MATERIAL MODEL OF FRP STRENGTHENED REINFORCED CONCRETE IN BIAXIAL STRESS STATE SUBJECTED TO MONOTONIC LOADING CONDITIONS

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The application of shear walls as a primary earthquake resistant mechanism in structural design is traditionally accepted and used over the last decades. However, very often, older structures having shear walls no longer comply with the contemporary standards and codes which raise the need for their strengthening and retrofit. Many strengthening measures and techniques have been designed and used. One of the newest and most promising is the use of externally bonded FRP strips and sheets. Intensive research has started in order to design the most economic and efficient technique for application of these materials. Different modelling approaches have been employed in these designs. This paper presents the attempt to formulate a new material model that could be used to model FRP strengthened RC member and its implementation into ANSYS. The model results are tested against the available experimental data in order to verify its correctness and practical usability. The obtained results show satisfactory match when compared with the experimental data.

Keywords: Finite Element Method (FEM), ANSYS, reinforced concrete, strengthening, FRP

1. INTRODUCTION

The conventional earthquake resistant design of reinforced concrete structures advises use of shear walls as effective way to add earthquake resistance to the reinforced concrete frames. A problem arises with structures erected decades ago following design rules which are by today’s standards obsolete, inadequate and inefficient. Major earthquake events from around the world have shown the design deficiencies of these structures by inducing extensive damages in the structural members. Many of the old shear wall buildings are at risk of suffering damages from a major earthquake mostly due to their insufficient in-plane stiffness, flexural and shear strengths and ductility owing to the older design codes which didn't adequately estimate the demands that major earthquakes impose on the structures. This problem is ever increasing as the existing structures are getting older and their members gradually deteriorate.

Many different methods of seismic strengthening and repair of shear wall structures have been developed and tested in the last thirty years. Recently, state-of-the-art strengthening and retrofit techniques increasingly utilize externally bonded fiber reinforced polymer (FRP) composites, which offer unique properties in terms of strength, lightness, chemical resistance, and ease of application. Such techniques are most attractive for their fast execution and low labor costs.

Only recently have researchers attempted to simulate the behavior of reinforced concrete strengthened with FRP composites using the finite element method. The majority of the studies that included numerical modeling of FRP strengthened RC members with FEM use element overlaying, where one-, two- or even three dimensional elements (solid or layered) that represent the FRP material are superimposed over the concrete elements, either with (ex. Khomwan and Foster [7]; Wong and Vecchio [12]) or without (ex. Kheyroddin and Naderpour [6]) interface elements that represent the influence of the adhesive material or the bond between the FRP and the concrete.

A different approach is presented in this paper. An attempt is made to formulate a new material model which will simplify the modeling of FRP strengthened reinforced concrete members. The newly formulated material model is implemented into ANSYS [1] and tested using available experimental data.

2. MODEL FORMULATION

In the analysis of RC structures plane stress problems make up a large majority of practical cases. Therefore, the numerical model presented here is based on the inelastic model for cyclic biaxial loading of reinforced concrete of Darwin and Pecknold [3] which was designed to be used for such type of structures (shear walls, beams, slabs, shear panels, shells, reactor containment vessels).

2.1 CONCRETE

The concrete is treated as incrementally linear, elastic material, which means that during each load increment the material is assumed to behave elastically. It is also considered to exhibit stress-induced orthotropic material behavior. The constitutive relationship for incrementally linear orthotropic material with reference to the principal axes of orthotropy can be written as:

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\tau_{12} \end{Bmatrix} = D_C \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{Bmatrix} \tag{1}$$

with D_C being:

$$D_C = \frac{1}{1-\nu^2}\begin{bmatrix} E_1 & \nu\sqrt{E_1E_2} & 0 \\ \nu\sqrt{E_1E_2} & E_2 & 0 \\ 0 & 0 & (1-\nu^2)G \end{bmatrix} \tag{2}$$

where $d\sigma_i$ and $d\varepsilon_i$ are the stress and strain increments, E_1 and E_2 are initial concrete stiffness modules in principal directions, $\nu = \sqrt{\nu_1\nu_2}$ is the "equivalent" poison ratio, $G = \frac{1}{4(1-\nu^2)}(E_1 + E_2 - 2\nu\sqrt{E_1E_2})$ is the shear modulus and D_C is the concrete constitutive matrix in the principle directions. Before it can be used in the finite element procedure, the concrete constitutive matrix is transformed to global coordinates using:

$$D_C^{'} = T^T D_C T \tag{3}$$

where T is the strain transformation matrix (Cook, 1974). At the moment when the principle tensile stress exceeds the concrete tensile strength a "crack" forms perpendicular to the principle stress direction. This is modelled by reducing the values of E and ν to zero. This has an effect of creating a "smeared" rather than discrete crack. The constitutive equation for the cracked concrete then takes the form:

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\tau_{12} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & \frac{E_2}{4} \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{Bmatrix} \tag{4}$$

If the tensile strength in the other principle direction is exceeded then a second crack occurs and the constitutive matrix is then reduced to $D_C = [0]$. In order to keep track of the material degradation, the concept of "equivalent uniaxial strain" is used. It allows derivation of the actual biaxial stress-strain curves from uniaxial curves. The equation suggested by Saenz [10] is frequently used for this purpose:

$$\sigma_i = \frac{\varepsilon_{ui} \cdot E_0}{1 + \left(\frac{E_0}{E_S} - 2\right)\frac{\varepsilon_{ui}}{\varepsilon_{ci}} + \left(\frac{\varepsilon_{ui}}{\varepsilon_{ci}}\right)^2} \tag{5}$$

where E_0 is the tangent modulus of elasticity at zero stress, E_S is the secant modulus at the point of maximum compressive stress (σ_{ci}), and ε_{ci} is the equivalent uniaxial strain at maximum compressive stress, Fig.1. The concrete biaxial strength envelope suggested by Kupfer and Gerstle [9], Fig. 2, is used to determine the value of σ_{ci}.

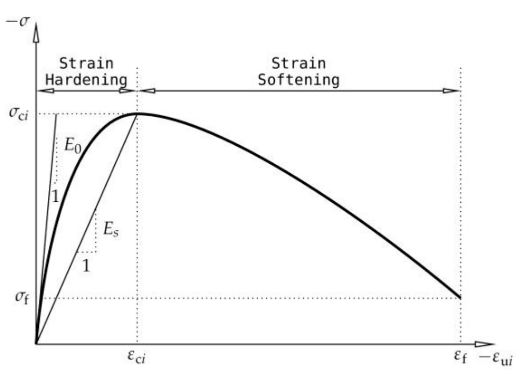

Figure 1. Equivalent Uniaxial Stress-Strain Curve in Compression [4].

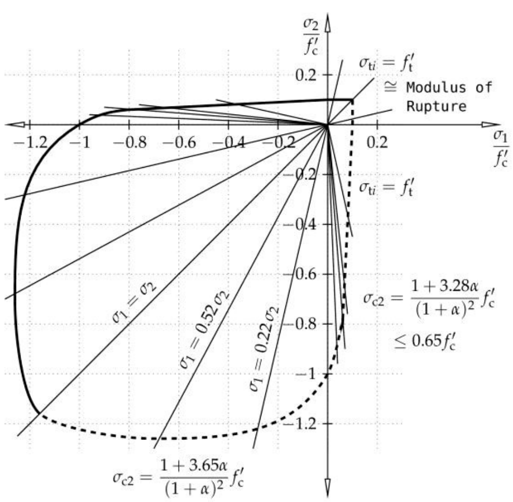

Figure 2. Analytical Biaxial Strength Envelope [9]

2.2 REINFORCING STEEL

Generally, the reinforcing steel can be modeled as discrete or distributed. The model presented here considers the reinforcing steel to be distributed, or "smeared", throughout the concrete. A simple, bilinear model with strain hardening is adopted for the stress-strain behavior of the steel. The constitutive matrix of the steel defined in the steel direction is

$$D_S = p_S \begin{bmatrix} E_{steel} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{6}$$

with E_{steel} the tangent stiffness of the steel and p_S the reinforcing ratio. Depending on the stress level in the steel, E_{steel} can be either equal to the initial steel stiffness E_S or reduced by a strain hardening stiffness ratio (δ), see Fig.3. Before using it in the composite material matrix, D_S is transformed to the global coordinates using the strain transformation matrix (T).

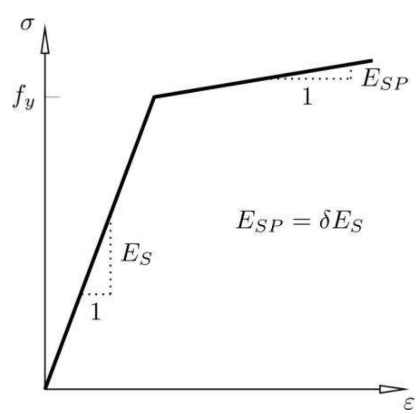

Figure 3. Stress-strain curve of reinforcing steel

2.3 FRP STRENGTHENING

The influence of the FRP strengthening is accounted for in the same fashion as the reinforcing steel. The material is treated as distributed, or "smeared" throughout the concrete. Its material behavior is assumed to be elastic-brittle, having abrupt failure after reaching its maximal strength (Fig.4). It is also capable of transmitting only tension stresses. The constitutive matrix of the FRP defined in the direction of the FRP fibers is therefore:

$$D_F = p_F \begin{bmatrix} E_F & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (7)$$

with E_F the tangent stiffness of the FRP and p_F the "strengthening" ratio. Before using it in the composite material matrix D_F must also transformed to the global coordinates using the strain transformation matrix (T).

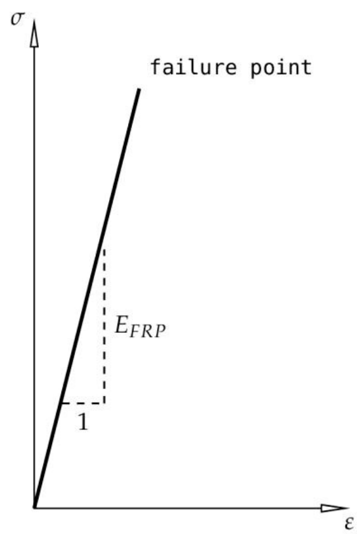

Figure 4. Stress-strain curve of FRP

2.4 COMPOSITE MATERIAL MATRIX

After defining the constitutive matrices of the constituent materials, the constitutive matrix of the composite material in the global coordinates is obtained by their summation:

$$D' = D'_C + \sum_{i=1}^{n} D'_{S,i} + \sum_{i=1}^{m} D'_{F,i} \quad (8)$$

where D', D'_C, $D'_{S,i}$ and $D'_{F,i}$ are the constitutive matrices if the composite material, concrete, steel and FRP in global coordinates, respectively, n is the number of different reinforcing steels and m is the number of different FRPs used for strengthening.

3. VERIFICATION

The material model briefly described in the second section was coded and implemented into ANSYS in order to test its correctness and usability by comparing the results from numerical analyses with the available experimental data from the literature.

3.1 GARDEN AND HOLLAWAY (1998)

Garden and Hollaway [5] performed four point bending tests on 1 m long RC beams strengthened with CFRP. The CFRP plates with a thickness of 0.82 mm were attached to the soffit of the beam. The beam with designation $3_{\mathrm{U,1.0m}}$ was analyzed here (Fig. 5). Three different sections or parts of the beam (top, bottom and middle) were defined in order to accommodate the steel reinforcement and FRP strengthening placement in the actual beam cross-section (Fig.6). Only half of the beam was model making use of the beam and load symmetry. The material properties for the reinforcing steel and the FRP strengthening for the three parts of the beam model are given in Table 1. The concrete definition for the whole beam was the same, having uniaxial compression strength of $f'_C = 43\,\mathrm{MPa}$, uniaxial tensile strength of $f'_t = 3\,\mathrm{MPa}$, initial modulus of elasticity $E_0 = 40\,\mathrm{GPa}$, equivalent uniaxial strain at maximum strength $\varepsilon_{cu} = -0.0022$ and equivalent Poisson's ratio $\nu = 0.2$.

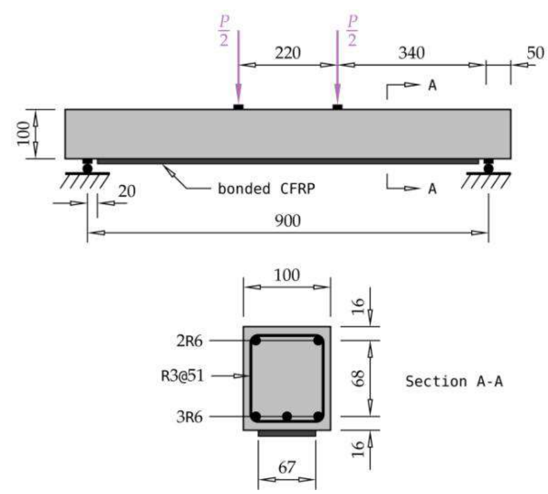

Figure 5. Test specimen $3_{\mathrm{U,1.0m}}$ [5]

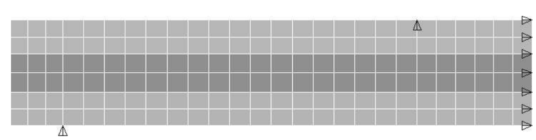

Figure 6. FEM model of test specimen $3_{\mathrm{U,1.0m}}$ in ANSYS

Displacement control analysis was performed by applying series of vertical displacements at the point where the load was applied in the actual test. The obtained load-deflection curve is shown in Fig. 7. While the beam stiffness in the initial load increments was apparently overestimated, the rest of the curve stays very close to the shape of the experimental curve.

Table 1. Material parameters for the $3_{\mathrm{U,1.0m}}$ beam used in the analysis

Part	Steel #1 (horizontal)			
	f_y	E_S	δ	p_S
	MPa	GPa	%	%
Bottom	350	215	0	2.7
Top	350	215	0	2.7
Middle	-	-	-	-
Part	Steel #2 (horizontal)			
Bottom	350	215	0	1.7
Top	350	215	0	1.7
Middle	350	215	0	1.7
Part	FRP			
	E_S		ε_F	p_F
	GPa		%	%
Bottom	110		1.2	1.7
Top	-		-	-
Middle	-		-	-

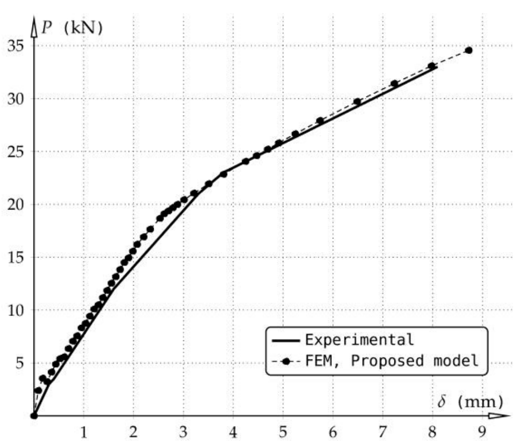

Figure 7. Load versus mid-span deflection of $3_{\mathrm{U,1.0m}}$ beam

3.2 WONG (2001)

Wong [11] has conducted test on three large-scale beams. They were then analyzed by Wong and Vecchio [12]. The beams were designed with only tension reinforcement. No shear steel reinforcement was used. Instead, CFRP strips were glued to the side surfaces to act as shear strengthening. The geometry of the three beam specimens is shown in Fig. 8. The CFRP fabric used for strengthening was composed of graphite fibers oriented in the longitudinal direction and Kevlar 49 weft in the perpendicular direction. The material was tested to obtain its material properties. Tensile strength of 1090 MPa and ultimate strain of 0.011 were recorded. Strips of this material with a width of 200 mm were bonded on the side surfaces (not wrapped around the beam) at a central distance of 300 mm between each other. The beams were tested under monotonic three-point loading until failure.

In the FEM analyses performed by Wong and Vecchio [12], 2D elements were used to represent the concrete. The elements were double noded with one set of nodes used for the concrete elements, while the second set was used to attach the truss elements that represented the FRP. Then the coincident nodes were connected by contact or link elements representing the bond.

The finite element model used here was build using 4-node quadrilateral Plane182 elements. Only half of the each test specimen was modelled making use of its symmetry (Fig. 9). To properly model the longitudinal steel reinforcement, the beams were divided into two parts - upper and lower part, with the lower part being of height of 130 mm and containing the smeared longitudinal reinforcement. The material properties of the two parts for each of the beams are presented in Tables 2, 3 and 4. The available experimental data was used to calibrate the models. The resulting load-deflection curves are shown in Fig. 10. They are compared with the recorded experimental data as well as with the results of the FEM analysis performed by Wong and Vecchio [12]. It can be seen that the obtained results closely follow the experimental curves especially in the deflection range up to the steel yielding point. The failure in the models occurred due to concrete crushing at the top point in the symmetry axes, i.e. the location where the load is applied. The performed analyses showed different results in predicting the point of failure which was mostly influenced by the finite element and load step sizes.

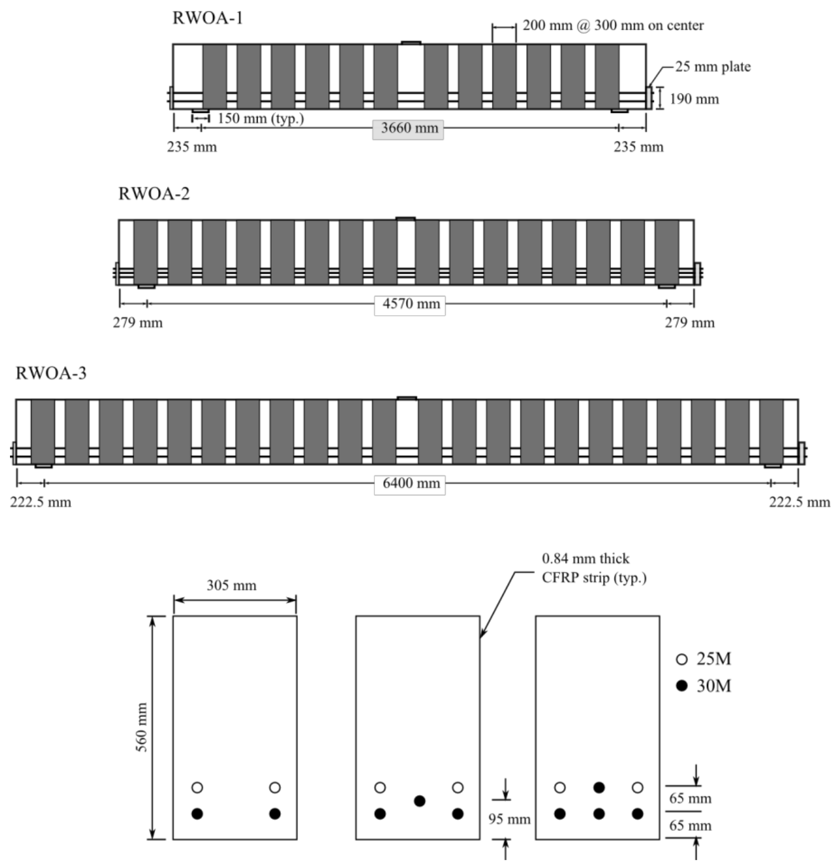

Figure 8. Elevation views and cross section details of RWOA beams [11]

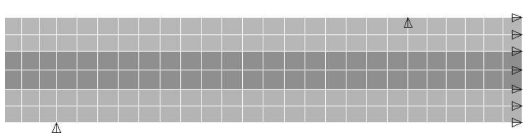

Figure 9. FEM mesh model of RWOA beams in ANSYS

Table 2. Material Parameters for the concrete of the RWOA Beams used in the Analysis

Beam	f'_C	E_0	f_t	ε_{cu}	ν
RWOA	MPa	GPa	MPa	%	%
1	23	18	4	-0.35	0.2
2	26	20	4	-0.35	0.2
3	44	25	4	-0.35	0.2

Table 3. Material Parameters for the steel in the lower part of the RWOA Beams, used in the Analysis (the upper part of the beams does not contain any steel material)

Beam	f_y	E_S	δ	p_S
RWOA	MPa	GPa	%	%
1	430	200	1	5.6
2	400	200	1	7.3
3	400	200	1	8.9

Table 4. Material Parameters for the FRP of the RWOA Beams used in the Analysis

Beam	E_0	ε_F	p_F
RWOA	GPa	%	%
1	100	1.1	0.2
2	100	1.1.	0.2
3	100	1.1	0.2

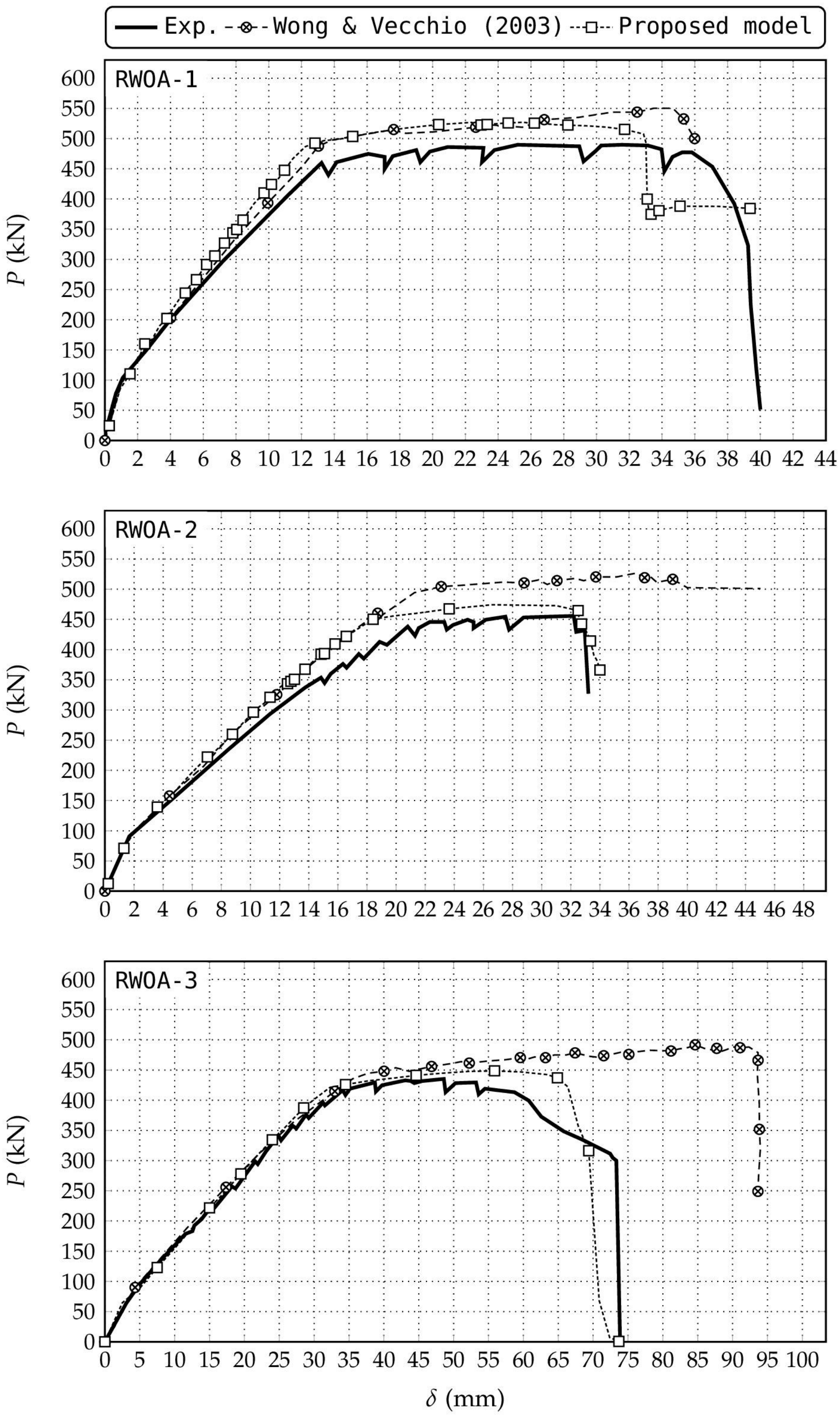

Figure 10. Load versus mid span deflection for RWOA beams [11]

4. SUMMARY AND CONCLUSIONS

The paper present an attempt to formulate material model which will correctly simulate the behavior of reinforced concrete members in plane stress strengthened with FRP materials. The presented model builds up on the concepts of an earlier model reinforced concrete model of Darwin and Pecknold. It uses the uniaxial strain approach in modelling of biaxially loaded reinforced concrete and the distributed approach of modelling the cracking behavior and the reinforcing steel.

The proposed model is subsequently implemented into the code of the general finite element method program ANSYS as a user material model in order to test its results against the available experimental data. Two different analyses were presented: An analysis of RC beam strengthened for bending by externally bonded FRP strips on the soffit side if the beam and analyses of three RC beams strengthened by externally bonded FRP wraps on their sides. The results are compared against the experimentally obtained data as well as against the numerical results from another finite element analysis performed by other authors employing more traditional approach into finite element modelling if such problems. Based on the presented results, it can be concluded that the proposed model is able to correctly simulate the behavior of the RC beams strengthened with FRP in different configurations. Its ANSYS implementation enables its use in both research and practical purposes, facilitating the further research in this field as well as the practical applications in the construction industry.

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