16TH EUROPEAN CONFERENCE ON EARTHQUAKE ENGINEERING
THESSALONIKI 18 - 21 JUNE 2018

NUMERICAL MODEL OF BIAXIALLY LOADED REINFORCED CONCRETE STRENGTHENED WITH FIBER REINFORCED POLYMERS

Vladimir VITANOV[1]

ABSTRACT

The use of shear walls as a primary earthquake resistant mechanism in structural design is traditionally accepted and used over the last decades. However, very often, older structures having shear walls no longer comply with the contemporary standards and codes which raise the need for their strengthening and retrofit. Many different methods of seismic strengthening and repair of shear wall structures have been developed and tested. One of the most recent strengthening and retrofit techniques involves the use of externally bonded fiber reinforced polymer (FRP) composites, which offer unique properties such as high strength, low mass, chemical resistance, and ease of application.

This paper present a material model of reinforced concrete strengthened with FRP. The proposed model is implemented into ANSYS as a user material model in order to test its results against the available experimental data. Results of the analysis of monotonically loaded RC beams as well as cyclically loaded shear walls are presented. The results are compared against the experimentally obtained data as well as against the numerical results from other finite element analyses employing more traditional approach in the finite element modelling. Based on the presented results, it can be concluded that the proposed model is able to adequately simulate the behavior of the RC members strengthened with FRP in different configurations. Its ANSYS implementation enables its use in both research and practical purposes, facilitating the further research in this field as well as the practical applications in the construction industry.

Keywords: FRP; Strengthening; Numerical Model, ANSYS, Reinforced Concrete

1. INTRODUCTION

The conventional earthquake resistant design of reinforced concrete structures advises use of shear walls as effective way to add earthquake resistance to the reinforced concrete frames. A problem arises with structures erected decades ago following design rules which are by today's standards obsolete, inadequate and inefficient. Major earthquake events from around the world have shown the design deficiencies of these structures by inducing extensive damages in the structural members. Many of the old shear wall buildings are at risk of suffering damages from a major earthquake mostly due to their insufficient in-plane stiffness, flexural and shear strengths and ductility owing to the older design codes which didn't adequately estimate the demands that major earthquakes impose on the structures. This problem is ever increasing as the existing structures are getting older and their members gradually deteriorate.

Many different methods of seismic strengthening and repair of shear wall structures have been developed and tested in the last thirty years. Recently, state-of-the-art strengthening and retrofit techniques increasingly utilize externally bonded fiber reinforced polymer (FRP) composites, which offer unique properties in terms of strength, lightness, chemical resistance, and ease of application. Such techniques are most attractive for their fast execution and low labor costs.

Only recently have researchers attempted to simulate the behavior of reinforced concrete strengthened

---

[1]Assoc.Prof., Ss,Cyril and Methodius University, Skopje, the former Yugoslav Republic of Macedonia, v.vitanov@gf.ukim.edu.mk

with FRP composites using the finite element method. The majority of the studies that included numerical modeling of FRP strengthened RC members with FEM use element overlaying, where one-, two- or even three dimensional elements (solid or layered) that represent the FRP material are superimposed over the concrete elements, either with (ex. Khomwan and Foster, 2004; Wong and Vecchio, 2003) or without (ex. Kheyroddin and Naderpour, 2008) interface elements that represent the influence of the adhesive material or the bond between the FRP and the concrete.
A different approach is presented in this paper. An attempt is made to formulate a new material model which will simplify the modeling of FRP strengthened reinforced concrete members. The newly formulated material model is implemented into ANSYS and tested using available experimental data.

2. MODEL FORMULATION

In the analysis of RC structures plane stress problems make up a large majority of practical cases. Therefore, the numerical model presented here is based on the inelastic model for cyclic biaxial loading of reinforced concrete of Darwin and Pecknold (1974) which was designed to be used for such type of structures (shear walls, beams, slabs, shear panels, shells, reactor containment vessels).

2.1 Concrete

The concrete is treated as incrementally linear, elastic material, which means that during each load increment the material is assumed to behave elastically. It is also considered to exhibit stress-induced orthotropic material behavior. The constitutive relationship for incrementally linear orthotropic material with reference to the principal axes of orthotropy can be written as:

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\tau_{12} \end{Bmatrix} = \frac{1}{1-\nu^2} \begin{bmatrix} E_1 & \nu\sqrt{E_1E_2} & 0 \\ \nu\sqrt{E_1E_2} & E_2 & 0 \\ 0 & 0 & (1-\nu^2)G \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{Bmatrix} = D_C \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{Bmatrix} \qquad (1)$$

where $d\sigma_i$ and $d\varepsilon_i$ are the stress and strain increments, E_1 and E_2 are initial concrete stiffness modules in principal directions, $\nu = \nu_1 \cdot \nu_2$ is the "equivalent" poison ratio, $G = \frac{1}{4(1-\nu^2)}(E_1 + E_2 - 2\nu\sqrt{E_1E_2})$ is the shear modulus and D_C is the concrete constitutive matrix in the principle directions. Before it can be used in the finite element procedure, the concrete constitutive matrix is transformed to global coordinates using:

$$D_C' = T^T D_C T \qquad (2)$$

where T is the strain transformation matrix (Cook, 1974). At the moment when the principle tensile stress exceeds the concrete tensile strength a "crack" forms perpendicular to the principle stress direction. This is modeled by reducing the values of $E$and ν to zero. This has an effect of creating a "smeared" rather than discrete crack. The constitutive equation for the cracked concrete then takes the form:

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\tau_{12} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & \frac{E_2}{4} \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{Bmatrix} \qquad (3)$$

If the tensile strength in the other principle direction is exceeded then a second crack occurs and the constitutive matrix is then reduced to $D_C = [0]$. In order to keep track of the material degradation, the concept of "equivalent uniaxial strain" is used. It allows derivation of the actual biaxial stress-strain curves from uniaxial curves. The equation suggested by Saenz (1964) is often used for this purpose:

$$\sigma_i = \frac{\varepsilon_{ui} \cdot E_0}{1+\left(\frac{E_0}{E_S}-2\right)\frac{\varepsilon_{ui}}{\varepsilon_{ci}}+\left(\frac{\varepsilon_{ui}}{\varepsilon_{ci}}\right)^2} \tag{4}$$

where E_0 is the tangent modulus of elasticity at zero stress, E_s is the secant modulus at the point of maximum compressive stress (σ_{ci}), and ε_{ci} is the equivalent uniaxial strain at maximum compressive stress. The concrete biaxial strength envelope suggested by Kupfer and Gerstle (1973), is used to determine the value of σ_{ci}.

2.2 Reinforcing Steel

Generally, the reinforcing steel can be modeled as discrete or distributed. The model presented here considers the reinforcing steel to be distributed, or "smeared", throughout the concrete. A simple, bilinear model with strain hardening is adopted for the stress-strain behavior of the steel. The constitutive matrix of the steel defined in the steel direction is

$$D_S = p_S \begin{bmatrix} E_{Steel} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{5}$$

with E_{Steel} the tangent stiffness of the steel and p_S the reinforcing ratio. Depending on the stress level in the steel, E_{Steel} can be either equal to the initial steel stiffness E_S or reduced by a strain hardening stiffness ratio (δ). Before using it in the composite material matrix, D_S is transformed to the global coordinates using the strain transformation matrix (T).

2.3 FRP Strengthening

The influence of the FRP strengthening is accounted for in the same fashion as the reinforcing steel. The material is treated as distributed, or "smeared" throughout the concrete. Its material behavior is assumed to be elastic-brittle, having abrupt failure after reaching its maximal strength. It is also capable of transmitting only tension stresses. The constitutive matrix of the FRP defined in the direction of the FRP fibers is therefore:

$$D_F = p_F \begin{bmatrix} E_F & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{6}$$

with E_F the tangent stiffness of the FRP and p_F the "strengthening" ratio. Before using it in the composite material matrix D_F must also transformed to the global coordinates using the strain transformation matrix (T).

2.4 Composite material matrix

After defining the constitutive matrices of the constituent materials, the constitutive matrix of the composite material in the global coordinates is obtained by their summation:

$$D' = D'_C + \sum_{i=1}^{n} D'_{S,i} + \sum_{i=1}^{m} D'_{F,i} \tag{7}$$

where D', D'_C, D'_S and D'_F are the constitutive matrices if the composite material, concrete, steel and FRP in global coordinates, respectively, n is the number of different reinforcing steels and m is the number of different FRPs used for strengthening.

3. ANSYS IMPLEMENTATION

The proposed material model briefly described in the previous section was coded and implemented into ANSYS in order to test its correctness and usability by comparing the results from numerical analyses with the available experimental data from the literature. The model is implemented in modular fashion (Figure 1) so the same material model can be used to model both strengthened and unstrengthened parts of the model.

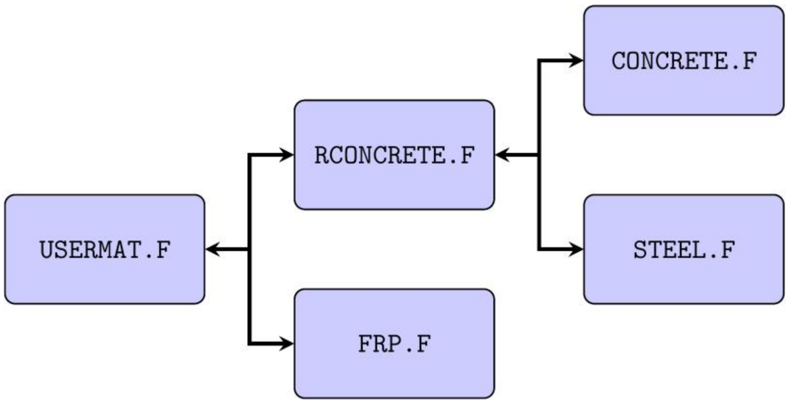

Figure 1. FEM mesh model of RWOA beams in ANSYS

4. VERIFICATION

4.1. Monotonic loading of RC beams

The results of the analysis of three-large scale RC beams tested by Wong (2001) are presented here. These were also analyzed by Wong and Vecchio (2003). The beams were designed with only tension reinforcement. No shear steel reinforcement was used. Instead, CFRP strips were glued to the side surfaces to act as shear strengthening. The CFRP fabric used for strengthening was composed of graphite fibers oriented in the longitudinal direction and Kevlar 49 weft in the perpendicular direction. The material was tested to obtain its material properties. Tensile strength of 1090 MPa and ultimate strain of 0.011 was recorded. Strips of this material with a width of 200 mm were bonded on the side surfaces (not wrapped around the beam) at a central distance of 300 mm between each other. The beams were tested under monotonic three-point loading until failure.

In the FEM analyses performed by Wong and Vecchio (2003), 2D elements were used to represent the concrete. The elements were double-noded with one set of nodes used for the concrete elements, while the second set was used to attach the truss elements that represented the FRP. Then the coincident nodes were connected by contact or link elements representing the bond.

The finite element model used here was build using 4-node quadrilateral Plane182 elements. Only half of each test specimen was modeled making use of its symmetry (Figure 2). To properly model the longitudinal steel reinforcement, the beams were divided into two parts - upper and lower part, with the lower part being of height of 130 mm and containing the smeared longitudinal reinforcement. The material properties of the two parts for each of the beams are presented in Table 1. The available experimental data was used to calibrate the models. The resulting load-deflection curves are shown in Figure 3. They are compared with the recorded experimental data as well as with the results of the FEM analysis performed by Wong and Vecchio (2003). It can be seen that the obtained results closely follow the experimental curves especially in the deflection range up to the steel yielding point. The failure in the models occurred due to concrete crushing at the top point in the symmetry axes, i.e. the location where the load is applied. The performed analyses showed different results in predicting the point of failure which was mostly influenced by the finite element and load step sizes.

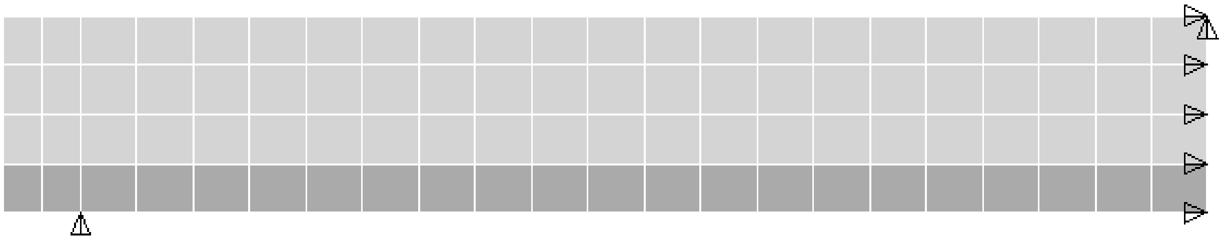

Figure 2. FEM mesh model of RWOA beams in ANSYS

Table 1. Material Parameters for the RWOA Beams used in the Analysis

	Concrete					Steel				FRP		
	f_c' MPa	E_0 GPa	f_t MPa	ε_{cu} %	ν	f_y MPa	E_S GPa	δ %	p_S %	E_F GPa	ε_F %	p_F %
RWOA 1												
upper	23	18	4	-0.35	0.2					100	1.1	0.2
lower	23	18	4	-0.35	0.2	430	200	1	5.6	100	1.1	0.2
RWOA 2												
upper	26	20	4	-0.35	0.2					100	1.1	0.2
lower	26	20	4	-0.35	0.2	400	200	1	7.3	100	1.1	0.2
RWOA 3												
upper	44	25	4	-0.35	0.2					100	1.1	0.2
lower	44	25	4	-0.35	0.2	400	200	1	8.9	100	1.1	0.2

4.2 Cyclic loading of RC Walls

A series of reinforced concrete shear wall specimens were tested in cyclic loading conditions (Lombard, 1999). The walls were constructed using 40 MPa concrete with identical reinforcement of 400 MPa, 10 mm reinforcing bars. The height of the walls from the base of the panel to the center of the cap beam is 2 m, the length is 1.5 m and the thickness is 10 cm. The vertical reinforcement consists of five pairs of 10 mm bars, spaced at 40 cm for a reinforcement ratio of 0.8%. The horizontal steel consisted of five pairs of 10 mm bars, spaced at 40 cm for a reinforcement ratio of 0.5%. Three of the test specimens included a control wall and two strengthened walls. The control wall was tested in its original state which provided a baseline for the evaluation of the repair and strengthening techniques. The two strengthened shear walls were strengthened by applying 0.11 mm carbon fiber sheets to the walls without pre-damage. The carbon fiber sheets had an elastic tensile modulus of 230 GPa and failure strain of 1.5%. The first specimen was strengthened with one vertical layer of FRP externally bonded to each face of the wall (Wall 1). The second specimen had one horizontal and two vertical FRP layers on each face of the wall (Wall 2). Both specimens were not loaded until the strengthening was applied.
This paper presents the results of the analysis of the specimens Wall 1 и Wall 2. For the FEM model in this case triangular as well as quadrilateral meshes were tested (Figure 4). The preliminary analyses showed that using triangular mesh generally led to better solutions. A mesh of triangular, 6-node Plane183 elements with average size of 25 cm was used for the final results.

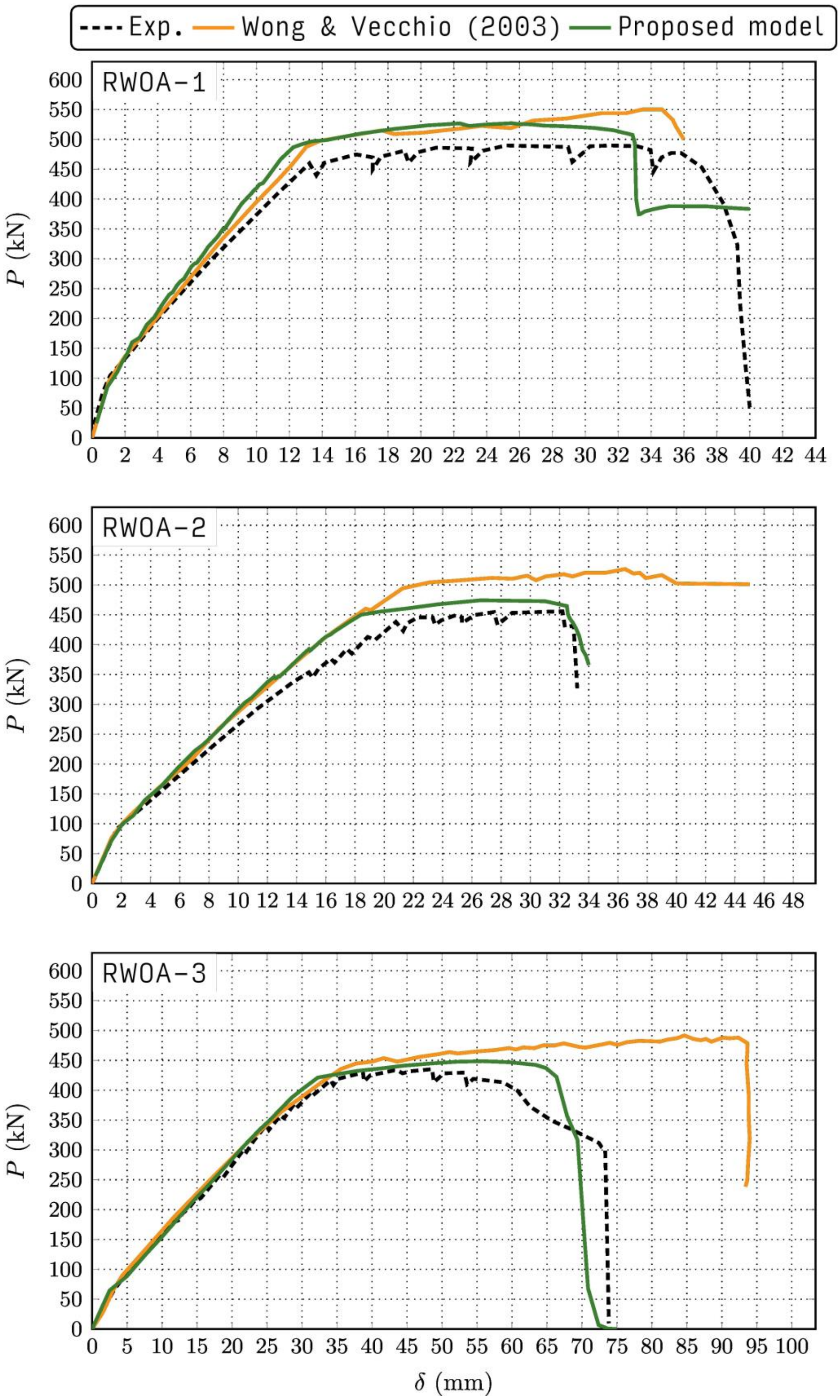

Figure 3. Load versus mid span deflection for RWOA beams

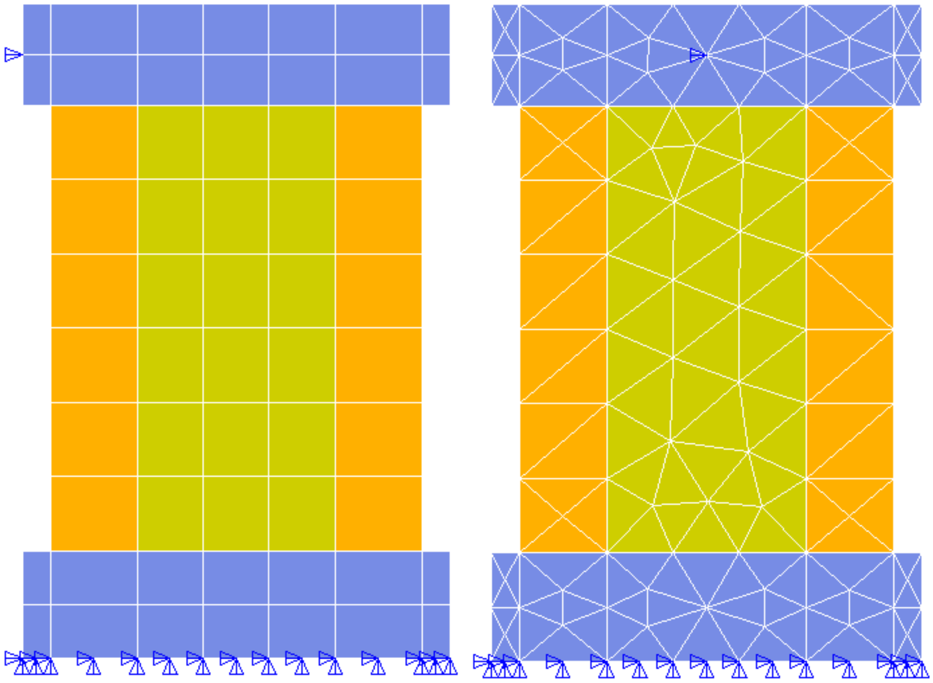

Figure 4. Quadrilateral and Triangular Element Mesh of the FEM Model

Five different sections of the wall with different properties were defined: top and bottom beam, two side section (‘columns’) and a middle section (‘panel’) (Figure 4). Since the top and the bottom beam are significantly stiffer that the wall and their actual purpose is to provide the load transfer and anchorage for the tested wall, they were modelled as linear-elastic with very high elasticity modulus. The confining effect of the stirrups in the ‘columns’ was approximately accounted for by slightly increasing the concrete compressive strength in those regions, taking it to be 46 MPa in the ‘columns’, and 40 MPa in the ‘panel’. The other concrete parameters were taken as: tensile strength of 4 MPA, initial elasticity modulus of 35 GPa, equivalent uniaxial strain of 0.35% and equivalent Poisson’s ratio of 0.2. The steel material parameters were taken as: yield strength of 400 GPa, elasticity modulus of 200 GPa and strain hardening stiffness ratio of 1.8. The reinforcement ratio in vertical direction is 0.8%, and in horizontal direction 3% (in the ‘columns’) and 0.5% (in the ‘panel’). The FRP material parameters were taken as: elasticity modulus of 230 GPA, ultimate strain at failure 1.5% and “strengthening” ratio of 0.22 for both the ‘columns’ and the ‘panel’ for each applied layer of the FRP strengthening, in the corresponding direction.

The cyclic load was applied at the middle of the top beam as a series of small displacements. The force and displacement at the same point were taken as results of the performed analyses. These were compared with the available experimental data.

The resulting hysteretic loops are shown Figure 5 for Wall 1 and Figure 6 for Wall 2 (with Figure 7 and Figure 8 showing individual loops developed at greater deflections where the model shows distinct inelastic behaviour). To measure how the numerical results compare to the experimental data the energy dissipated at each cycle (which corresponds to the area of the hysteretic loop) was calculated. The calculated energy dissipation is given in Table 2 for Wall 1 and Table 3 for Wall 2. The results indicate quite good correspondence with the experimentally acquired data.

It should also be noted that although the cyclic loading analyses yielded good results, the solution showed significant sensitivity on the input parameters (element type and size, load step sizes, material data). Non-convergent load-step solutions frequently occurred leading to premature failure of the model. To obtain good and stable solution the model needed to be calibrated by performing several parametric analyses which would yield the most appropriate set of input parameters. As the final results show, once stable solution is reached, the simulation shows satisfactory correspondence to the test results.

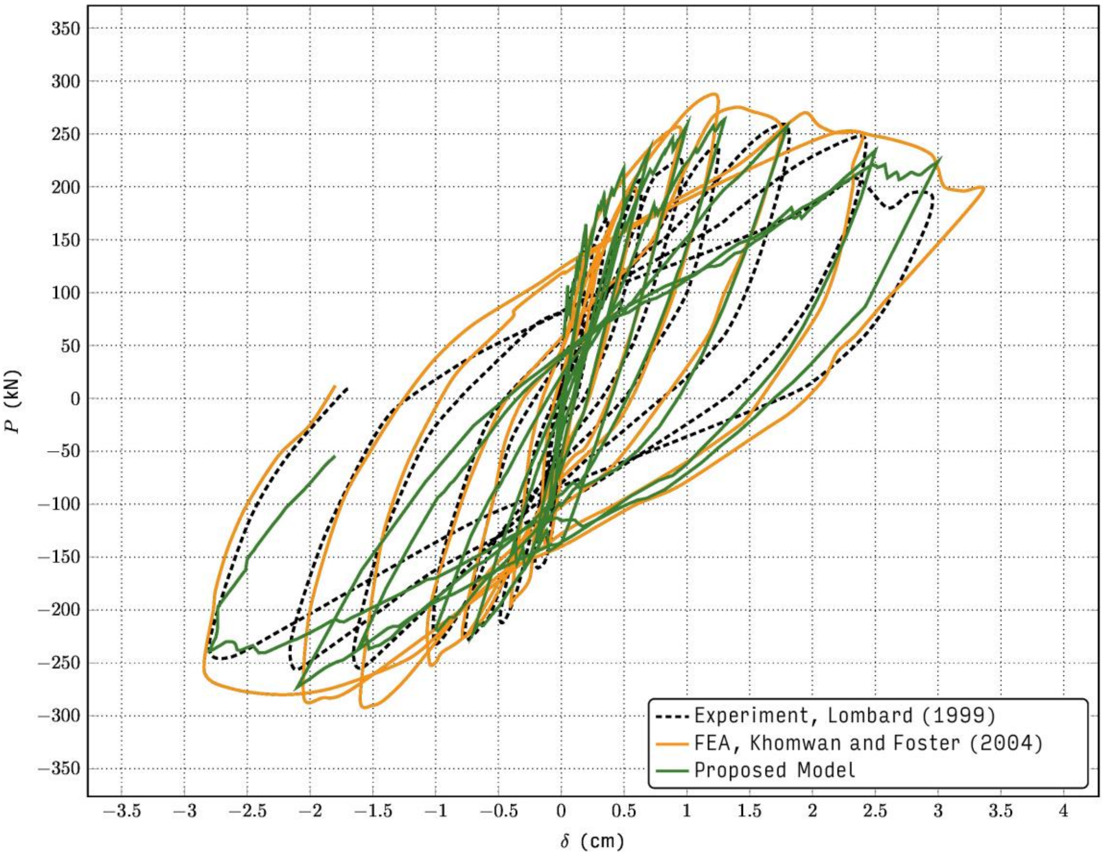

Figure 5. Load-Deflection Curves for Test Wall 1

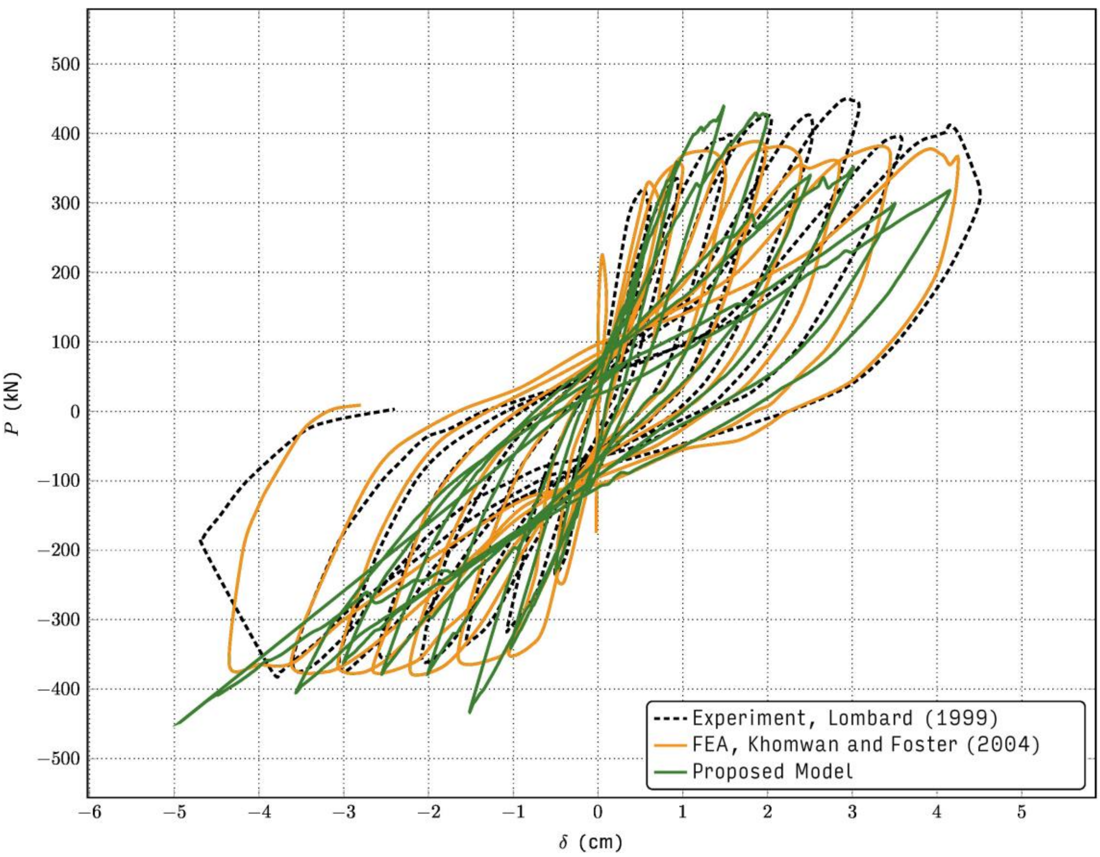

Figure 6. Load-Deflection Curves for Test Wall 2

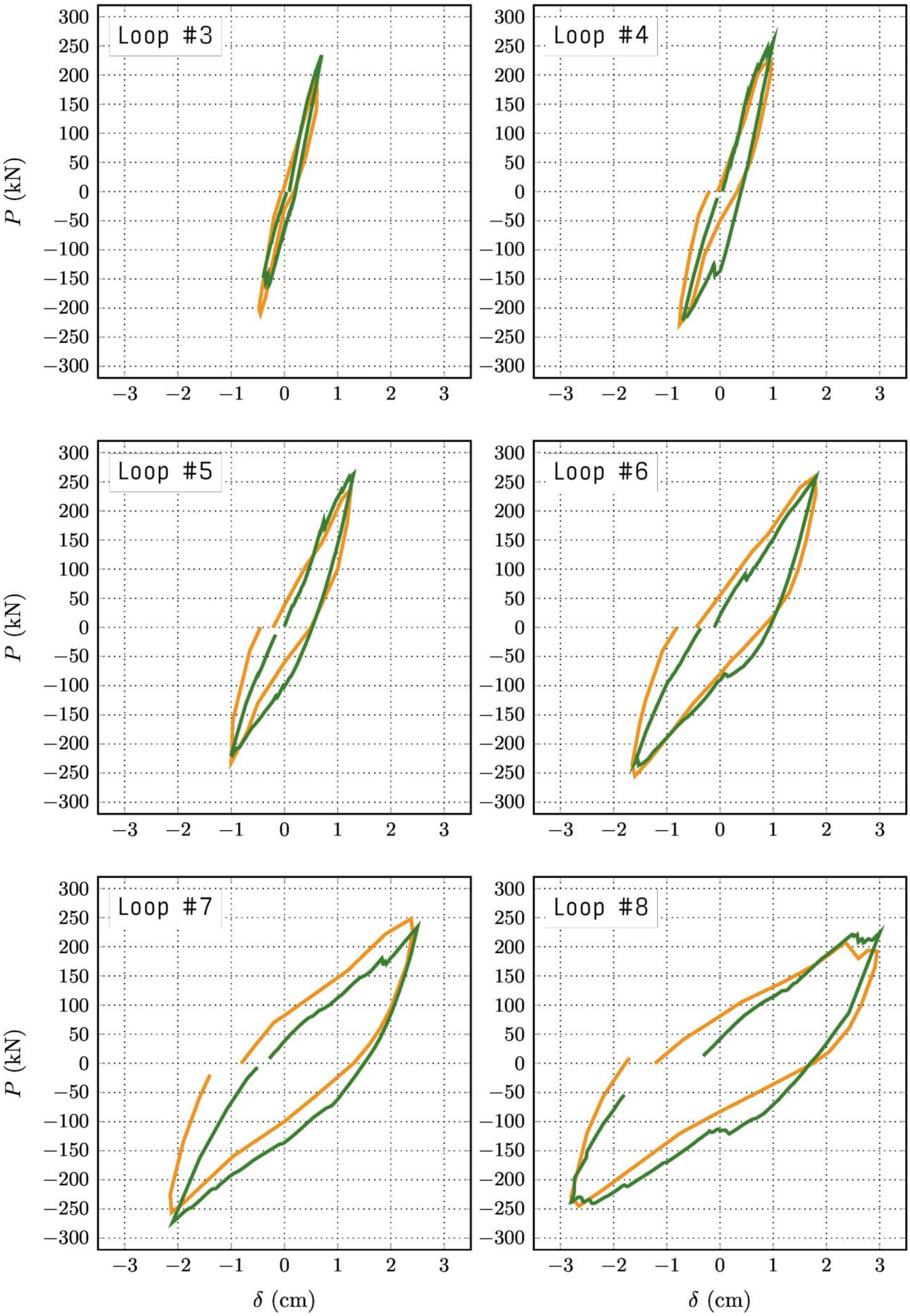

Figure 7. Load-Deflection Curves for Test Wall 1 (Orange Line – Experimental Results, Green Line – Numerical Results)

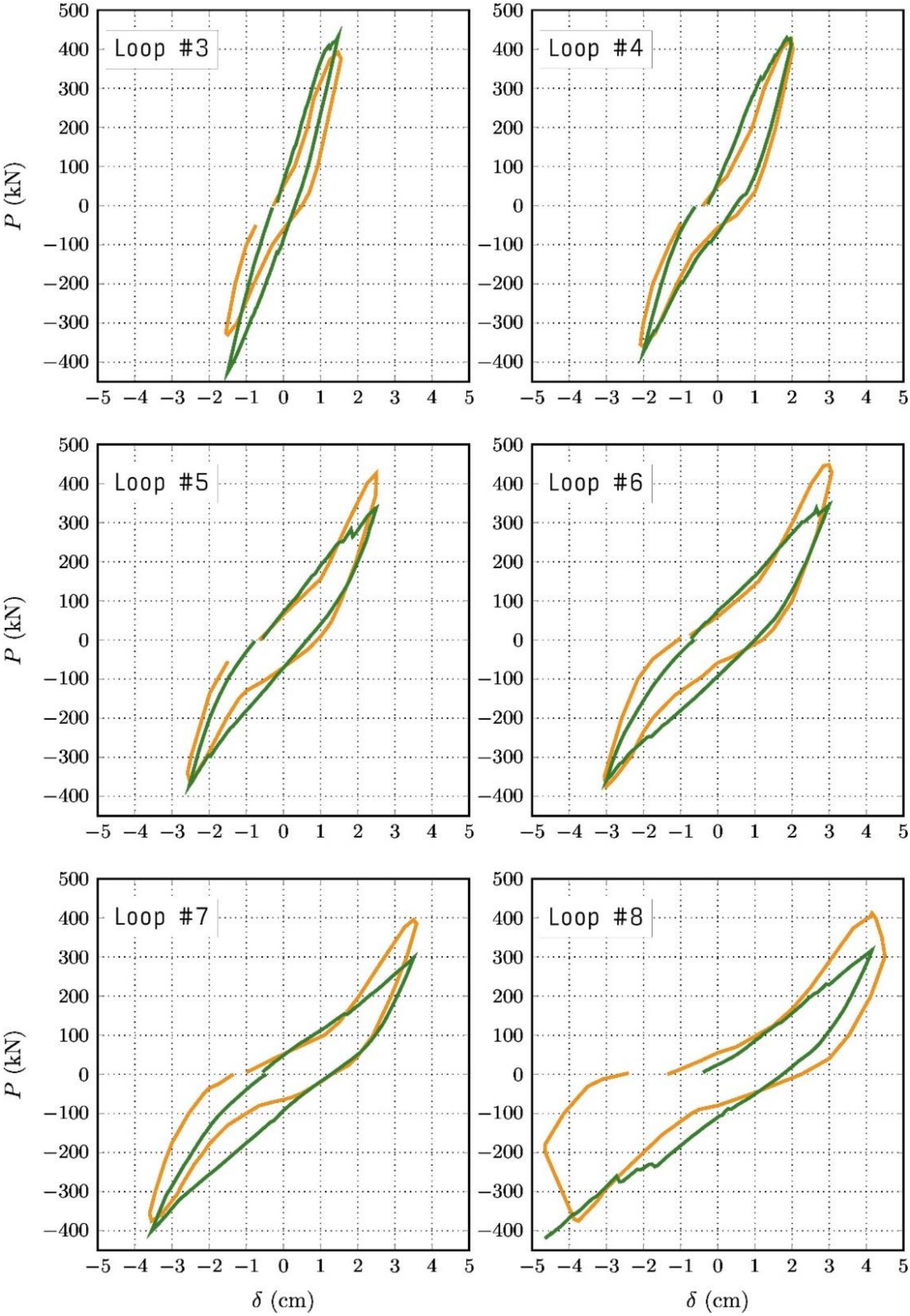

Figure 8. Load-Deflection Curves for Test Wall 2 (Orange Line – Experimental Results, Green Line – Numerical Results)

Table 2. Comparison of Energy Dissipation per Cycle for Wall 1 (in Nm)

Loop #	Experimental	FEM	Ratio	Difference
3	622.25	494.81	0.80	20%
4	1412.25	1442.94	1.02	2%
5	2268.10	1964.96	0.87	13%
6	4603.00	3511.48	0.76	24%
7	6960.88	5797.95	0.83	17%
8	8660.70	8161.54	0.94	6%

Table 3. Comparison of Energy Dissipation per Cycle for Wall 2 (in Nm)

Loop #	Experimental	FEM	Ratio	Difference
3	3880.50	3477.32	0.90	10%
4	5327.65	5194.67	0.98	2%
5	6527.80	6000.68	0.92	8%
6	8649.65	7623.38	0.88	12%
7	8718.05	7681.69	0.88	12%
8	17520.80	7279.18	0.42	58%

5. CONCLUSIONS

The paper presents an attempt to formulate material model which will correctly simulate the behavior of reinforced concrete members in plane stress strengthened with FRP materials. The presented model builds up on the concepts of an earlier reinforced concrete model of Darwin and Pecknold. It uses the uniaxial strain approach in modeling of biaxially loaded reinforced concrete and the distributed approach of modelling the cracking behavior as well as the reinforcing steel and the applied FRP strengthening. The proposed model is subsequently implemented into the code of the general finite element method program ANSYS as a user material model in order to test its results against the available experimental data.

Analyses of three RC beams strengthened by externally bonded FRP wraps on their sides were performed and presented herein. The results are compared against the experimentally obtained data as well as against the numerical results from another finite element analysis performed by other authors employing more traditional approach into finite element modelling of such problems. It can be concluded that the proposed model successfully predicts the behavior of the RC beams strengthened with FRP and subjected to monotonic loading conditions.

The results shown in Tables 2 and 3 indicate that the numerical model underestimates the energy dissipation for about 13.66% on average for the Wall 1 specimen and 8.8% on average for Wall 2 (excluding the erroneous result of the loop #8 where the numerical analysis did not reach convergence before completing the full cycle) compared to the experimental results. Considering the highly inelastic nature of the simulated processes, this can be considered as a good result. The model also predicts the ultimate forces and displacement as well as stiffness degradation in each cycle quite favorably (Figures 7 and 8). However, it must also be pointed out that during the extensive testing of the proposed model, some drawbacks could be identified. Mostly that the model showed significant sensitivity to the values of the input parameters, while the simulation times were very high. These issues must be addressed before the model can be applied and used in real world applications.

Based on the presented results, it can be concluded that the proposed model is able to correctly simulate the behavior of the RC beams and walls strengthened with FRP. Its ANSYS implementation enables its use in both research and practical purposes, facilitating the further research in this field as well as the practical applications in the construction industry.

6. REFERENCES

ANSYS Inc., ANSYS®Multiphysics, Release 9.0

Darwin, D. and Pecknold, D.A.W. (1974). Inelastic Model for Cyclic Loading of Reinforced Concrete, Civil Engineering Studies SRS-409, University of Illinois, Urbana Champaign, Illinois, 169 pages.

Cook, R. D. (1974). Concepts and applications of finite element analysis. New York: John Wiley & Sons.

Kheyroddin, A. & Naderpour, H. (2008). Nonlinear finite element analysis of composite RC shear walls, *Iranian Journal of Science & Technology, Transaction B, Engineering*, 32(B2), 79–89.

Khomwan, N. and Foster, S. J. (2004). Finite Element Modelling of FRP Strengthened Beams and Walls. Technical Report UNICIV Report R-432, The University of New South Wales, School of Civil and Environmental Engineering, Kensington, Sydney 2052 Australia, 68 pages.

Kupfer, H. and Gerstle, K.H. (1973). Behavior of Concrete Under Biaxial Stresses, *Journal of Engineering Mechanics Division*, 852-866.

Lombard, J. (1999). *Seismic strengthening and Repair of Reinforced Concrete Shear Walls Using Externally Bonded Carbon Fibre Tow Sheets*, Master Thesis, Ontario, Canada, (1999)

Saenz, L. (1964). Discussion of "Equation for Stress-Strain Curve of Concrete" by Desayi and Krishman, *ACI Journal*, 1229-1235

Wong, R. S. Y. (2001). Towards modelling of reinforced concrete members with externally-bonded fibre reinforced polymer (FRP) composites. Master's thesis, University of Toronto, Toronto, Canada.

Wong, R.S.Y. and Vecchio, F.J. (2003). Towards modeling of reinforced concrete members with externally bonded fiber-reinforced polymer composites. *ACI Structural Journal* **100:1**,47-55.