COMPARISON OF 4D SURFACES DEFINED BY TWO, THREE AND FOUR VARIABLES

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Abstract

The paper is a continuance of the previous researches of 4D surfaces defined by two and three variables. In this paper a geometric, a mathematical and computer algorithm is presented for the projections of 4D surfaces which are defined with functions with four variables. The presentation of 4D surfaces is a presentation of 4D points in 4D geometric space; points are transformed in 3D and 2D space and are previewed on the display. The determined points are connected in mesh of horizontal and vertical isolines. The computer program is created that can analyze the 4D surfaces projections easily and quickly. The graphical presentations of the projections of 4D surfaces composed of four variables are more complex than of the projections of 4D surfaces composed of two and three variables.

Key words: 4D geometry, 4D space, 4D surfaces

1. INTRODUCTION

1.1. 4D Point

A point is defined with a number of coordinates or parameters depending of the space where it's found. In 1D space it is determined with one coordinate $A(x)$, in 2D space with two coordinates $A(x,y)$, in 3D space with three coordinates $A(x,y,z)$, in 4D space with four coordinates $A(x,y,z,w)$. If the point A of the 4D space $A(x,y,z,w)$ is projected in 3D space, there are 4 projections on the coordinate hyperplane $A(x,y,z)$, $A(x,y,w)$, $A(x,z,w)$ and $A(y,z,w)$ and 6 projections on the coordinate plane $A(x,y)$, $A(x,z)$, $A(y,z)$, $A(x,w)$, $A(y,w)$ and $A(z,w)$ (fig.1).

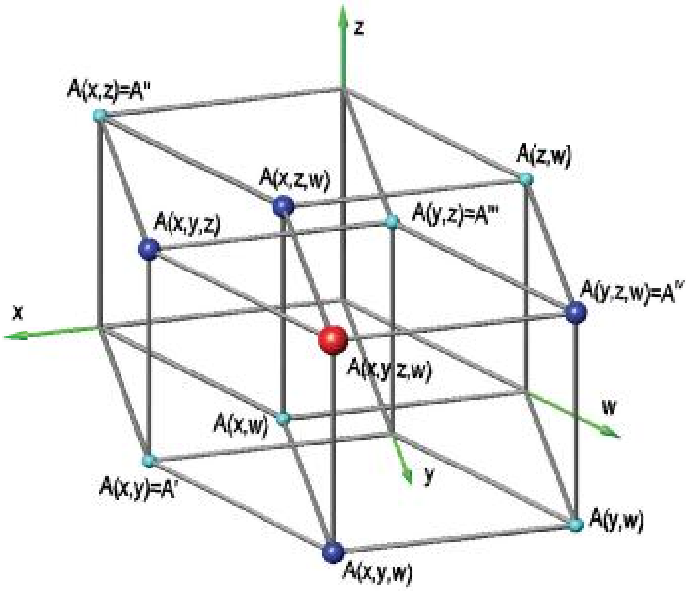

Fig.1. *4D point*

1.2. 4D Surfaces and computer algorithm

4D surfaces are surfaces in which the position of each point is determined by four coordinates. The presentation of 4D surfaces is to present points in 4D space. The 4D point *T(x,y,z,w)* is transformed into 3D and 2D point *T(x,y)* and is shown on the screen. For creation computer algorithms, matrices are used for 4D transformations - scaling, translation and rotation.
The 4D surface is defined as 4D function

f(x,y,z,w)=0,

that can be set with 4 variables

x(x,y,z,w)=0; y(x,y,z,w)=0; z(x,y,z,w)=0 and *w(x,y,z,w)=0*,

then, the coordinates of the 4D points *T(x,y,z,w)* will be determined and will be transformed and connected by using a simple algorithm:

```
Begin
s1=x2-x1/bragli;
s2=y2-y1/brrad;
        for(i=0; i<s1; i++){
                for(j=0; j<s2; j++){
                function = f(x,y,z,w);
                line(T(x[i],y[j]), T(x[i+1],y[j+1]));
                        }}
```

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end

The 4D points are connected to a mesh of horizontal (*bragli*) and vertical (*brrad*) isolines. Through the user's menu of the designed computer program we set the limits of variables $x1<x<x2$, $y1<y<y2$, $z1<z<z2$ and $w1<w<w2$, coordinate hyperplane in which projecting is performed (fig.2.).

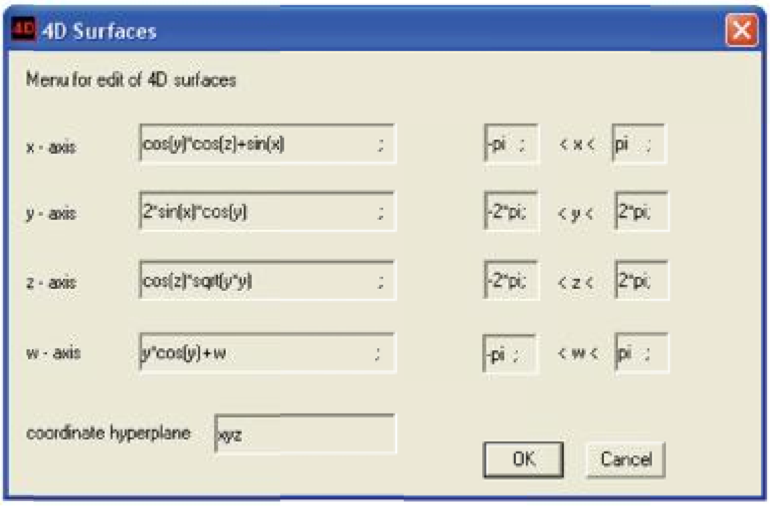

Fig.2. *Menu for setting 4D surfaces*

2. COMPARISON OF 4D SURFACES DEFINED BY TWO, THREE AND FOUR VARIABLES

The comparing of the 4D surfaces composed of two or three variables and 4D surfaces composed of four variables can be done only visually, because we have no knowledge for the layout of the surfaces. Specifically, we'll compare the projections of 4D surfaces composed of two or three variables for which we have knowledge from the previous examinations and 4D surfaces composed of four variables.

We will examine 4D surface composed of 2 variables (x,y):

$$f(x,y,z,w)=(\, y\cos(x), y\sin(x), \sin(x)+\cos(y), \sin(x)+\cos(y))$$

within the limits

for $-\pi < x < \pi$ *and* $-\pi < y < \pi$

projected on the coordinate hyperplane *xyz* that obtaines a surface shown in figure 3.

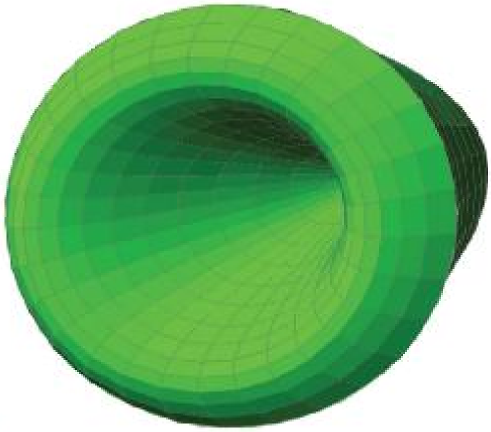

Fig.3. *4D surface set with function f(x,y,z,w)=(* $y\cos(x), y\sin(x), \sin(x)+\cos(y), \sin(x)+\cos(y)$*)*
for $-\pi < x < \pi$ *and* $-\pi < y < \pi$

If you introduce one more variable in the function that defines the 4D surface, there will be more complicated surface.
We will examine 4D surface composed of 3 variables (x,y,z):

f(x,y,z,w)=($y\cos(x), y\sin(x), \sin(z)+\cos(z), \sin(x)+\cos(y)$*)*

within the limits

for $-\pi < x < \pi$ *,* $-\pi < y < \pi$ *and* $-\pi < z < \pi$

projected on the coordinate hyperplane *xyz* that obtaines a surface shown in figure 4.

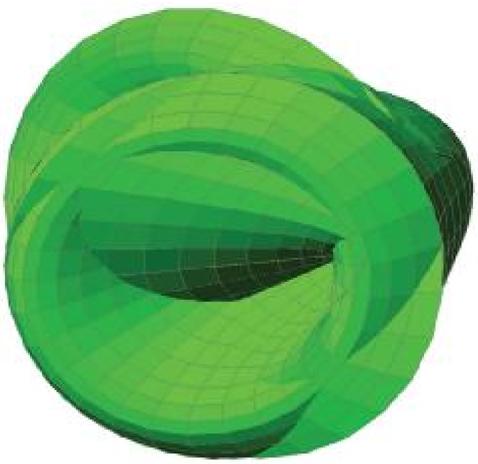

Fig.4. *4D surface set with function*
f(x,y,z,w)=($y\cos(x), y\sin(x), \sin(z)+\cos(z), \sin(x)+\cos(y)$*)*

for $-\pi < x < \pi$, $-\pi < y < \pi$ *and* $-\pi < z < \pi$

If you introduce one more variable in the function that defines the 4D surface, there will be more complicated surface.
We will examine 4D surface composed of 4 variables *(x,y,z,w)*:

$f(x,y,z,w)=(y\cos(x), y\sin(x), \sin(z)+\cos(z), \sin(w)+\cos(w))$

within the limits

for $-\pi < x < \pi$, $-\pi < y < \pi$, $-\pi < z < \pi$ *and* $-\pi < w < \pi$

projected on the coordinate hyperplane *xyz* that obtaines a surface shown in figure 5.

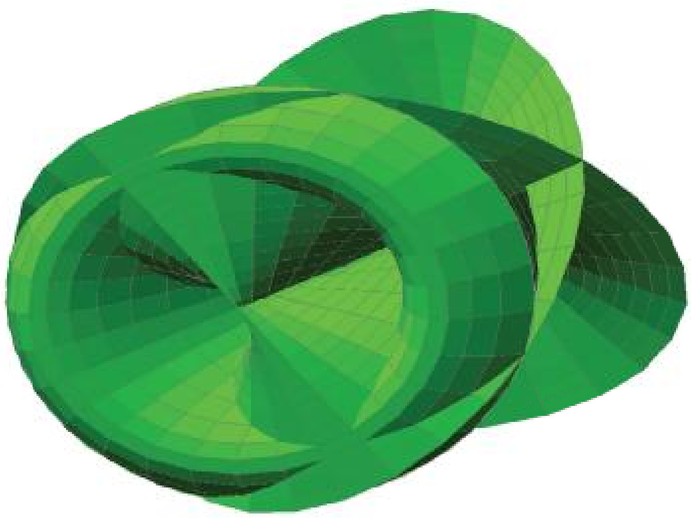

Fig.5. *4D surface set with function*
$f(x,y,z,w)=(y\cos(x), y\sin(x), \sin(z)+\cos(z), \sin(w)+\cos(w))$
for $-\pi < x < \pi$, $-\pi < y < \pi$, $-\pi < z < \pi$ *and* $-\pi < w < \pi$

In the next examples we'll analyze and compare 4D surfaces composed with three and four variables.
4D surface composed with three variables *(x,y,z)*:

$f(x,y,z,w)=(\cos(x), \sin(y), \cos(z)\sin(x), \sin(x))$

within the limits

for $-\pi < x < \pi$, $-\pi < y < \pi$ *and* $-\pi < z < \pi$

projected on the coordinate hyperplane *xyz* that obtaines a surface shown in figure 6.

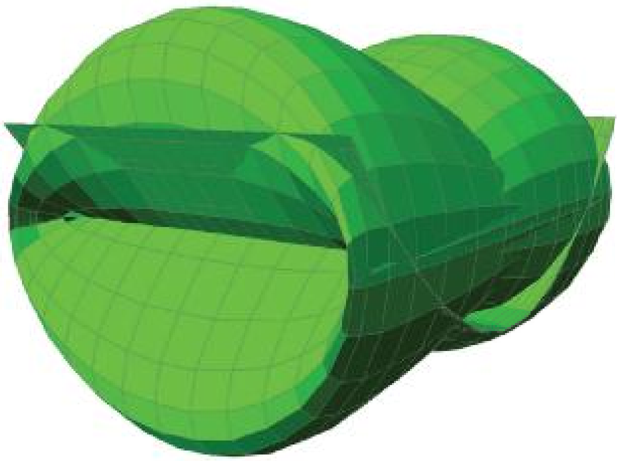

Fig.6. *4D surface set with function*
f(x,y,z,w)=$(\cos(x), \sin(y), \cos(z)\sin(x), \sin(x))$
for $-\pi < x < \pi$, $-\pi < y < \pi$ *and* $-\pi < z < \pi$

In the next example a change is presented by *x* axis, ie instead of *cos(x) cos(w)* is placed.
4D surface composed with 4 variables *(x,y,z,w)*:

f(x,y,z,w)=$(\cos(w), \sin(y), \cos(z)\sin(x), \sin(x))$

within the limits

for $-\pi < x < \pi$, $-\pi < y < \pi$, $-\pi < z < \pi$ *and* $-\pi < w < \pi$

projected on the coordinate hyperplane *xyz* that obtaines a surface shown in figure 7.

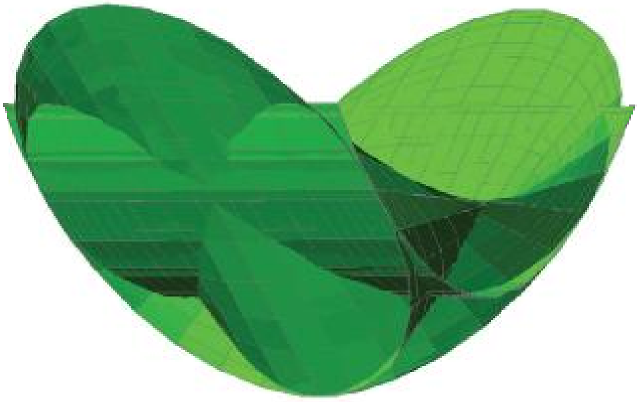

Fig.7. *4D surface set with function*
f(x,y,z,w)=$(\cos(w), \sin(y), \cos(z)\sin(x), \sin(x))$
for $-\pi < x < \pi$, $-\pi < y < \pi$, $-\pi < z < \pi$ *and* $-\pi < w < \pi$

In the next example a change is presented by *w* axis, ie instead of *sin(x) sin(w)* is placed and 4D surface composed with 4 variables *(x,y,z,w)* will be formed:

f(x,y,z,w)=$(\cos(x), \sin(y), \cos(z)\sin(x), \sin(w))$

within the limits

for $-\pi < x < \pi$, $-\pi < y < \pi$, $-\pi < z < \pi$ *and* $-\pi < w < \pi$

projected on the coordinate hyperplane *xyz* that obtaines a surface shown in figure 8.

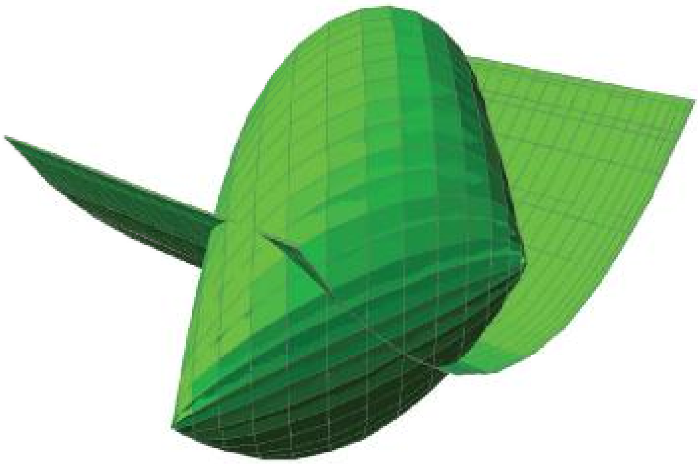

Fig.8. *4D surface set with function*
f(x,y,z,w)=$(\cos(x), \sin(y), \cos(z)\sin(x), \sin(w))$
for $-\pi < x < \pi$, $-\pi < y < \pi$, $-\pi < z < \pi$ *and* $-\pi < w < \pi$

If change is created by *x* axis instead of *cos(x) cos(w)* is placed, change is created by *w* axis instead of *sin(x) sin(w)* is placed and 4D surface composed with 4 variables *(x,y,z,w)* will be formed:

$f(x,y,z,w)=(\cos(w),\sin(y),\cos(z)\sin(x),\sin(w))$

within the limits

for $-\pi < x < \pi$, $-\pi < y < \pi$, $-\pi < z < \pi$ and $-\pi < w < \pi$

projected on the coordinate hyperplane *xyz* that obtaines a surface shown in figure 9.

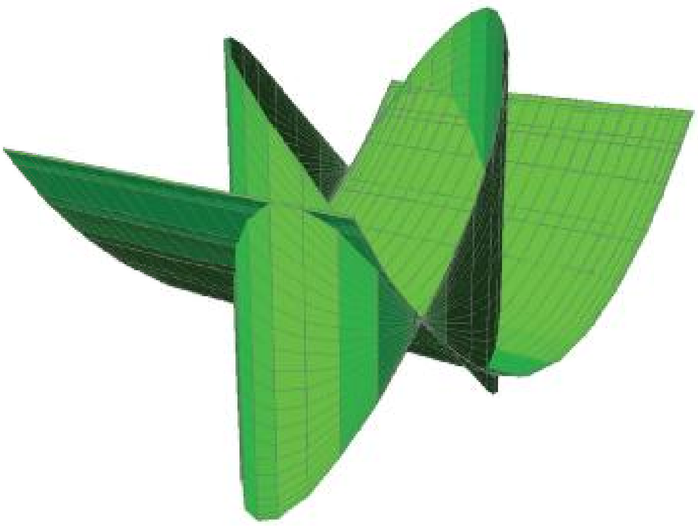

Fig.9. *4D surface set with function*
$f(x,y,z,w)=(\cos(w),\sin(y),\cos(z)\sin(x),\sin(w))$
for $-\pi < x < \pi$, $-\pi < y < \pi$, $-\pi < z < \pi$ and $-\pi < w < \pi$

4D surface composed with three variables *(x,y,z)*:

$f(x,y,z,w)=(\cos(x),\sin(y),\cos(z),\sin(x))$

within the limits

for $-\pi < x < \pi$, $-\pi < y < \pi$ and $-\pi < z < \pi$

projected on the coordinate hyperplane *xyz* that obtaines a surface shown in figure 10.

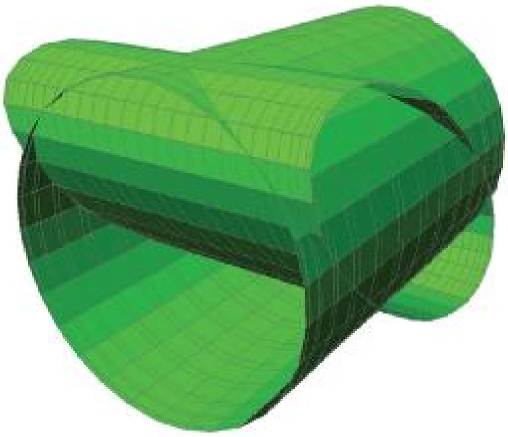

Fig.10. *4D surface set with function*
$f(x,y,z,w)=(\cos(x), \sin(y), \cos(z), \sin(x))$
for $-\pi < x < \pi$, $-\pi < y < \pi$ *and* $-\pi < z < \pi$

If change is created by *w* axis instead of *sin(x) sin(w)* is placed and 4D surface composed with 4 variables *(x,y,z,w)* will be formed:

$f(x,y,z,w)=(\cos(x), \sin(y), \cos(z), \sin(w))$

within the limits

for $-\pi < x < \pi$, $-\pi < y < \pi$, $-\pi < z < \pi$ *and* $-\pi < w < \pi$

projected on the coordinate hyperplane *xyz* that obtaines a surface shown in figure 11.

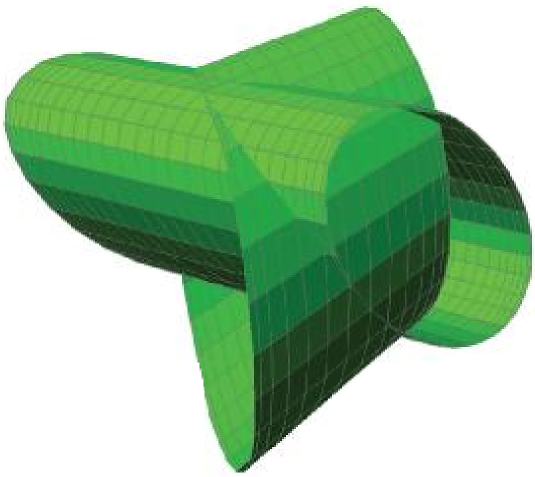

Fig.11. *4D surface set with function*
$f(x,y,z,w)=(\cos(x), \sin(y), \cos(z), \sin(w))$
for $-\pi < x < \pi$, $-\pi < y < \pi$, $-\pi < z < \pi$ *and* $-\pi < w < \pi$

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3. GEOMETRIC ANALYSIS OF THE 4D SURFACES COMPLEXITY

The geometric analysis is consisted of presentation of the projections of complex 4D surfaces composed of 4 variables projected in *xyz* coordinate hyperplane (fig.12. and fig.13.).

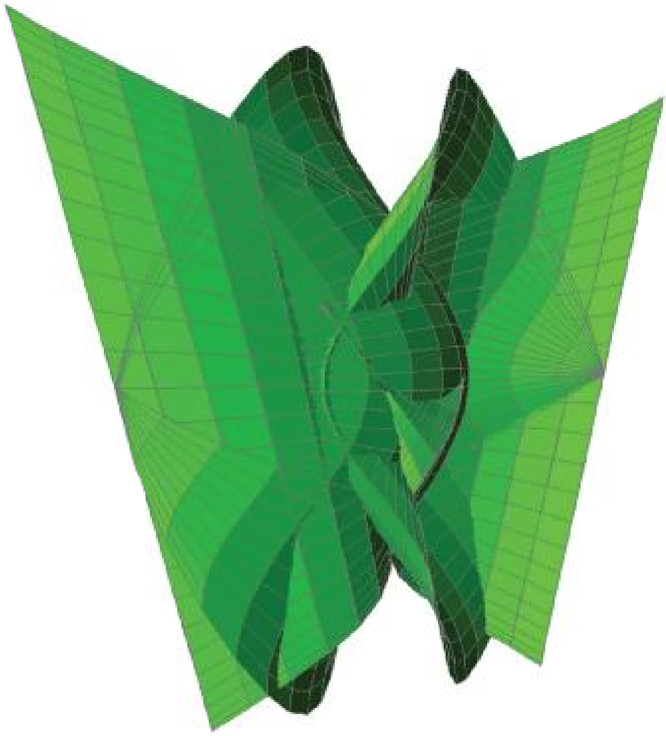

Fig.12. *4D surface set with function*
$f(x,y,z,w)=(\sin(x)\cos(y), \sin(z)\cos(w), y\sin(x), w\cos(z))$
for $-\pi < x < \pi$, $-\pi < y < \pi$, $-\pi < z < \pi$ *and* $-\pi < w < \pi$

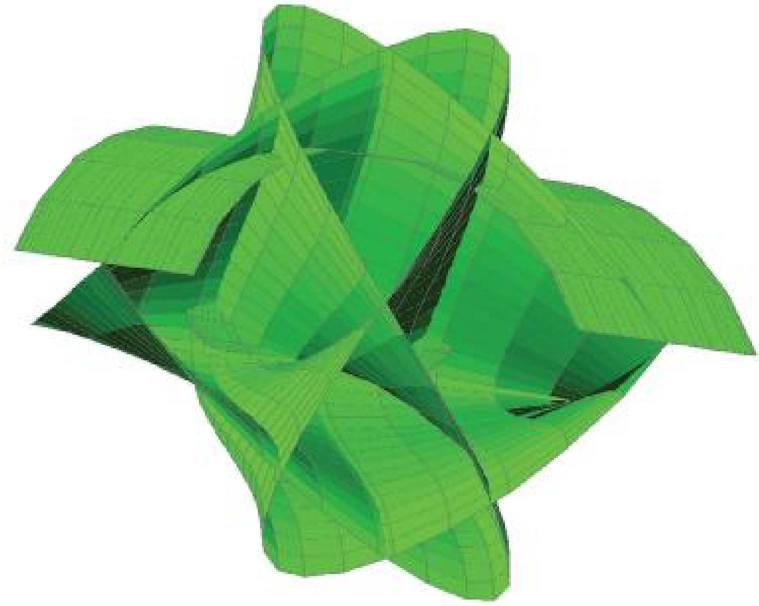

Fig.13. *4D surface set with function*
f(x,y,z,w)=$(\sin(x), \cos(y), y\sin(z), z\cos(w))$
for $-\pi < x < \pi$, $-\pi < y < \pi$, $-\pi < z < \pi$ *and* $-\pi < w < \pi$

4. CONCLUSION

This paper can contribute the understanding of 4D space, more precisely, with the geometric algorithm of 4D surfaces which are set with functions with four variables are processed projections. The computer program is created that can analyze of the 4D surfaces projections easily and quickly. The graphical presentations of the projections of 4D surfaces composed of four variables are more complex than of the projections of 4D surfaces composed of two and three variables.

5. REFERENCES

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