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• Нови подходи •
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THE $\gamma \longrightarrow \varepsilon^+ + \varepsilon^-$ PROCESS. A USEFUL AND INTUITIVE ANALOGY TO RELATIVISTIC QUANTUM MECHANICAL EXPLANATION IN UNDERGRADUATE RADIOCHEMISTRY COURSES

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Abstract. In many radiochemistry textbooks and courses, the reaction $\gamma \longrightarrow e^+ + e^-$ is usually referred to as a "conversion (transformation) of the γ-ray (*i.e.* γ-photon) into an electron-positron pair", which is simply incorrect. Appropriate explanation of the mentioned process should be based on relativistic quantum mechanics, particularly on Dirac's theory. Dirac's theory is, on the other hand, rather involved for those who lack a profound knowledge in relativistic quantum mechanics. The authors offer a thought experiment as a suitable analogy that helps understand the basics of the contemporary theory that lies behind the reaction in question.

Introduction

Essentially all of contemporary chemical science is undoubtedly heavily interconnected with many areas of physics. It is therefore becoming more and more important to achieve a more profound knowledge of physics even in undergraduate chemistry studies. However, most chemistry students lack a solid mathematical background so that a rigorous (and therefore correct) description of a number of physical phenomena that have a certain chemical importance is impossible. The previous statements are especially true when one comes across quantum mechanics and related problems. It is quite interesting to note how various subareas of quantum mechanics interpenetrate into different topics of chemistry (i.e. physical chemistry). Let us mention, within this context, several illustrative examples. Time dependent quantum mechanical methods are becoming increasingly popular for studying the chemical reaction dynamics [1–3 and references therein]. Also, relativistic quantum mechanics has already become a routine tool for studying systems in which relativistic phenomena are very important, so that they can not be overlooked [4–6]. As these areas of quantum mechanics require even more profound knowledge of mathematics, it is really becoming more and more complicated for a chemist (at least for those educated in a standard way) to follow and understand on sufficiently high level these modern trends.

It is not the main purpose of the present paper to suggest which specific mathematical knowledge is required for all the mentioned purposes, but to point at something else. Although it is perhaps impossible to perform every quantum mechanical derivation in a rigorous way (at least in standard undergraduate chemistry studies) and therefore often only a qualitative description of the final results is taught, it is necessary to keep the *correct* interpretation of the conclusions arising from quantum mechanics. For this purpose, analogies with systems that are well known and understood by chemists can be of certain interest. It is one such intuitive and interesting analogy that we point to in the present paper.

In a number of nuclear and radiochemistry textbooks, the process:

$$\gamma \longrightarrow e^{+} + e^{-}$$

is usually referred to as "*conversion (transformation) of the γ-ray (i.e. γ-photon) into an electron-positron pair*" [see, e.g. 7]. This statement is simply incorrect from the viewpoint of contemporary quantum mechanics. An appropriate explanation of the mentioned process should be based on relativistic quantum mechanics, particularly on Dirac's theory [8,9]. Dirac's theory is, on the other hand, rather involved for those lacking a profound knowledge in relativistic quantum mechanics. However, let us recall (qualitatively) what should the rigorous description of this process involve. According to Dirac's theory, the (electromagnetic) vacuum, as a basic state of matter should be thought of as a "sea" of electrons occupying all possible quantum states with negative energies. These particles are therefore "invisible" in standard physical experiments (hence the term "virtual"). However, when a quantum of γ radiation (with sufficient energy) interacts with the vacuum, it may "excite" an electron from a state with $E < 0$, into state with $E > 0$. The excited electron would therefore become observable, and would behave as any ordinary electron. Within the electromagnetic vacuum, on the other hand, a “hole” remains, which would behave identically as the excited electron, but with a very important difference – it would have an opposite charge than the electron. This “hole” in the negative energy states within the electromagnetic vacuum is the well-known *positron* or the electron anti-particle. The whole process is schematically depicted in Fig. 1. Therefore, in the light of the rigorous Dirac’s theory no conversion of the gamma quantum to an electron–positron pair occurs, but instead, a virtual particle constituting the electromagnetic vacuum is excited into a state with positive energy.

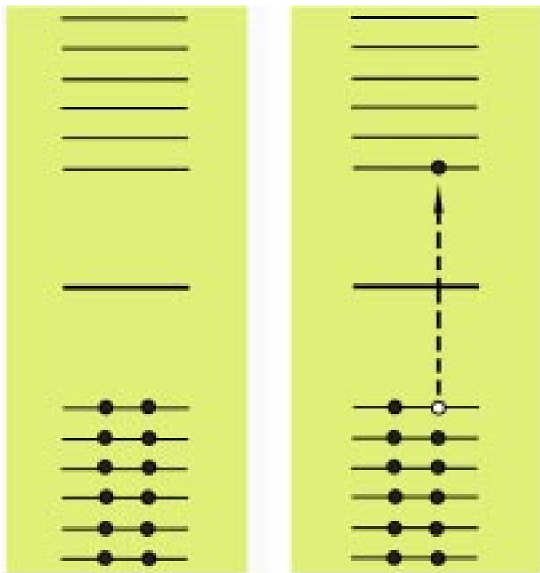

Figure 1. The ground state of the vacuum with all electrons occupying negative energy states (left), and excitation of the vacuum with a γ quantum, promoting an electron to the positive energy states and leaving a “hole” (positron) in the states with negative energy

There is at least one excellent book [10] where the fundamentals of Dirac’s theory are presented in a simplified way and it could help beginners to understand the basic concept, particularly the notion of the filled negative energy states. In addition to this one, and in order to allow for a correct qualitative description of the results arising from relativistic quantum mechanics, using a language which is more common for a chemist, we propose the following thought experiment.

The thought experiment

The basic idea behind this thought experiment is that in some other situation that chemists are familiar with, no one would speak of a conversion (transformation) of photons into pairs, but the process would have been named properly.

We consider a transparent tube filled with hydrogen, under low enough pressure, and sufficiently high temperature, so that the degree of dissociation of molecular hydrogen is considerably high. Voltage is applied to the gas in the tube through electrodes placed at the ends of the tube (Fig. 2). The system is continuously irradiated *varying the radiation wavelength* (starting from very high to lower values). Since (as a result of the experimental conditions), sufficiently large number of hydrogen atoms is present in the tube, when the incident radiation wavelength achieves sufficiently low value so that the condition $h\nu = E_i$ is fulfilled (where $\nu = c/\lambda$, ν is the incident radiation frequency, and E_i is the hydrogen atom first ionization energy), current flow through the circuit will be indicated by the galvanometer G.

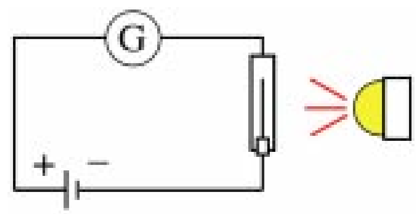

Figure 2. The hydrogen atoms in the tube (under high temperature, low pressure) are irradiated with EM radiation of varying frequency/wavelength. Once the energy of the quanta is equal to the ionization energy of the H-atoms, the galvanometer indicates a current flow. Does this mean that "*Photons are being converted into proton–electron pairs?*"

It is quite clear to every chemist that the underlying reason for the detected current flow through the circuit is the ionization of hydrogen atoms within the tube, as a result of the interaction of photons with hydrogen atoms. Should we explain this as a result of a conversion of a photon into electron–ion pair? Of course, not! There is no doubt that the incident radiation expels the electrons from hydrogen atoms, in which they are characterized with negative energies (binded states) to free particle states (*i.e.* states with positive energy). Why should then one speak about a conversion of a γ photon into an electron–positron pair?

The analogy of this thought experiment and the process of excitation of electromagnetic vacuum by a gamma quantum is obvious and intuitive. We believe (and also know from experience) that this analogy might be very useful for students attending an undergraduate nuclear and radiochemistry course for a more thorough (and correct) understanding of the electromagnetic vacuum and the processes of its excitation.

Conclusion

We believe that the proposed thought experiment could be very useful in teaching undergraduate Radiochemistry courses (basing on the authors' experience). By the proposed analogy (a thought experiment with a result well-known to chemists) the process $\gamma \longrightarrow e^+ + e^-$ can be correctly understood, and correctly explained and named. Thus, even without a rigorous mathematical explanation of the phenomenon in question (basing on Dirac's theory), it can be *correctly* qualitatively described and understood by undergraduate chemistry students.

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$\gamma \rightarrow \varepsilon^{+} + \varepsilon^{-}$: ЕДНО РЕЛАТИВИСТИЧНО КВАНТОВОМЕХАНИЧНО ОБЯСНЕНИЕ В ОСНОВНИЯ КУРС ПО РАДИОХИМИЯ

Резюме. В много учебници по радиохимия процесът $\gamma \rightarrow \varepsilon^{+} + \varepsilon^{-}$ се разглежда като превръщане на γ-лъчите (т.е. γ-фотони) в електрон-позитронова двойка. Това обаче не е достататъчно прецизно. Обяснението на този процес трябва да се търси на релативистична квантовомеханична основа, в частност чрез теорията на Дирак. Тази теория рядко се излага пред аудитория, която не е запозната в дълбочина с релативистичната кванова механика. Авторите предлагат мислен експеримент, основан на една аналогия, чрез който обучаващите се могат да получат представа за тази сложна материя.

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