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PROJECTIONS OF 4D SURFACES

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A b s t r a c t: A geometric and mathematical model, as well as a computer algorithm of 4D surfaces' projections are presented in this paper. The definition of the 4D surfaces' projections is provided by the functions with two variables. 4D surfaces presentation is reduced with presentation of 4D points in 4D geometric space. The points are visible on the display after the transformation in 3D and 2D space. Determined points are connected in the mesh of horizontal and vertical isolines. With the analysis of 4D surfaces the analogy between 3D surfaces and 4D surfaces is confirmed.

Key words: 4D geometry; 4D space; 4D surface

1. 4D POINT

The point is defined with a determined number of coordinates or parameters regarding the space in which it exists. In 1D space the point is determined with one coordinate *A(x)*, in 2D space with two coordinates *A(x,y),* in 3D space with three coordinates *A(x,y,z)*, as well as the point in 4D space is determined with four coordinates *A(x,y,z,w)*. If the point *A* of 4D space *A(x,y,z,w)* is projected in 3D space, 4 projections are obtained on the coordinate hyperplanes *A(x,y,z)*, *A(x,y,w)*, *A(x,z,w)* and *A(y,z,w)* and 6 projections on the coordinate planes *A(x,y), A(x,z), A(y,z), A(x,w), A(y,w)* and *A(z,w)* (Fig.1).

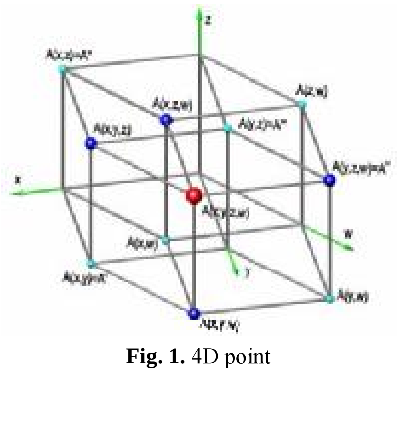

Fig. 1. 4D point

2. 4D SURFACES AND COMPUTER ALGORITHM

4D surfaces are surfaces in which the position of each point is determined with four coordinates. The presentation of 4D surfaces is reduced to the presentation of points in 4D space. 4D point *T(x,y,z,w)* is transformed into 3D and 2D point *T(x,y)* and shown on the screen. For the computer algorithm creation matrices for 4D transformations – scaling, translation and rotation are used.

Matrix for scaling

$$\begin{vmatrix} Sx & 0 & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 & 0 \\ 0 & 0 & Sz & 0 & 0 \\ 0 & 0 & 0 & Sw & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix},$$

matrix for translation

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ Tx & Ty & Tz & Tw & 1 \end{vmatrix},$$

matrix for the rotation around the *xy* plane

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}.$$

Matrices for the rotation around other planes, *xz, xw, yz, yw* and *zw,* are obtained with the analogy to the previous.

4D surface is defined by 4D function

$$f(x,y,z,w) = 0,$$

that can be set with 2 variables

$$z(x,y) = 0 \quad \text{and} \quad w(x,y) = 0,$$

after that, the coordinates of 4D points *T(x,y,z,w)* could be determined. 4D points are transformed and connected using a simple algorithm:

```
Begin
s1=x2-x1/bragli;
s2=y2-y1/brrad;
   for(i=0; i<s1; i++){
     for(j=0; j<s2; j++){
        function = f(x,y,z,w);
           line(T(x[i],y[j]), T(x[i+1],y[j+1]));
                    }}
end
```

4D points could be connected to a mesh of horizontal (*bragli*) and vertical (*brrad*) isolines. The user interface of the designed computer program provides setting of the border variables $x_1<x<x_2$ and $y_1<y<y_2$ and a coordinate hyperplane in which the projecting would be performed (Fig. 2).

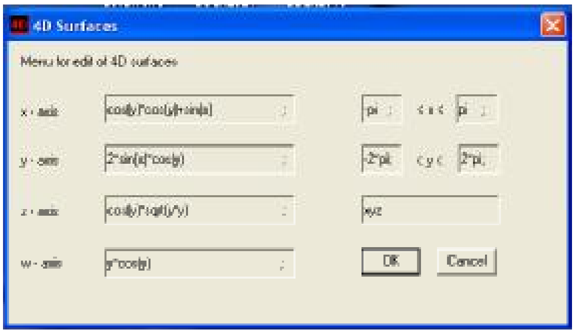

Fig. 2. User interface for setting 4D surfaces

3. THE COMPUTER PROGRAM VERIFICATION

The computer program verification has been performed with setting of simple 2D and 3D structures. The analogy has been performed between 2D structures – parabola, 3D structures – hyperbolical paraboloid and 4D structures – 4D hyperbolical paraboloid.

The basic equation of a parabola is

$$x^2 = 2py\,.$$

If $2p = 1$, then the equation is reduced to $x^2 = y$, from which 2D points are produced.

$$f(x,y,z,w) = (x, x^2, 0, 0);\ \ x \in R$$

Within the limits $-\pi < x < \pi$ and $-\pi < y < \pi$ the geometric shape on the fFig. 3 is produced.

Fig.3. Parabola $f(x,y,z,w) = (x,x^2,0,0)$ for $-\pi < x < \pi$ and $-\pi < y < \pi$

The hyperbolical paraboloid is obtained with sliding of hyperbola along the parabola. This is presented with canonical type of equation as

$$\frac{x^2}{p} - \frac{y^2}{q} = 2z\,.$$

If $p = q = \frac{1}{2}$, then the equation is reduced to $z = x^2 - y^2$ from which 3D points are produced.

$$f(x,y,z,w) = (x, y, x^2 - y^2, 0);\ \ x, y \in R$$

Within the limits of $-\pi < x < \pi$ and $-\pi < y < \pi$ the geometric shape of the Fig. 4 is produced.

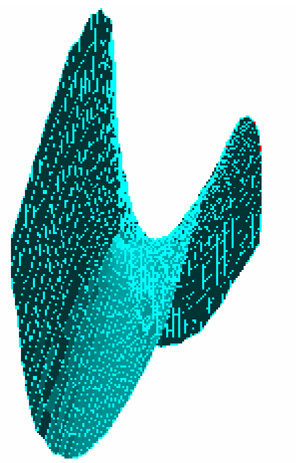

Fig. 4. Hyperbolical parboloid $f(x,y,z,w) = (x,y,x^2-y^2,0)$ for $-\pi < x < \pi$ and $-\pi < y < \pi$

The projection of the hyperbolical paraboloid on the *xz* coordinate plane is parabola, as well as the projection on the *yz* coordinate plane is hyperbola. According to the analogy, projection of the 4D hyperbolical paraboloid on the *xyz* coordinate hyperplane is hyperbolical paraboloid. That means that *x, y, z* coordinates must have value like the hyperbolical paraboloid has and then the *w* coordinate is added. From this statements the next equation (function) is derived

$$f(x,y,z,w) = (x, y, x^2 - y^2, 2xy);\ \ x, y \in R$$

Within the limits of $-\pi < x < \pi$ and $-\pi < y < \pi$ the geometric shape on the Fig. 5 is produced.

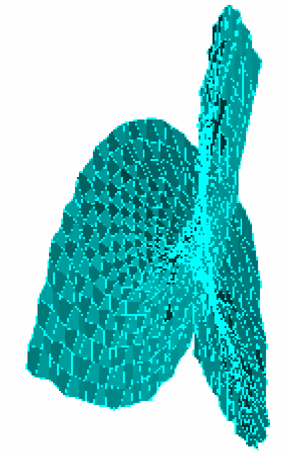

Fig. 5. 4D hyperbolical paraboloid $f(x,y,z,w) = (x,y,x^2 - y^2,2xy)$ for $-\pi < x < \pi$ and $-\pi < y < \pi$

4. GEOMETRIC ANALYSIS OF 4D SURFACES

Geometric analysis is consisted of presentation of the complex 4D surfaces projections on the *xyz* coordinate hyperplane (Fig. 6 – Fig. 9).

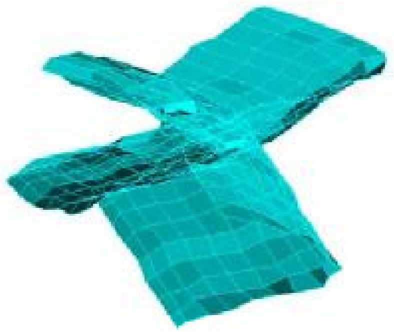

Fig. 6. 4D surface set with function f(x,y,z,w)=($\sqrt{x^2}, x\cos(x), \sqrt{y^2}, y\sin(y)$) for $-\pi < x < \pi$ and $-\pi < y < \pi$

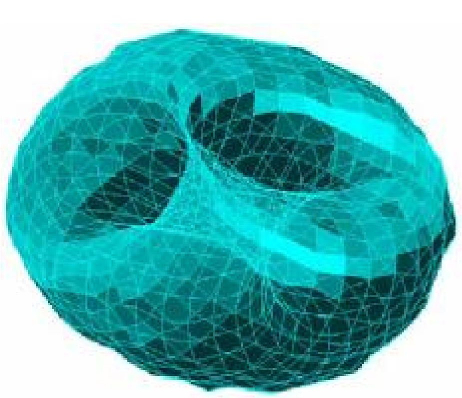

Fig. 7. 4D surface set with the function $f(x,y,z,w) = (\sin(x), \cos(x), \sin(y), \cos(y))$ for $-\pi < x < \pi$ and $-\pi < y < \pi$

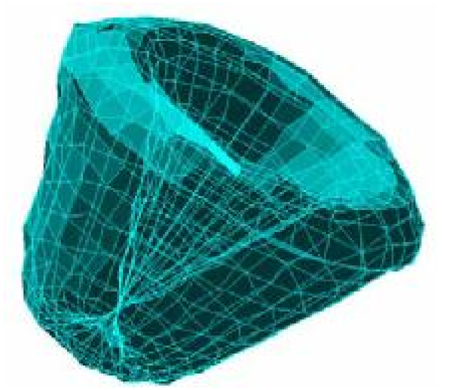

Fig. 8. 4D surface set with the function $f(x,y,z,w) = (x\cos(x), y\sin(x), \sin(y) + \cos(y), \sin(y) + \cos(y))$ for $-\pi < x < \pi$ and $-\pi < y < \pi$

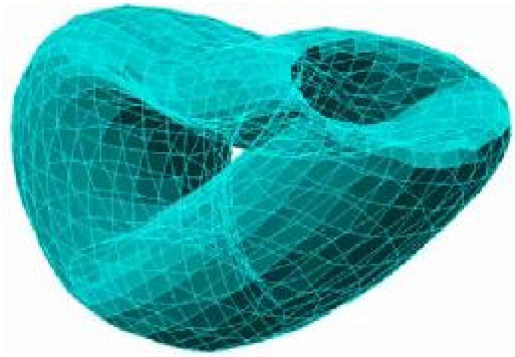

Fig. 9. 4D surface set with the function $f(x,y,z,w) = (\cos(x), \sin(x), \cos(y)\sqrt{y^2}, \sin(y))$ for $-\pi < x < \pi$ and $-\pi < y < \pi$

5. CONCLUSION

The contribution of this paper consists of the way of perception of 4D space. More precisely, 4D surfaces projections are geometrically processed using the function with two variables. The created computer program, based on the presented geometric algorithm, has an ability to analyze the 4D surfaces' projections in an easer and faster way. Verification using analogy has shown that the computer program gives accurate results and can be used for presentation of 4D surfaces.

REFERENCES

[1] T. Banchoff.: *Computer Animation and the Geometry of Surfaces in 3D and 4D Space*, International Congress, Helsinki, 1978, pp.1005–1013.

[2] C. Hinton: *Fourth Dimension*, Kessinger Publishing; Washington, 1997.

[3] H. Manning: *The Fourth Dimension Simply Explained*, Dover Publications, New York, 2005.

[4] B. Филиппов: *Начертательная геометрия многомерного пространства и ЕЕ приложения*, Ленинград, 1979.