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Evaluation of crack width in reinforced concrete beams subjected to variable load

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ABSTRACT In this paper, the influence of loading histories of variable (imposed) actions on the behavior of reinforced concrete beams and especially the crack width was analyzed. For the evaluation of long-term effects (effects due to creep and shrinkage in concrete structures), quasi-permanent combination of actions was used to verify the reversible limit state. An experimental program and analytical research was performed to compare the experimentally obtained results of crack width and results of proposed calculation models given in EN 1992-1-1 Eurocode 2 and in the fib Model code 2010. For two specific loading histories, of series of beams D and E, the quasi-permanent coefficient ψ_2 was defined using the quasi-permanent combination of actions. These loading histories were consisting of long-term permanent action G and repeated variable action Q. The variable load was applied in cycles of loading/unloading for 24 and 48 hours in the period of 330 days. A total of eight reinforced concrete beams, dimensions 15/28/300 cm were tested.

1 INTRODUCTION

Cracks can be usually observed on the concrete surface during service life of concrete structures and causes nonlinear behavior of concrete structures exceeding the tensile strength of concrete. Beside their great influence on serviceability, cracks are also associated to durability, permeability and aesthetics issues.

There are various types of cracks, essentially defined by the principal cause or mechanism, but a few of them can be controlled by the designer. Usually restrained deformations from shrinkage or temperature movements and loading can be treated by the designer (Beeby & Narayanan 1995).

Structural cracks in hardened concrete are caused by actions (flexure, tension, shear, torsion and internal micro cracks due to severe stress zones).

Variable actions such as imposed loads for buildings are those arising from occupancy. According to the categories of use, imposed loads have great importance for areas for storage and industrial use, garages and vehicle traffic areas. Because of nature of variable loads, they have phenomenon of appearance in different time intervals that cannot be predicted and that are acting like random variables during the service life of structure (Arangjelovski 2011).

Repeated variable actions cause significant increase in concrete and reinforcement strain, increasing the crack width and deflections, reduction of tension stiffening and increase in bond-slip. The long-term effects usually include the creep of concrete, shrinkage and increase of strain due to repeated load. (Balazs 1997).

Taking into account the time dependency of load effects, two types of serviceability limits states should be satisfied: irreversible and reversible limit states. Evaluation and verification of serviceability limit state is expressed by the equation (Gulvanessian, Calgaro & Holicky 2002):

$$E_d \leq C_d \tag{1}$$

where: C_d = limiting design value of serviceability criterion and E_d = design value of the effects of actions specified in the serviceability criterion, determined on the basis of the relevant combination.

They are associated with the characteristic, frequent and quasi permanent combinations of actions.

The frequent combination of actions is normally used for reversible limit states such as the quasi-permanent combination of actions and it is usually expressed as:

$$\sum_{j\geq 1} G_{k,j} + P + \psi_{1,1} Q_{k,1} + \sum_{i>1} \psi_{2,i} Q_{k,i} \tag{2}$$

where: $G_{k,j}$ = permanent actions; P = prestressing action; $Q_{k,1}$ = leading variable action; $Q_{k,i}$ = accompanying variable actions; ψ_1 = frequent factor and ψ_2 = quasi permanent factor depending on the type of action.

The quasi-permanent combination which is used for the assessment of long-term effects in simplified form is written as:

$$\sum_{j\geq 1} G_{k,j} + P + \sum_{i\geq 1} \psi_{2,i} Q_{k,i} \tag{3}$$

where: $G_{k,j}$ = permanent load, $Q_{k,i}$ = variable load and ψ_2 = quasi permanent factor depending on the type of action.

Recommended values of frequent factor and quasi-permanent factor ψ_1 and ψ_2, needed for evaluation

Table 1. Recommended values of $\psi 1$ and $\psi 2$ factor for buildings (Table A1.1 of EN1990:2002).

Action	ψ_1	ψ_2
Imposed loads in buildings, category (see EN1991-1-1)		
Category A: domestic, residential areas	0.5	0.3
Category B: office areas	0.5	0.3
Category C: congregation areas	0.7	0.6
Category D: shopping areas	0.7	0.6
Category E: storage areas	0.9	0.8
Category F: traffic area, vehicle weight ≤ 30 kN	0.7	0.6
Category G: traffic area, 30 kN $<$ vehicle weight ≤ 160 kN	0.5	0.3
Category H: roofs	0.0	0.0

of reversible serviceability limit state, are given in the Table 1 for the area of interest according to EN 1990:2002+A1:2005.

Extensive experimental program was performed in order to define the factors ψ_1 and ψ_2 for the evaluation of reversible limit state for crack width for two specific loading histories. Loading histories are consisted of sustained permanent action G and repeated variable load Q applied in cycles loading and unloading for 24 and 48 hours respectively. The quality of concrete was class C30/37 and C60/75.

In this paper the experimental results, from measured maximum crack spacing and crack width, are analyzed by the crack control models given in EN1992-1-1 Eurocode 2 and in the fib Model Code for Concrete Structures 2010.

2 MODELS FOR CALCULATION OF CRACK WIDTH

2.1 *General*

When the elements are subjected to flexure, cracks are formed in reinforced concrete members when the tensile deformations from loads or restraint forces reach the tensile deformation capacity of concrete.

In the case of flexural member, the crack formation phenomenon is often subdivided into two phases: the crack formation phase and the stabilized cracking phase. (Borosnyoi and Balazs 2005).

This phenomenon is presented on the following figures 1 and 2.

A rigorous formulation of crack width can be obtained by the integration of the actual strains of reinforcement and that of concrete between cracks, based on accumulated slips:

$$w = \int_0^{S_r} \left[\varepsilon_s(x) - \varepsilon_c(x)\right] dx \tag{4}$$

or

$$w_k = \int_0^{2ls,\max} \left(\varepsilon_s - \varepsilon_c\right) \cdot dx \tag{5}$$

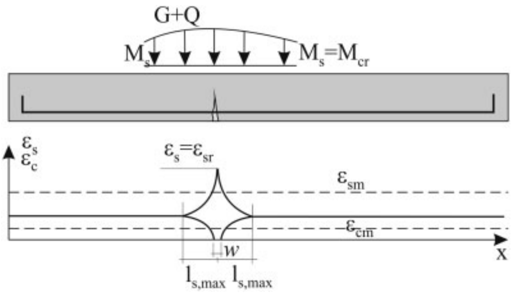

Figure 1. Crack formation phase.

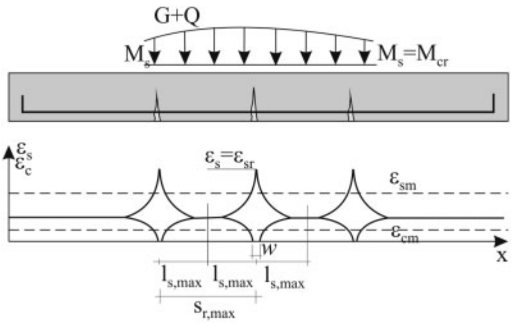

Figure 2. Stabilized cracking phase.

For the crack formation stage, characteristic value of crack width is:

$$w_k = 2 \cdot l_{s,\max} \cdot \left(\varepsilon_{sm} - \varepsilon_{cm}\right) \tag{6}$$

For the stabilized crack formation stage value of crack width is:

$$w_k = s_{r,\max} \cdot \left(\varepsilon_{sm} - \varepsilon_{cm}\right) \tag{7}$$

2.2 *Model of EN1992-1-1 Eurocode 2 Design of concrete structures—part 1-1: General rules and rules for buildings 2004*

For the calculation of crack width of reinforced concrete elements following expression may be used:

$$w_k = S_{r,\max}\left(\varepsilon_{sm} - \varepsilon_{cm}\right) \tag{8}$$

where $S_{r,max}$ = maximum crack spacing; ε_{sm} = mean strain in the reinforcement under the relevant combinations of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening; and ε_{cm} = mean strain in the concrete between cracks.

The difference of the main strains in the reinforcement and concrete ε_{sm}-ε_{cm} may be calculated from the expression:

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}\left(1 + \alpha_e \rho_{p,eff}\right)}{E_s} \geq 0.6 \frac{\sigma_s}{E_s} \tag{9}$$

where σ_s = stress in the tension reinforcement assuming a cracked section; α_e = ratio of E_s/E_c; $\rho_{p,eff} = A_s/A_{c,eff}$ and k_t = factor dependent on the duration of load ($k_t = 0.6$ for short term loading and $k_t = 0.4$ for long term loading).

For the calculation of maximum crack spacing may be calculated from the following expression:

$$s_{r,\max} = k_3 c + k_1 k_2 k_4 \phi / \rho p, eff \qquad (10)$$

where ϕ = bar diameter, c = cover to the longitudinal reinforcement; k_1 = coefficient which takes account the bond properties of the bonded reinforcement ($k_1 = 0.8$ for high bond bars and $k_1 = 1.6$ for bars with an effectively plain surface), k_2 = coefficient which takes account of the distribution of strains ($k_2 = 0.5$ for bending and $k_2 = 1.0$ for pure tension), $k_3 = 3.4$ and $k_4 = 0.425$.

2.3 *Fib Model Code for Concrete Structures 2010*

For all stages of cracking, the design crack width w_d may be calculated:

$$w_d = 2 l_{s,\max} \left(\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs} \right) \qquad (11)$$

where $l_{s,max}$ = denotes the length over which slip between concrete and steel occurs; ε_{sm} = average steel strain over the length $l_{s,max}$; ε_{cm} = average concrete strain over the length $l_{s,max}$; and ε_{cs} = strain of concrete due to shrinkage.

For the length $l_{s,max}$ the following expression applies:

$$l_{s,\max} = k \cdot c + \frac{1}{4} \cdot \frac{f_{ctm}}{\tau_{bms}} \cdot \frac{\varphi_s}{\rho_{s,ef}} \qquad (12)$$

where k = empirical parameter to take the influence of the concrete cover into consideration ($k = 1$); c = concrete cover; and τ_{bm} = mean bond strength between steel and concrete (Table 1).

The relative mean strain ε_{sm}-ε_{cm}-ε_{sh} follows from:

$$\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs} = \frac{\sigma_s - \beta \cdot \sigma_{sr}}{E_s} - \eta_r \cdot \varepsilon_{sh} \qquad (13)$$

where: σ_s = steel stress in a crack; σ_{sr} =maximum steel stress in a crack in the crack formation stage, which for pure tension is:

$$\sigma_{sr} = \frac{f_{ctm}}{\rho_{s,ef}} \left(1 + \alpha_e \rho_{s,ef} \right) \qquad (14)$$

where:

$$\rho_{s,ef} = \frac{A_s}{A_{c,ef}} \qquad (15)$$

where: $A_{c,ef}$ = effective area of concrete in tension; α_e = modular ratio = E_s/E_c; β = an empirical coefficient to assess the mean strain over $l_{s,max}$ depending on the type of loading; η_r = coefficient for considering the shrinkage contribution; and ε_{sh} = shrinkage strain.

The value for τ_{bm} and coefficients β and η_r can be taken from Table 2.

Table 2. Values for τ_{bms}, β and η_r for deformed reinforcing bars.

	Crack formation stage	Stabilized cracking stage
Short term instantaneous loading	$\tau_{bms} = 1.8 f_{ctm}(t)$ $\beta = 0.6$ $\eta r = 0$	$\tau_{bms} = 1.8 f_{ctm}(t)$ $\beta = 0.6$ $\eta r = 0$
Long term repeated loading	$\tau_{bms} = 1.35 f_{ctm}(t)$ $\beta = 0.6$ $\eta r = 0$	$\tau_{bms} = 1.8 f_{ctm}(t)$ $\beta = 0.4$ $\eta r = 0$

2.4 *Calculation of stresses and deformations of reinforced concrete elements in service*

For the proposed models of crack width, it is necessary to take into account loading histories. This can be viewed as a succession of stages consisting of permanent and variable actions giving rise to instantaneous stress variations. Again for the purposes of simplification, we may consider a succession of one stage consisting of permanent actions and one stage consisting of variable actions. Finally, it is necessary to know the stresses in the cracked section in order to perform the necessary control of cracking conditions and those on maximum stresses in serviceability conditions. (Balazs, Beeby et.al, 1997).

The variable actions give rise to the variations in stresses, modify the stresses and strains due to permanent loads. This causes displacement of the neutral axis when dialing with cracked cross section of the member. (Balazs, Beeby et.al, 1997).

In this paper for calculation of stresses and deformation, including time dependent effect for permanent and variable load, the Method based on the age adjusted effective modulus (AAEM) and superposition's of fictitious load effects was used.

The final strains in the section were obtained by summing up the strains related to long-term actions to the elastic strains due to instantaneous variation in load effect.

3 EXPERIMENTAL PROGRAM

3.1 *Description*

An experimental program was proposed to analyze long-term behavior of reinforced concrete elements under action of different types of loading histories. In this paper experimental results from testing of 12 beams were given for the series of beams *D*, *E* and *F*. The experimental program is given in Table 3.

Series of beams *D* and *E* were consisted of combination of action of long-term permanent load with intensity *G* and repeated variable load *Q* which was applied in cycles of loading/unloading for 24/48 hours respectively for a period of 330 days. The idea of loading and unloading is given in Figure 3.

Beams from series F were used for measuring free shrinkage of reinforced concrete in same period of 330 days.

Table 3. Experimental program.

Series	Type of load	Loading cycle
A	Short-term load	/
B	Permanent load "G"	/
C	Permanent load "G" and quasi-permanent load "Q/2"	$G + \psi_2 Q = G + 0.5Q$
D	Permanent load "G" and variable load "Q"	Loading/unloading for $\Delta_{t1} = 24$ hours.
E	Permanent load "G" and variable load "Q"	Loading/unloading for $\Delta_{t2} = 48$ hours.
F	Shrinkage	/

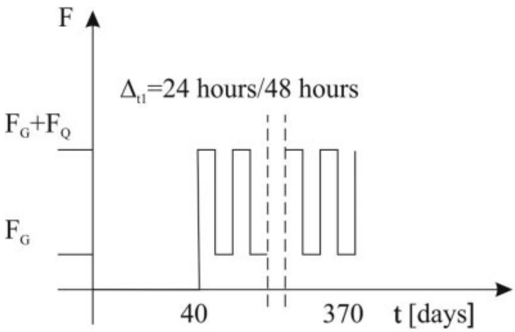

Figure 3. Loading history for the elements of series D and E.

Table 4. Design values of actions.

Actions		Intensity kN
Permanent action	"G"	2×4
Variable action	"Q"	2×7.6
Service load	"G+Q"	2×11.6

Design characteristics of actions are given in Table 4. The self-weight of the beam is uniform load of 1 kN/m.

In each series of reinforced concrete beams, the dimensions were width/height/length=15/28/300cm. Series of beams *D* were divided in *D*1 and *D*2 made from ordinary concrete class C30/37 and D3 and D4 were made of high-strength concrete class C60/75. This was also applied to series *E* and *F*.

Details of the beams and test set up for the experiment are provided on Figure 4. All specimens were cast from the same batch of concrete and all specimens were loaded at concrete age of 40 days.

The measured compressive strength, tensile splitting strength and elastic modulus of concrete class C30/37 at the age of loading at 40 days were $f_{ck} = 31.9$ MPa, $f_{ct,sp} = 2.9$ MPa and $E_{cm} = 30483$ MPa. For concrete class C60/75 at the same age results were $f_{ck} = 66.4$ MPa, $f_{ct,sp} = 5.3$ MPa and $E_{cm} = 39470$ MPa. Measured values of concrete properties at age of 370-day for concrete C30/37 were $f_{ck} = 34.1$ MPa, $f_{ct,sp} = 3.5$ MPa, $E_{cm} = 33150$ MPa, total

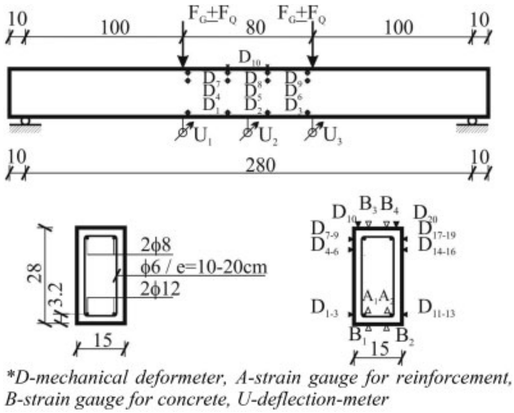

Figure 4. Reinforced concrete beams, dimensions, detail of reinforcement and test set up.

shrinkage (as a sum of autogenous and drying shrinkage) $\varepsilon_c = 0.000647.5$ and creep coefficient $\varphi_c = 1.723$. For concrete C60/75, properties were $f_{ck} = 75.5$ MPa, $f_{ct,sp} = 5.3$ MPa, $E_{cm} = 41230$ MPa, total shrinkage $\varepsilon_c = 0.000683$ and creep coefficient $\varphi_c = 0.703$.

Deformed reinforcement, diameter of 12 mm, was used with yield strength of $f_{0.2} = 400$ MPa, tensile strength $f_m = 719$ MPa and modulus of elasticity $E_{sm} = 200200$ MPa.

Throughout the period of 330 days the beams were carefully monitored in the middle of the span to record time-dependent deflections a, gradual development of cracks, number of cracks, l_{smax} – maximum crack spacing, w_k – characteristic crack width and σ_s – steel stress in a crack.

The tests were performed out at the Laboratory of the Faculty of Civil Engineering, University "Ss. Cyril and Methodius" in Skopje. The environmental conditions in the laboratory were relative constant value of humidity RH = 63% and temperature T = 17°C.

More details of the experimental program, mix design of concrete classes C30/37 and C60/75, experiment results for properties of concrete and measured deflections in the middle of the span are given in papers from Arangjelovski, Markovski & Mark 2012 and 2014.

3.2 *Results from measured crack parameters*

At the start of the experiment at concrete age of $t = 40$ days, first the beams were loaded by the permanent load *G* which didn't cause cracks in the section, then the variable load *Q* was applied and which cause cracks in the beams. First the crack width w_{G+Q} $(t = 40)$ was measured approximately in the middle of the span, and then after unloading at the level of permanent load *G* crack width w_G $(t = 40)$ was measured.

The values of initial crack width, obtained at loading at age of concrete of $t = 40$ days and final crack width measured at concrete age of $t = 370$ days for series *D* are given in Table 5, and for series *E* in Table 6.

Table 5. Experimentally measured crack width w for series "D" beams.

Level of actions	Crack width w D1-C30/37 mm	D2-C30/37 mm	D3-C60/75 mm	D4-C60/75 mm
w_G ($t_0 = 40$)	0.08	0.06	0.05	0.04
w_G ($t = 370$)	0.14	0.12	0.05	0.04
w_{G+Q} ($t_0 = 40$)	0.17	0.15	0.10	0.07
w_{G+Q} ($t = 370$)	0.23	0.22	0.12	0.08

Table 6. Experimentally measured crack width w for series "E" beams.

Level of actions	Crack width w E1-C30/37 mm	E2-C30/37 mm	E3-C60/75 mm	E4-C60/75 mm
w_G ($t_0 = 40$)	0.08	0.12	0.06	0.05
w_G ($t = 400$)	0.13	0.16	0.09	0.08
w_{G+Q} ($t_0 = 40$)	0.15	0.17	0.08	0.07
w_{G+Q} ($t = 400$)	0.18	0.22	0.12	0.12

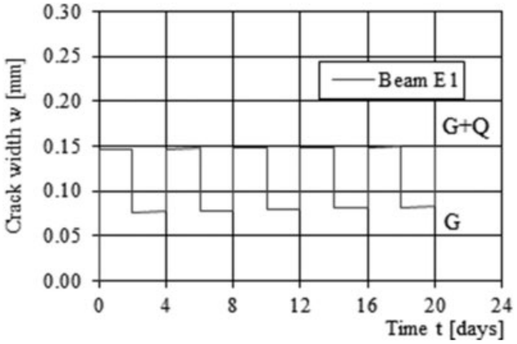

Figure 5. Diagram crack width w – time t for beam D1 concrete class C30/37.

Characteristic relation of crack width w – time t for the beam E1, for the first 20 days of loading/unloading in cycles of 48 hours is given in Figure 5.

The typical diagram of relation crack width w – time t for series of beams D and E were given in Figures 6–8. One representative diagram was given for beam D1 made of concrete C30/37 and one for beam D3 made of high-strength concrete C60/75 (Figure 6 and Figure 7). The same was done and for the beams of series E, the diagrams of crack width w- time t for E1 and E3 are given in Figure 8 and Figure 9.

Because of the type of loading histories (repeated loading and unloading) the diagram of the measured crack w during time t has a form of an area defined by the limits of permanent load G and by the permanent load G and variable load Q.

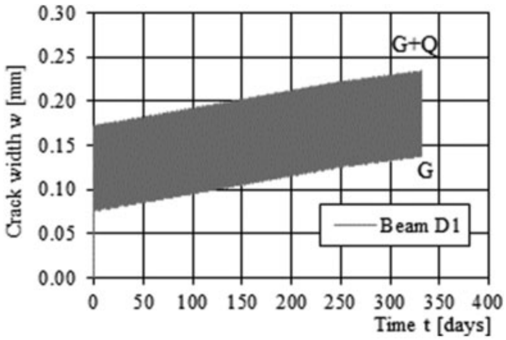

Figure 6. Diagram crack width w – time t for beam D1 concrete class C30/37.

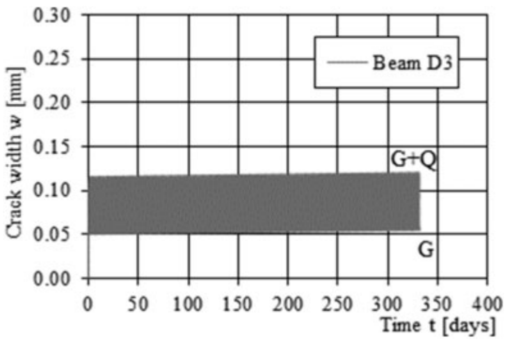

Figure 7. Diagram crack width w – time t for beam D3 concrete class C60/75.

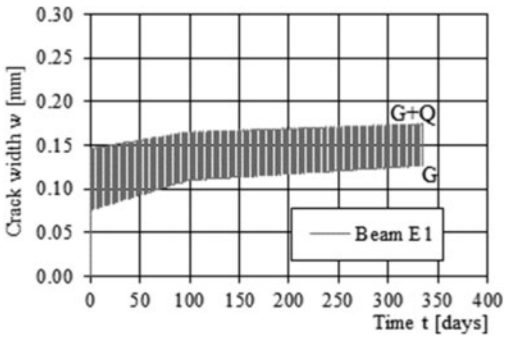

Figure 8. Diagram crack width w – time t for beam E1 concrete class C30/37.

Experimental results from measuring n – number of cracks and $S_{r,max}$ maximum crack spacing are given in Table 7 and Table 8 for series of beams D and E respectively.

Steel stress in the crack was measured by mechanical deflect meter on the concrete surface and by strain gauge on the reinforcement.

The experimental results for the measured steel stress in the crack in the crack situated in the middle of the span for the series of beams D and E are given in tables 9 and 10. Steel stress in the crack was measured by mechanical deflect meter and by strain gauge.

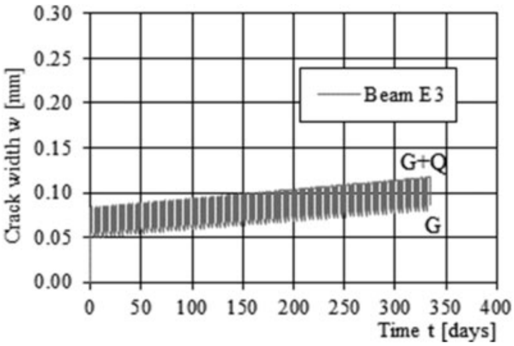

Figure 9. Diagram crack width w – time t for beam E3 concrete class C60/75.

Table 7. Experimentally measured maximum crack spacing $S_{r,max}$ for series "D" beams.

		Maximum crack spacing $S_{r,max}$			
Beams	No. of cracks n	D1-C30/37 mm	D2-C30/37 mm	D3-C60/75 mm	D4-C60/75 mm
D1	6	182	168	194	238
D2	7	214	270	182	180
D3	4	202	152	182	178
D4	4	164	214		
		208	268		
			202		
Mean value of $S_{r,max}$		194	212	186	199

Table 8. Experimentally measured maximum crack spacing $S_{r,max}$ for series "E" beams.

		Maximum crack spacing $S_{r,max}$			
Beams	No. of cracks n	E1-C30/37 mm	E2-C30/37 mm	E3-C60/75 mm	E4-C60/75 mm
E1	6	212	132	202	192
E2	6	172	248	200	186
E3	4	190	212	182	194
E4	4	250	160		
		190	222		
Mean value of $S_{r,max}$		203	195	195	191

For the purpose of using the model for the calculation of crack width according to fib Model Code 2010, also the free shrinkage was investigated in the experimental program on separate series of beams F also made from concrete C30/37 and high-strength concrete C60/75.

The experimental results for the shrinkage deformation ε_{cs} during the period of $t = 370$ days are given in Table 11 and in Figure 10 for the beam F1 and F3 as representative diagram.

Table 9. Experimentally measured σ_s-steel stress in a crack for series "D" beams.

	Steel stress in the crack σ_s			
Level of actions	D1-C30/37 MPa	D2-C30/37 MPa	D3-C60/75 MPa	D4-C60/75 MPa
$\sigma_{s,G}$ ($t_0 = 40$)	173.8	110.1	96.9	174.6
$\sigma_{s,G+Q}$ ($t = 400$)	217.4	143.3	99.7	206.6
$\sigma_{s,G}$ ($t_0 = 40$)	270.3	167.8	146.5	262.7
$\sigma_{s,G+Q}$ ($t = 400$)	337.5	215.0	161.4	300.3

Table 10. Experimentally measured σ_s-steel stress in a crack for series "E" beams.

	Steel stress in the crack σ_s			
Level of actions	E1-C30/37 MPa	E2-C30/37 MPa	E3-C60/75 MPa	E4-C60/75 MPa
$\sigma_{s,G}$ ($t_0 = 40$)	130.9	167.4	58.1	60.5
$\sigma_{s,G+Q}$ ($t = 400$)	136.9	190.6	142.1	72.1
$\sigma_{s,G}$ ($t_0 = 40$)	203.4	249.0	88.9	96.1
$\sigma_{s,G+Q}$ ($t = 400$)	215.4	285.9	221.1	114.9

Table 11. Experimentally measured shrinkage ε_{cs} for series "F" beams.

	Shrinkage ε_{cs}			
Days	F1-C30/37 $[10^{-3}]$	F2-C30/37 $[10^{-3}]$	F3-C60/75 $[10^{-3}]$	F4-C60/75 $[10^{-3}]$
$t_0 = 40$	0.184	0.158	0.070	0.080
$t = 370$	0.220	0.250	0.160	0.116

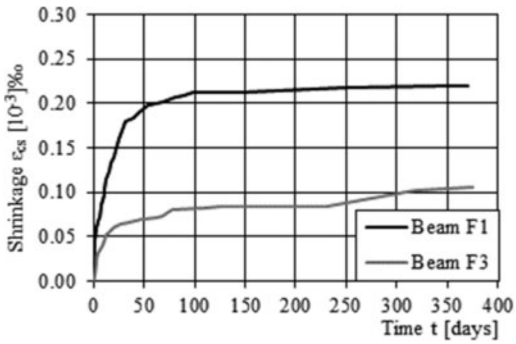

Figure 10. Diagram shrinkage ε_{cs} – time t for beam F1 concrete class C30/37 and for beam F3 concrete class C60/75.

4 ANALYTICAL ANALYSIS

For the analytical analysis of the experimentally obtained results, evaluation of the crack parameters was performed using the calculation model of EN1992-1-1 Eurocode 2 Design of concrete structures – Part 1-1: General rules and rules for buildings

Table 12. Comparison of experimentally obtained mean value for maximum crack spacing $S_{r,max}$ and analytical results for series “D” and “E” beams.

Beams	Maximum crack spacing $S_{r,max}$			
	D1-C30/37 mm	D2-C30/37 mm	D3-C60/75 mm	D4-C60/75 mm
Mean value of $S_{r,max,experiment}$	194	212	186	199
	E1-C30/37	E2-C30/37	E3-C60/75	E4-C60/75
Mean value of $S_{r,max,experiment}$	203	195	195	191
EN1992-1-1 EC2 $S_{r,max}$	187	187	190	190
fib Model Code 2010				
l_{smax}	101.5	101.5	103.8	103.8
$S_{r,max}$	203.0	203.0	207.6	207.6

Table 13. Comparison of experimentally obtained mean value for crack width $w$and analytical results for series "D" and “E” beams.

Level of actions Experiment	Crack width w			
	D1-C30/37 mm	D2-C30/37 mm	D3-C60/75 mm	D4-C60/75 mm
w_{G+Q} ($t_0 = 40$)	0.17	0.15	0.10	0.07
w_{G+Q} ($t = 370$)	0.23	0.22	0.12	0.08
	E1-C30/37	E2-C30/37	E3-C60/75	E4-C60/75
w_{G+Q} ($t_0 = 40$)	0.15	0.17	0.08	0.07
w_{G+Q} ($t = 370$)	0.18	0.22	0.12	0.12
EN1992-1-1 EC2				
w_{G+Q} ($t_0 = 40$)	0.17	0.17	0.11	0.11
w_{G+Q} ($t = 370$)	0.20	0.20	0.13	0.13
fib Model Code 2010				
w_{G+Q} ($t_0 = 40$)	0.15	0.15	/	/
w_{G+Q} ($t = 370$)	0.17	0.17	/	/

2004 and fib Model Code for Concrete Structures 2010.

In both models, for the serviceability limit states design, combination of actions was used to verify irreversible serviceability limit state using characteristic combination and for reversible serviceability limit state using frequent and quasi-permanent combination. Irreversible serviceability limit state was used to verify the value of final crack width w at age of concrete $t = 370$ for the action of permanent load G and variable load Q. To verify the reversible limit state including time effects from shrinkage and creep of concrete, quasi-permanent combination of action was used to verify time-dependent final crack width at the level of permanent load G, which is of interest to define the quasi-permanent coefficient ψ_2. For the calculation of stresses in the cross section the AAEM method and principle of superposition was used.

Varying the factor ψ_2 in the quasi-permanent combination of actions, we tried to obtain similar results between the experiment and models for calculation of crack parameters.

4.1 *Analysis of results for maximum crack spacing $S_{r,max}$*

First, analysis of maximum crack spacing $S_{r,max}$ was performed to verify the experimental results and analytical results using both crack models. The results of the comparison are given in table 12.

The crack model given in Eurocode 2 underestimated maximum crack spacing about 6.4%–8.6% from the experimental results for the beams made from concrete C30/37, but for the beams made from concrete C60/75 we have obtained similar result with neglecting difference.

Using the crack model from fib Model Code for Concrete Structures 2010, this model gives similar results for the maximum crack spacing for the beams made from concrete C30/37 and overestimated the results for the beams made from concrete C60/75.

4.2 *Analysis of results for maximum crack width w*

Analysis of comparison the experimental results and calculated crack width w using both models are given in Table 13.

The both crack models using characteristic combination of actions to verify irreversible limit state gives proper prediction of the crack width w for the beams made of concrete C30/37. Calculation of the crack width was performed for the level of load as sum of permanent load G and variable load Q at the time of loading $t = 40$ days and for $t = 370$ days.

In the analysis using the fib Model Code 2010 crack model we could not calculated the crack width w for the beams using concrete C60/75 because the calculated steel stress in the crack σ_s was lower than σ_{sr} maximum steel stress in a crack in the crack formation stage $\sigma_s < \sigma_{sr}$. One explanation for this problem may be that higher mechanical properties of high-strength concrete enables formation of cracks at the level of combination of actions as a sum of permanent G and variable load Q. The service load, flexure moment $M = 12.6$ kNm, is very close to cracking moment $M_{cr} = 11.6$ kNm, which suggest that the crack formation stage will last during the whole period of loading.

4.3 *Analysis of results for quasi-permanent coefficient $\psi 2$*

For the reversible serviceability limit state, quasi-permanent combination of actions was used to verify the crack width at the level of permanent load G, which is from prime interest for the designers. Eurocode 2

Table 14. ψ_2 factors for series of beams D and E made of ordinary concrete C30/37 and high-strength concrete C60/75.

Series	Permanent action G kN	Variable action Q kN	ψ_2 factor	Quasi-permanent load $G+\psi_2 Q$ M [kNm]	Crack width w mm
Beams made from concrete C30/37					
D	4	7.6	0.70	10.3	0.13
E	4	7.6	0.85	11.5	0.15
Beams made from concrete C60/75					
D	4	7.6	0.55	9.2	0.05
E	4	7.6	0.70	10.3	0.07

crack model was used to obtain the same crack width with experimental ones.

The results from the analysis are given in the Table 14.

5 CONCLUSIONS

From the experimental and analytical analysis of crack parameters, for beams subjected to permanent load G and repeated variable load Q following conclusions can be received:

- Using both crack models, given in the Eurocode 2 and in fib Model Code 2010, give good agreement with the experimental results.
- It is very important, especially when we use high-strength concrete for the reinforced concrete elements to ensure that the tensile steel stress in the crack σ_s are greater than maximum steel stress in the crack in the crack formation stage σ_{sr}. This condition was not satisfied using the fib Model Code 2010.
- Calculation of the crack width using the quasi-permanent combinations of actions to verify reversible serviceability limit states, at the level of permanent load, shows that for beams made of concrete C30/37 quasi-permanent factor ψ_2 is in range of $\psi_2 = 0.70$–0.85 which corresponds to the value $\psi_2 = 0.8$ for storage areas, as proposed in the Eurocode 2.
- For the beams made of concrete C60/75, verification of crack width using quasi-permanent combination of actions shows that the quasi-permanent factor is in range of $\psi_2 = 0.55$–0.70. This values are lower than the proposed values in Eurocode 2, which indicates that because of higher mechanical properties we should use higher level of load intensity.

REFERENCES

Aitcin, P.C. 1998. High-Performance Concrete, London: E&FN SPON.

Arangelovski, T. 2011. Time-dependent behavior of reinforced high-strength concrete elements under action of variable loads, *Doctoral dissertation*, Skopje: University "St. Cyril and Methodius" and SEEFORM doctoral studies-DAAD program, English version.

Arangjelovski, T., Markovski, G. & Mark, P. 2012 Influence of repeated variable load on long-term behaviour of concrete elements, *Life-Cycle and Sustainability of Civil Infrastructure Systems*, *Proceedings of the third international symposium on life-cycle civil engineering Vienna*, pp.1382–1389, London: Taylor & Francis Group.

Arangjelovski, T., Markovski, G. & Mark, P. 2014 Influence of repeated variable load on long-term behaviour of concrete elements, *Journal of Civil Engineering and Architecture* Vol.8, No.3:302–314. New York: David Publishing Company.

Balazs, G.L. et al. 2013. Design for SLS according to fib Model Code 2010, Structural Concrete Journal of the fib Vol. 14, No.2:99–123. Berlin: Ernst & Sohn.

Bazant Z.P. 1972. Prediction of concrete creep effects using age-adjusted effective modulus method, *ACI Journal 69*:212–217. Farmington Hills: ACI.

Bazant Z.P. & and Baweja, S. 2000. Creep and Shrinkage Prediction Model for Analysis and Design of Concrete Structures: Model B3, *The Adam Neville Symposium: Creep and Shrinkage-Structural Design Effects, SP-194, American Concrete Institute 2000*, Akthem Al-Manaseer (ed.), Farmington Hills: ACI.

Beeby, A.W. & Narayanan, R.S. 1995. Designers' handbook to Eurocode 2. London: Thomas Telford.

Borosnyoi, A. & Balazs, G.L. 2005. Models for flexural cracking in concrete: the state of the art, *Structural Concrete Journal of the fib Vol.6*, No.2:53–62. London: Thomas Telford.

Caldarone, M.A. 2009. High-Strength Concrete (A practical guide), New York: Taylor & Francis.

CEB Bulletin d'information N^o 235. 1997. Serviceability Models-Behaviour and modelling in serviceability limit states including repeated and sustained loads. G. Balazs (eds). Lausanne.

CEB, CEB – FIP. 1991. Model Code 1990, London: Thomas Telford.

Fib, CEB-FIP. 2013. Fib Model Code for concrete structures 2010, Berlin: Ernst & Sohn.

Fischer, A., Kramp, M., Prietz, F., & Rosler, M. 2003. Stahlbeton nach DIN 1045-1, Berlin: Ernst & Sohn.

Ghali, A., Favre, R.& Elbadry, M. 2002. Concrete Structures (Stresses and Deformation), London: Spon Press.

Giussani, F. & Mota, F. 2005. Models for the Serviceability Limit States of R.C. Structures, Proceedings of 30th Conference on Our World in Concrete & Structures, Singapore, August 23–24, 2005, Singapore: CI Premier PTE LTD.

Gulvanessian, H., Calgaro, J.A. & Holicky, M. 2002. Designers' Guide to EN1990 Eurocode: Basis of structural design. London: Thomas Telford.

Nakov, D., Markovski G., Arangjelovski T & Mark P., "Determination of the coefficient $\psi 2$ to define long-term effects on concrete elements under variable load", *Scientific Journal of Civil Engineering*, *Vol. 4*, Issue 1, June 2015 Skopje.

Sakata, K. & Shimomura, T. 2004. Recent Progress in Research on and Code Evaluation of Concrete Creep and Shrinkage in Japan, *Journal of Advanced Concrete Technology Vol.2,* No.2: 133–140. Tokyo: JCI.

Zongjin, L., Leung, C. & Xi Y. 2009. Structural Renovation in Concrete. Abingdon: Taylor & Francis.