il}/I''_i, and that is valid for each branch *i-l* where $l \in \beta_i^{\text{imag}}$.

Thus, having results of power flow calculation it is easy to calculate value of IB'_{il}/I'_i and knowing load current components of IC'_i we can use (12) to calculate real part of all load current components in each branch *i-l* where $l \in \beta_i^{\text{real}}$. By analogy, using (13) it is possible to obtain imaginary part of all load current components in each branch *i-l* where $l \in \beta_i^{\text{real}}$. After obtaining all load current components in a branch we say that the total branch current is decomposed into corresponding load current components.

5. Procedure to calculate components of node current flows and branch currents

In this Section we develop a procedure aimed to obtain current components of node current flows and branch currents. For that purpose, we use equations developed in Section 4.

Substituting (12) and (13) into (11), we get for $i = 1, 2, \ldots, N$,

$$IC_i = IIG'_i + \sum_{k\in\alpha_i^{\text{real}}} \frac{IB'_{ki}}{I'_k}\cdot IC'_k + j\left(IIG''_i + \sum_{m\in\alpha_i^{\text{imag}}} \frac{IB'_{mi}}{I''_m}\cdot IC''_m\right). \tag{14}$$

After the separation of real and imaginary parts from (14), we rearrange them as follows

$$IC'_i - \sum_{k\in\alpha_i^{\text{real}}} \frac{IB'_{ki}}{I'_k}\cdot IC'_k = IIG'_i;\ i = 1, \ldots, N; \tag{15}$$

Table 2
Loss allocation for active power losses in five-node system (kW).

Load node	Generator node					Total
	1	2	3	4	5	
2	12.987		0.012	0.009		13.007
3	2.473			1.219	0.376	4.068
4						0
5			0.256	0.448		0.704
Total	15.460	0	0.268	1.676	0.376	17.779

Table 3
Loss allocation for reactive power losses in five-node system (kvar).

Load node	Generator node					Total
	1	2	3	4	5	
2	12.453		−0.010	0.050		12.494
3	1.187			0.307	1.312	2.806
4						0
5			−0.073	0.385		0.311
Total	13.640	0	−0.083	0.742	1.312	15.611

$$IC''_i - \sum_{m\in\alpha_i^{\text{imag}}} \frac{IB''_{mi}}{I''_m}\cdot IC''_m = IIG''_i;\ i = 1, \ldots, N \tag{16}$$

Note that in case $k \notin \alpha_i^{\text{real}}$ we have $IB'_{ki} = 0$ and in case $m \notin \alpha_i^{\text{imag}}$ we have $IB''_{mi} = 0$. Furthemore, if instead of writing the summation terms over set indices in (15) and (16), we will obtain the same results by applying consecutive summation for $k = 1, \ldots, N$ in (15) and $m = 1, \ldots, N$ in (16). This formal suggestion allows more concise writing and the systems of linear Eqs. (15) and (16) can be expressed in matrix form as

$$\mathbf{A}'\cdot\mathbf{IC}' = \mathbf{IIG}', \tag{17}$$

$$\mathbf{A}''\cdot\mathbf{IC}'' = \mathbf{IIG}'', \tag{18}$$

where elements of vectors $\mathbf{IC}'$, $\mathbf{IC}''$, $\mathbf{IIG}'$ and $\mathbf{IIG}''$ are IC'_i, $i = 1, \ldots, N$; IC''_i, $i = 1, \ldots, N$; IIG'_i, $i = 1, \ldots, N$ and IIG''_i, $i = 1, \ldots, N$, respectively; elements of $N \times N$ matrices $\mathbf{A}'$ and $\mathbf{A}''$ are as follows

$$A'_{ij} = \begin{cases} 1 & \text{for } i = j \\ -IB'_{ji}/I'_j & \text{for } j \in \alpha_i^{\text{real}} \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

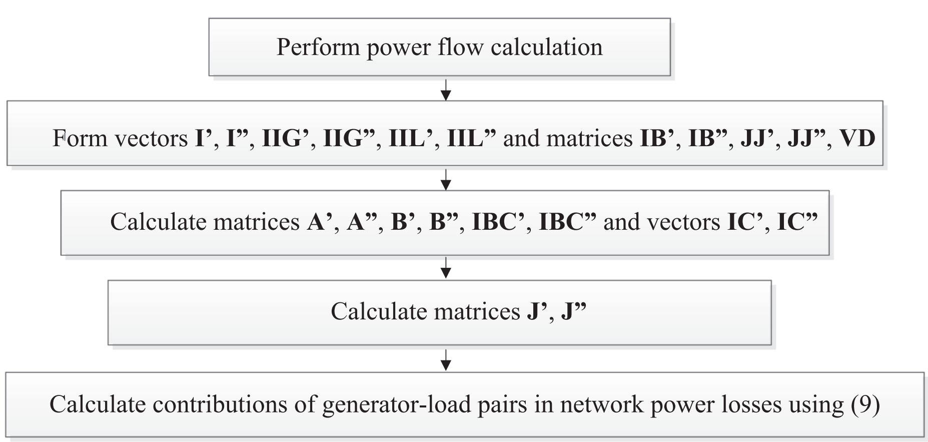

Fig. 4. Simple flow-chart of the proposed method.

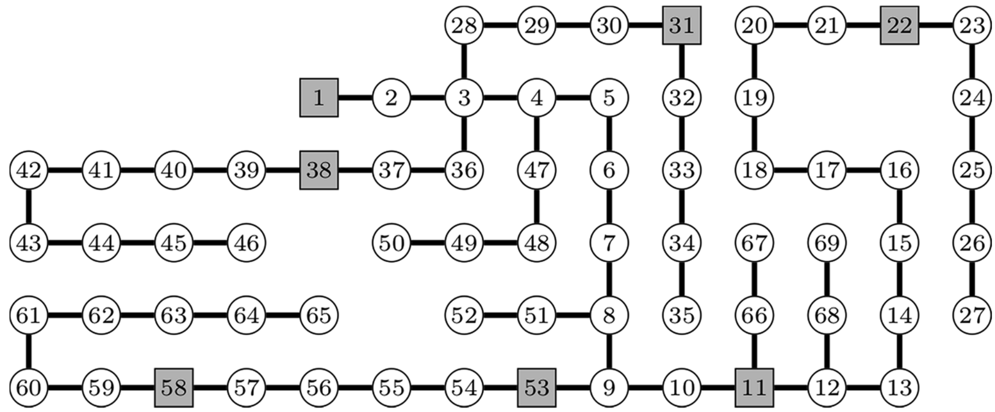

Fig. 5. Topology of 69-node distribution network.

$$A''_{ij} = \begin{cases} 1 & \text{for } i = j \\ -IB''_{ji}/I''_j & \text{for } j \in \alpha_i^{\text{imag}} \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

Setting $(\mathbf{A}')^{-1} = \mathbf{B}'$ and $(\mathbf{A}'')^{-1} = \mathbf{B}''$ relations (17) and (18) can be expressed as

$$\mathbf{IC}' = \mathbf{B}'\cdot\mathbf{IIG}' \tag{21}$$

$$\mathbf{IC}'' = \mathbf{B}''\cdot\mathbf{IIG}'' \tag{22}$$

From (21) we conclude that the contribution of generator at node k to the real part of the current flow at node i is equal to $B'_{ik}\cdot IIG'_k$. Consequently, for all real and imaginary parts of current flow components at node i we get, respectively,

$$IC'_i = \sum_{k=1}^{N} (B'_{ik}\cdot IIG'_k) \tag{23}$$

$$IC''_i = \sum_{k=1}^{N} (B''_{ik}\cdot IIG''_k) \tag{24}$$

Then from (12) and (23) we calculate real part of the current outflow components in line i-l (i.e. contribution of each generator in the network to the real part of the current in branch i-l, $i \in \alpha_l^{\text{real}}$) as

$$IBC'_{il} = \frac{IB'_{il}}{I'_i}\cdot \sum_{k=1}^{N} (B'_{ik}\cdot IIG'_k) \tag{25}$$

and from (13) and (24) we calculate imaginary part of current outflow components in line i-l (i.e. contribution of each generator in the network to the imaginary part of the current in branch i-l, $i \in \alpha_l^{\text{imag}}$) as

$$IC''_{i\text{ - }l} = \frac{IB''_{il}}{I''_i}\cdot \sum_{k=1}^{N} (B''_{ik}\cdot IIG''_k) \tag{26}$$

6. Calculation current components corresponding to generator-load pairs

The aim of this Section is to develop equations, which can be easily used to obtain current components corresponding to generator-load pairs. In order to do this, we first introduce net injected load current vector **IILC**. Even though total values of corresponding elements of vectors **IILC** and **IIL** are equal it is important to emphasize that these elements contain different components. Element of **IILC** in row i is the sum of current components each of them representing contribution of each individual generator in the node net injected load current. From

$$\frac{IIL'_i}{IILC'_i} = \frac{I'_i}{IC'_i} \text{ and } \frac{IIL''_i}{IILC''_i} = \frac{I''_i}{IC''_i}$$

follows that the elements of vector **IILC** can be expressed as

$$IILC'_i = \frac{IIL'_i}{I'_i}\cdot IC'_i \tag{27}$$

$$IILC''_i = \frac{IIL''_i}{I''_i}\cdot IC''_i \tag{28}$$

Further from (23) and (27), for $i = 1, 2, \ldots, N$, we get

$$IILC'_i = \frac{IIL'_i}{I'_i}\cdot \sum_{k=1}^{N} (B'_{ik}\cdot IIG'_k) \tag{29}$$

and from (24) and (28) we get

$$IILC''_i = \frac{IIL''_i}{I''_i}\cdot \sum_{k=1}^{N} (B''_{ik}\cdot IIG''_k) \tag{30}$$

From (29) the contribution of generator at node k to the real part of load current at node i is $(IIL'_i/I'_i)\cdot B'_{ik}\cdot IIG'_k$. Because for $i \neq k$ we have $JJ'_{ik} = 0$, and JJ'_{kk} may not be zero, in general case, for element J'_{ik} we may write

$$J'_{ik} = (IIL'_i/I'_i)\cdot B'_{ik}\cdot IIG'_k + JJ'_{ik}$$

Therefore, all elements of matrix $\mathbf{J}'$ can be calculate at once using

$$\mathbf{J}' = \text{diag}(\mathbf{IIL}''\oslash\mathbf{I}'')\cdot\mathbf{B}'\cdot\text{diag}(\mathbf{IIG}') + \mathbf{JJ}' \tag{31}$$

Note that in (31) and (32) symbol $\oslash$ denotes element-wise division.

Similarly, from (30) the contribution of generator at node k to the imaginary part of load current at node i is $(IIL''_i/I''_i)\cdot B''_{ik}\cdot IIG''_k$. Because for $i \neq k$ we have $JJ''_{ik} = 0$, and JJ''_{kk} may not be zero, in general case, for element J''_{ik} we may write

$$J''_{ik} = (IIL''_i/I''_i)\cdot B''_{ik}\cdot IIG''_k + JJ''_{ik}$$

All these contributions can be calculated at once and stored in matrix $\mathbf{J}''$ using

$$\mathbf{J}'' = \text{diag}(\mathbf{IIL}''\oslash\mathbf{I}'')\cdot\mathbf{B}''\cdot\text{diag}(\mathbf{IIG}'') + \mathbf{JJ}'' \tag{32}$$

Knowing elements of matrices $\mathbf{J}$ and $\Delta\mathbf{V}$ using (9) we can calculate contribution of each generator-load pair in the network power losses. Simple flow-chart of the proposed method is shown in Fig. 4.

Table 4
Loss allocation for active power losses in 69-node system (kW).

Load at node	Generator at node							Total
	1	11	22	31	38	53	58	
6	0.053				0.017			0.070
7	0.213				0.059			0.272
8	0.436				0.121			0.556
9	0.183				0.050			0.233
10	0.252				0.067			0.319
12	1.135	0.305			0.257			1.698
13	0.773	0.341			0.186			1.300
14	1.130	0.571			0.254			1.955
15	0.945	0.533			0.218			1.697
16	0.060	0.040			0.017			0.117
17	0.289	0.169			0.067			0.526
18	0.196	0.113			0.045			0.354
20	0.013	0.008			0.003			0.025
23	0.840	0.471	0.004		0.174			1.490
24	0.351	0.208	0.004		0.078			0.641
25	0.426	0.258	0.011		0.095			0.789
26	0.560	0.344	0.016		0.126			1.046
27	0.561	0.345	0.017		0.127			1.049
28	0.001				0.003			0.004
29	0.003				0.004			0.007
30	0.009				0.005			0.013
32	0.022			0.033	0.005			0.060
33	0.042			0.097	0.009			0.148
34	0.027			0.065	0.005			0.096
35	0.030			0.075	0.005			0.110
36					0.012			0.012
37	0.003				0.004			0.007
39					0.016			0.016
40					0.015			0.015
41					0.355			0.355
42					0.211			0.211
43					0.084			0.084
44					0.169			0.169
45					0.128			0.128
46					0.004			0.004
47	0.002				0.007			0.009
48	0.018				0.030			0.048
49	0.293				0.307			0.600
50	0.322				0.335			0.658
51	0.235				0.066			0.301
52	0.141				0.044			0.185
54	0.181				0.003	0.047		0.231
55	0.043				0.001	0.022		0.065
56	0.224				0.003	0.122	0.019	0.369
57	0.230					0.029	0.063	0.322
59	0.004					0.001	0.021	0.026
60	0.009					0.001	0.083	0.094
61	0.006					0.001	0.102	0.109
62	0.005					0.001	0.079	0.085
63	0.002						0.035	0.038
64	0.003						0.081	0.084
65	0.009					0.002	0.221	0.232
66	0.105	0.001			0.023			0.128
67	0.105	0.001			0.023			0.128
68	0.224	0.064			0.051			0.339
69	0.224	0.064			0.051			0.339
Total	10.937	3.837	0.052	0.270	3.940	0.225	0.706	19.967

7. Results for illustrative examples

In this Section we present results obtained applying the proposed method on the five-node system in Fig. 1 and the 69-node system from [9]. According to the proposed rules and equations for the network in Fig. 1 we get the following results (where all currents are expressed in amperes).

$\mathbf{IIL'} = [0 \;\; 71.28 \;\; 51.26 \;\; 0 \;\; 0]^{\mathrm{T}}$, $\mathbf{IIG'} = [81.78 \;\; 0 \;\; 0 \;\; 20.25 \;\; 20.51]^{\mathrm{T}}$,

$\mathbf{I'} = [81.78 \;\; 81.780 \;\; 51.26 \;\; 20.25 \;\; 20.51]^{\mathrm{T}}$,

$\mathbf{IIL'} \oslash \mathbf{I'} = [\,0 \;\; 0.8717 \;\; 1 \;\; 0 \;\; 0\,]^{\mathrm{T}}$,

$$\mathbf{JJ'} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 20.42 & 0 & 0 & 0 \\ 0 & 0 & 101.81 & 0 & 0 \\ 0 & 0 & 0 & 487.41 & 0 \\ 0 & 0 & 0 & 0 & 30.60 \end{bmatrix},$$

$$\mathbf{A'} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\frac{81.78}{81.78} & 1 & 0 & 0 & 0 \\ 0 & -\frac{10.49}{81.78} & 1 & -\frac{20.25}{20.25} & -\frac{20.51}{20.51} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$(\mathbf{A'})^{-1} = \mathbf{B'} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0.1283 & 0.1283 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The real parts of the load current components are obtained with (31), while the imaginary parts of the load current components obtained from (32) are thus giving the result, respectively

$$\mathbf{J'} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 71.28 & 20.42 & 0 & 0 & 0 \\ 10.49 & 0 & 101.81 & 20.25 & 20.51 \\ 0 & 0 & 0 & 487.41 & 0 \\ 0 & 0 & 0 & 0 & 30.60 \end{bmatrix},$$

$$\mathbf{J''} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -27.56 & -2.23 & -0.52 & -1.20 & 0 \\ 0 & 0 & -47.86 & 0 & 0 \\ 0 & 0 & 0 & -116.82 & 0 \\ 0 & 0 & -4.00 & -9.18 & 0 \end{bmatrix}$$

The components of power losses are calculated with (7) for a single generator-load pair or with (9) for all generator-load combinations. The results are presented in Tables 2 and 3. For example, the losses due to load at node 3 are

$$LD_{31} = (V_1 - V_3)\cdot J_{31}^{*} = (10 - 9.765\cdot e^{-j0.664^{\circ}})\cdot(10.49 + j0)^{*} = (2.473 + j1.187)\ \mathrm{kVA};$$

$$LD_{34} = (V_4 - V_3)\cdot J_{34}^{*} = (9.825\cdot e^{-j0.571^{\circ}} - 9.765\cdot e^{-j0.664^{\circ}})\cdot(20.25 + j0)^{*} = (1.219 + j0.307)\ \mathrm{kVA};$$

$$LD_{35} = (V_5 - V_3)\cdot J_{35}^{*} = (9.7828\cdot e^{-j0.288^{\circ}} - 9.765\cdot e^{-j0.664^{\circ}})\cdot(20.51 + j0)^{*} = (0.376 + j1.312)\ \mathrm{kVA}\ .$$

Note that in Table 2 there is no loss allocated to load at node 4 since the generator at the same node locally supplies it. Also, there is no loss allocated to generator at node 2 since load at node 2 locally consumes total generator current.

In addition, we consider a bigger test system with 69 nodes taken from [9]. This system is shown in Fig. 5, where generator nodes are marked with grey rectangles, while the data can be found in [15]. The results for this system are given in Table 4. There are no losses allocated to loads at nodes 11, 22, 31, 38, 53 and 58 since local generators supply them all.

From Tables 2 and 4 it is noticeable that there are no negative loss allocations. Such outcome is expected since all power transactions between each generator-load pair must cause losses on the corresponding supply path. In order to distribute power losses between network users many loss sharing principles may be applied but that is out of the scope of this paper. However, please note that the proposed method allows using any sharing principal.

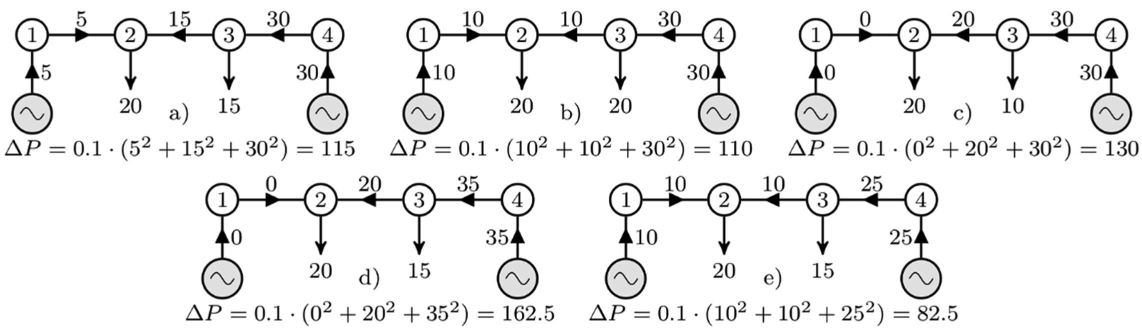

Fig. 6. Changes in power losses due to changes in load and generators currents.

8. Comments and discussions

In some cases, even connection of a load may decrease line losses and connection of a generator may increase line losses. As an illustration we use a modest example as follows. A simple network contains four nodes 1 (slack), 2, 3 and 4, and three lines 1–2, 2–3 and 3–4, where the resistance of each line is 0.1 pu (Fig. 6). Let us consider five cases which differ from the base case (a) by a change in current at one load or generator:

(a) Load currents at nodes 2 and 3 are 20 pu and 15 pu, respectively and generator current at node 4 is 30 pu. The total line losses are 115 pu.
(b) Load current at node 3 increases to 20 pu resulting in decrease of line losses to 110 pu.
(c) Load current at node 3 decreases to 10 pu causing increase of line losses to 130 pu.
(d) Generator current at node 4 increases to 35 pu causing rise of line losses to 162.5 pu.
(e) Generator current at node 4 decreases to 25 pu causing decrease of line losses to 82.5 pu.

These cases highlight that it is not correct to assume in advance that any increase of load current increases line losses, and that any increase of generator current decreases line losses, and vice versa. In other words, it is not correct to accredit negative losses to any DG or load, in advance.

In this work the problem under study are the line losses (and their distribution between generator-load pairs) in a predefined network taking into account all connected generators and loads, without taking into consideration which generator or load was connected last, meaning that we do not investigate changes in line losses before and after some connection or disconnection. Accordingly, the subject of the proposed method is evaluation of line losses corresponding to each generator-load pair considering all generators and loads that are already connected to the analysed network. Also, the proposed method indicates all lines participating in current flow from particular generator to particular load, evidencing corresponding losses in each of these lines. Therefore, we analyse only one condition of the network where line currents cause active power losses. In addition, in the proposed method not a single privilege is given to any network user.

In order to clarify the main substance of the proposed method let us first consider a network with only one DG intending to compare line losses in the network with and without the DG. For these two conditions it is necessary to make two separate line loss calculations. In both cases active power line losses cannot be negative. But the difference in the results may be positive or negative. In that case the difference in line losses (negative or positive) may be treated as an influence of the DG. But it is not so simple if there are more than one generator in the network. In that case, if we want to make comparison between two conditions, at first it is necessary to constitute precise description what are the two conditions we want to compare. Then, for each of the cases utilizing the proposed method we can calculate line losses corresponding to each generator-load pair in the network and get information which lines are engaged by each generator-load pair. Having results for the two cases, it is possible to compare line losses in the network for selected conditions. Furthermore, we point out that neither this method neither any other method directly gives results of any comparison. Consequently, it is not rational to recognise any negative active power line losses without knowing what two network states were compared. In the proposed method this principle was respected.

Application of the proposed method allows to get detailed information of the existing circumstances in the network, without discriminating any network user, and supposing that each network user wants to know these details in order to take them into consideration in the process of making corresponding decisions.

It should be emphasised that cost distribution of line losses between network users is not the subject of this work. However, it is rational to believe that knowledge of technical details in the network can be important and helpful for the network users in the process of discourse to make correct and fair agreement.

9. The code

The code for the method developed in this paper is written in MATLAB. It is given as an open-source in a GitHub repository along with all input data in [15].

10. Conclusions

This paper proposes an effective method for power loss analyses in distribution networks taking advantage of decomposition of total currents (obtained by power flow method) into components originated in individual generators.

The method takes advantage of expressing the losses as a product of the voltage drop in the branch and conjugate current components, which yields linear function in currents components. Consequently, there are no quadratic expressions and problems with non-separability of losses. Also, there is no additional assumptions, simplifications and need for normalization of allocated losses. The sum of allocated losses exactly matches with the total losses in the network obtained by power flow method. The method is exact, giving no privilege to any network user.

The method provides possibilities to determine network elements that are in use by each generator-load pair and to calculate corresponding power losses. In that way, the method can be of assistance in creating transparent obligations between generator owners, load owners and the distribution network operator.

Existence of PV nodes could be easily considered in any standard power flow computation and after that it does not make any additional problems in the implementation of the proposed method.

A disadvantage of the proposed method is that it is applicable to radial networks where influence of line susceptances may be neglected.

Declaration of Competing Interest

The authors declared that there is no conflict of interest.

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