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MATHEMATICAL DETERMINATION OF SYNTHETIC SURFACE'S PERPENDICULAR AND TANGENT LINES

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A b s t r a c t: In this paper, a geometric and a mathematical model with computer algorithm of perpendicular and tangent lines of the synthetic surfaces, composed of splines in two directions, are presented. Synthetic surfaces are used in an engineering design for describing parts with aerodynamic or other functional and esthetic shapes. Synthetic surfaces can be represented by the majority of contemporary CAD systems. The modern NC machines contain highly sophisticated controllers and processing tools, which provide complex synthetic surfaces manufacturing. During the machining process, machining tool's axis has to be placed perpendicularly to each point of the synthetic surface that is the product of machining. A mathematical model is necessary for determination of the tool's position during the machining process .

Key words: synthetic surface; spline

1. INTRODUCTION

In order to perform mechanical parts with perfect and complex geometric shapes modern the computer science applies so called synthetic surfaces. Synthetic surfaces are three dimensional surfaces presented as a synthesis of several synthetic curves in two directions, characterized with smooth curve – linear shape, such as synthetic surfaces constructed of spline curves in two directions [1]. The synthetic surface can be presented with a simple parametric expression [2].

The parametric expression of the synthetic surface is explained with continual functions with two parameters: *t* and *u*, that can be changed from minimum to maximum value. Changing the values of the *t* and *u* parameters causes a definition of different curve's points. That means that *t* and *u* are acting like local parametric coordinates of the synthetic surface's points. Parameters *t* and *u* are changing in the interval of 0 to 1. The parametric expression of the synthetic surface is:

$$P(t,u) = [xyz]^T = [x(t,u)\, y(t,u)\, z(t,u)]^T ,$$

where $0 \le u \le 1$ and $0 \le t \le 1$,
or the summary of the net of control points in the perpendicular and parallel directions:

$$P(t,u) = \sum_{i=0}^{n} \sum_{j=0}^{m} \left[x_{[i][j]}(t,u)\, y_{[i][j]}(t,u)\, z_{[i][j]}(t,u) \right].$$

Neighboring control points are connected with spline curves in two directions:

$$f_{(xy)}(t) = \sum_{i=0}^{n} \left[x_{[i]}(t)\, y_{[i]}(t)\, z_{[i]}(t) \right]$$

$$f_{(xz)}(u) = \sum_{j=0}^{m} \left[x_{[j]}(u)\, y_{[j]}(u)\, z_{[j]}(u) \right]$$

Spline curves are determined with points obtained through the next mathematical expressions, that depend on *t* and *u* parameters:

$$x = x_0 B_0(t) + x_1 B_1(t) + x_2 B_2(t) + x_3 B_3(t)$$
$$y = y_0 B_0(t) + y_1 B_1(t) + y_2 B_2(t) + y_3 B_3(t)$$
$$z = z_0 B_0(t) + z_1 B_1(t) + z_2 B_2(t) + z_3 B_3(t)$$

where

x, y, z – coordinates of the points

$B_0(t)$ – blending function

t – parameter

The number of added points depends on the type of the applied curve, more precisely it depends on the function of the surface's control points connection. Depending on the connection function, there are several kinds of spline curves: *B-spline, Cubic B-spline*, *Bezier* etc.

2. MATHEMATICAL EXPRESSIONS FOR THE TANGENT AND PERPENDICULAR LINES OF THE SYNTHETIC SURFACE

The synthetic surface is bounded with parts of spline curves, defined with the values of the parameters: $t = 0$, $t = 1$, $u = 0$ and $u = 1$ (Fig. 1).

The synthetic surface could be defined as a set of points through which the spline is passing [3]. There are also determined border values for the points. For the four-edged polygon, the border values are determined with 16 vectors and 4 spline curves, four position vectors for the points $P(0,0)$, $P(0,1)$, $P(1,0)$, $P(1,1)$, eight tangent vectors (two for each point) and four vectors of curvature (one for each point).

For the different operations and applications with synthetic surfaces, it is necessary to calculate the tangent line (T_t and T_u), the perpendicular line (N) and the curvature (Z) for the determined points.

The tangent lines for a point of a surface are determined as tangents of parametric curves that are passing through that point. The parametric expression for the curve is obtained from the general equation $P(t,u)$, for $t = t_i$, and the other for $u = u_j$, which means that t_i and u_j are the parametric coordinates of the surface's points (Fig. 1). The tangent lines T_u and T_t are determined with the first extract of both the curves in an accidental point. The surface perpendicular line is determined as a vector product of both the tangent lines in an accidental point. The surface curvature Z in an accidental point is determined as a double extract of t and u.

If a surface is presented as a function of $f(x,y,z) = 0$, then the equation of the tangent surface in the point $M(x,y,z)$ is:

$$(X-x)\frac{\partial f}{\partial x} + (Y-y)\frac{\partial f}{\partial y} + (Z-z)\frac{\partial f}{\partial z} = 0 .$$

A line perpendicular (perpendicular) to the tangent surface in the point M is a perpendicular line of the surface:

$$\frac{X-x}{\frac{\partial f}{\partial x}} = \frac{Y-y}{\frac{\partial f}{\partial y}} = \frac{Z-z}{\frac{\partial f}{\partial z}} .$$

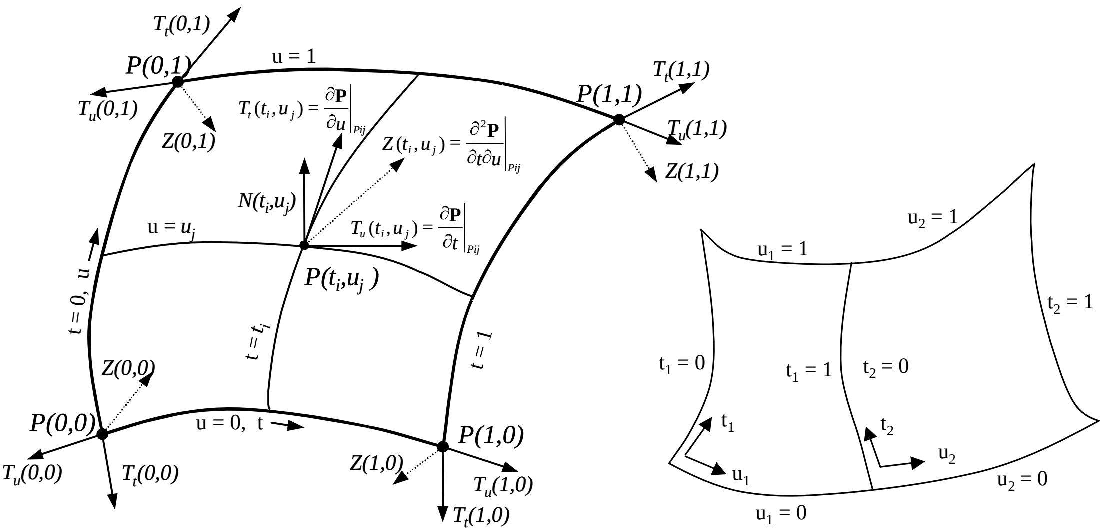

Fig. 1. Synthetic surface bounded with 4 spline curves and two connected synthetic surfaces

3. COMPUTER ALGORITHM FOR DETERMINATION OF THE SYNTHETIC SURFACE IS TANGENT AND PERPENDICULAR LINES

The problem of NC machine tool positioning perpendicularly to the surface is more complicated because the designed surface is not always a synthetic surface, nor a geometric function. The computer algorithm for providing a tangent line and a perpendicular line has to involve all possible variants of designed surfaces, and it has to be universal. Also, it has to work with imported surfaces, created in any graphical package [4].

An analysis of a synthetic surface has been made. The surface is created using synthesis of ***Cubic B-spline*** curves in two directions. The first spline curve $f_{(xy)}$ lays in the *xy* plane and moves along the other spline curve $f_{(xz)}$ that lays in the *xz* plane (Fig. 2).

The geometric algorithm for determination of tangent and perpendicular lines in a determined synthetic surface consists of several operations:

– section of synthetic surface with *yz* plane,

– construction of tangent and perpendicular lines of the obtained section curve $f_{(yz)}$,

– section of synthetic surface with *xz* plane,

– construction of tangent and perpendicular lines of the obtained section curve $f_{(xz)}$ (Fig. 3),

– construction of synthetic surface perpendicular line (Fig. 4).

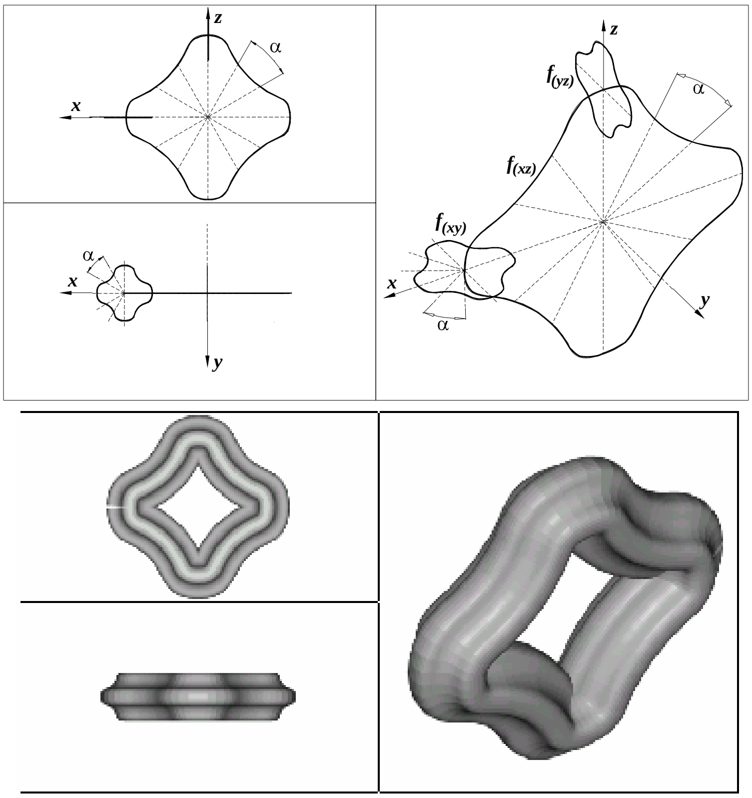

Fig. 2. Synthetic surface designed from Cubic B-spline curves $f_{(xy)}$ and $f_{(xz)}$

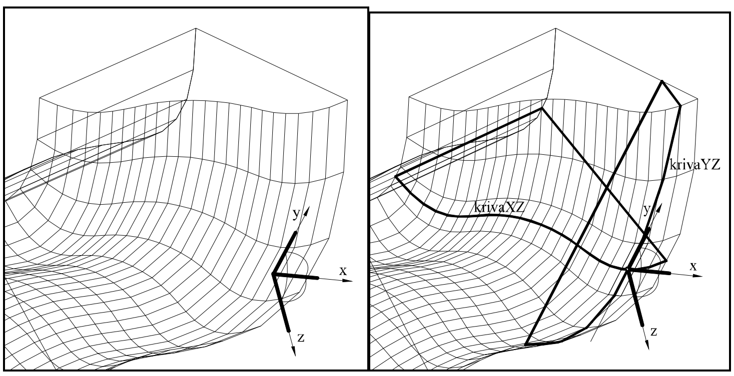

Fig. 3. Cutting curves $f_{(xz)}$ and $f_{(yz)}$ of a synthetic surface

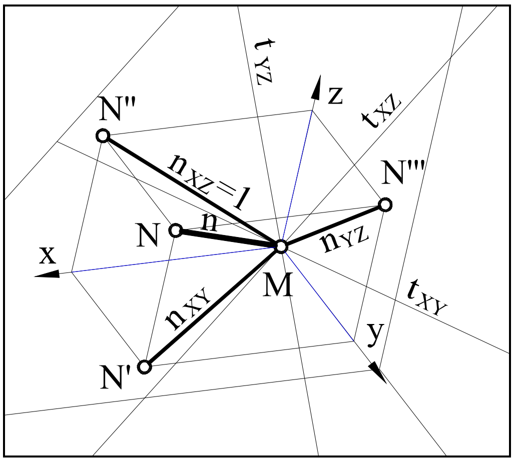

Fig. 4. Construction of a synthetic surface perpendicular line in the M point

The computer algorithm has the same structure as the geometric one, which is consisted of the same operations. On the base of the presented algorithms, a computer program for determination of the perpendicular and tangent lines for each point of a specified surface is created. This software is created as an upgrade of a commercial graphical package, using the C++ language. The procedures of the program enable a surface created in any graphical package to be imported and necessary elements for the NC tool path generation to be determined as perpendicular and tangent lines for each point of the imported surface.

Procedure 1. Section of a synthetic surface with *yz* plane

```
ads_command     (RTSTR, "_section",
                RTENAME, Mdel,
                RTSTR, "",          } < section of a surface (Mdel) with YZ plane>     RTSTR, "YZ",
                RTSTR, "",
                RTNONE);
Res = ads_entlast (CurveYZ);          <obtained section curve yZ>
ads_command     (RTSTR, "_ucs",
                RTSTR, "y",
                RTSTR, "90",        } <back to User Coordinate System>
                RTSTR, "",
                RTNONE);
```

Procedure 2. Section of a synthetic surface with *xz* plane

```
ads_command     (RTSTR, "_section",
                RTENAME, Mdel,
                RTSTR, "",          } < section of a surface (Mdel) with XZ plane>
                RTSTR, "XZ",
                RTSTR, "",
                RTNONE);
Res = ads_entlast (CurveXZ);          <obtained section curve XZ>
ads_command     (RTSTR, "_ucs",
                RTSTR, "x",
                RTSTR, "90",        } <back to User Coordinate System>
                RTSTR, "",
                RTNONE);
```

4. CONCLUSIONS

Actuality of the presented research results from a rising necessity for the NC machining of products with complex geometry. NC machining technology uses NC programs that specify the tool path for the product's machining. The NC tool path specifies the position of the tool for each point of the product's surface. The position of the tool is different for each point of the surface and its axis has to be perpendicular to the product's surface. This research includes a computer program, created as an upgrade of a commercial graphical package, using the C++ language. This program has an ability to import a surface created in any graphical package and to compute necessary elements for the NC tool path generation: perpendicular and tangent lines for each point of the imported surface. Procedures for determination of perpendicular and tangent lines for each point of a surface are made on the base of presented geometric and computer algorithms, using simple mathematic expressions.

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Р е з и м е

МАТЕМАТИЧКО ОПРЕДЕЛУВАЊЕ НА НОРМАЛА И ТАНГЕНТА НА СИНТЕТИЧКА ПОВРШИНА

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Клучни зборови: синтетичка површина; сплајн

Во овој труд е презентиран геометриски и математички модел со компјутерски алгоритам за определување на нормала и тангента на синтетичка површина, одредена со сплајн-криви во два правца. Синтетичките површини се користат во инженерскиот дизајн за опишување делови со аеродинамичен облик или други функционални или естетски облици. Синтетичките површини можат да бидат креирани со повеќето современи CAD-системи. Современите NC машини содржат високо софистицирани контролери и алати кои овозможуваат обработка на сложени синтетички површини. За време на процесот на производство оската на алатот за обработка мора да биде поставена нормално на секоја точка од синтетичката површина која се обработува. Притоа е нопходен математички модел за одредување на позицијата на алатот за време на процесот на обработка.